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DISCONTINUITY AND WEAK COMPATIBILITY IN FIXED POINT CONSIDERATION OF GREGUS TYPE IN CONVEX METRIC SPACES

Abstract: In this paper, we prove common fixed point theorems of Gregus type for three discontinuous and weak compatible mappings in convex metric spaces. We improve, extend and generalize some well known results by many authors

Key words: common fixed point, compatible mapping, convex metric space, W-affine mapping.

1. Introduction

The notion of convex metric spaces was initially introduced by Takahashi [31]. He and others gave some fixed point theorems for nonexpansive mappings in convex metric spaces (Ding [5], Fu and Huang [8], Huang [13],Huang and He [15], Li [23], Niampally, Singh and Whitfield [27], Rhoades, Singh and Whitfield [28], Kalinde and Mishra [21]).

Machado [24], Tallman [32] Naimpally and Singh [26] Hadzic [10], [11], [12], Anderson, Singh and Whitfield [1] Beg and Mishra [2], Huang and Cho [14] and many others have studied convex metric spaces and fixed point theorems

Jungck [16] proved a common fixed point theorem for commuting maps generalizing the Banach's fixed point theorem.

Sessa [29] defined a generalization of commutativity, which is called weak commutativity and proved common fixed point theorem for weakly commuting mappings. Further, Jungck [17] introduced more generalized commutativity, so called compatibility, which is more general than that of weak commutativity. Since then various fixed point theorems for compatible mappings satisfying contractive type conditions and assuming continuity of at least one of the mappings in the compatible pair in complete metric spaces, have been obtained by many authors

It has been known from the paper of Kannan [22] that there exists maps that have a discontinuity in the domain but which have fixed points. However, in all the known cases the maps involved were continuous at the fixed point.

In 1998, Jungck and Rhoades [20] introduced the notion of weakly compatible maps and showed that compatible maps are weakly compatible but converse need not be true.

Recently, Singh and Mishra [30], Chugh and Kumar [3] proved some interesting results in metric spaces for weakly compatible maps without assuming any mapping continuous On the other hand Gregus [9] proved a fixed point theorem in Banach spaces, which is called Gregus fixed point theorem and then many authors have obtained some fixed point theorems of Gregus type.

Huang and Cho [14] studied common fixed point theorems in convex metric spaces. The main result of Huang and Cho [14] is as follows:

Theorem 1. *Let I and T be compatible mappings of K into itself satisfying the following condition:*

$$\begin{aligned} d^p(Tx,Ty) \;&\leq\; ad^p(Ix,Iy) + bmax\{d^p(Tx,Ix), d^p(Ty,Iy)\} \\ &\quad + c\max\{d^p(Ix,Iy), d^p(Tx,Ix), d^p(Ty,Iy)\} \end{aligned}$$

for all $x, y \in K$, where $a, b, c > 0$, $p \geq 1$, $a+b+c = 1$ and $\max\{\frac{(1-b)^2}{a}, b+c\} < (2-2^{1-p})(2^p-1)^{-1}$. If I is W-affine and continuous in K and $T(K) \subset I(K)$, then T and I have a unique common fixed point z in K and T is continuous at z.

In this paper, we extend the results of Huang and Cho [14] for three mappings. We improve the results of Huang and Cho [14] by relaxing the compatibility to weak compatibility and by removing the assumption of continuity in convex metric spaces. We also improve and generalize some main results in [4], [6], [7], [8], [9] [18], [19], [23], [25].

2. Preliminaries

In this section, we give some definitions and a lemma for our main results.

Definition 1. *Let (X, d) be a metric space and $J = [0,1]$. A mapping $W : X \times X \times J \to X$ is called a convex structure on X if for each $(x, y, \lambda) \in X \times X \times J$ and $u \in X$,*

$$d(u, W(x,y,\lambda)) \leq \lambda d(u,x) + (1-\lambda)d(u,y).$$

A metric space X together with a convex structure W is called a convex metric space.

Definition 2. *A nonempty subset K of X is said to be convex if, $W(x, y, \lambda) \in K$ for all $(x, y, \lambda) \in K \times K \times J$.*

Obviously, a Banach space or any convex subset of a Banach space is a convex metric space. But there are many examples of convex metric spaces which are not embedded in any Banach spaces. For further information on convex metric spaces, we refer to [31].

Definition 3. *Let (X, d) be a convex metric space and let K be a convex subset of X. A mapping $S : K \to K$ is said to be W-affine if*

$$SW(x, y, \lambda) = W(Sx, Sy, \lambda) forall (x, y, \lambda) \in K \times K \times J.$$

Definition 4 ([17])**.** *Let (X, d) be a metric space and let $S, T : X \to X$ be two mappings. S and T are said to be compatible if, whenever $\{x_n\}$ is a sequence in X such that Sx_n, $Tx_n \to t \in X$, then $d(STx_n, TSx_n) \to 0$.*

Definition 5 ([19])**.** *A pair of mappings S and T is called weakly compatible pair if they commute at coincidence points.*

Example 1. Let $X = [0, 2]$ with the metric d defined by $d(x, y) = |x-y|$, for all $x, y \in X$.

Define $S, T : X \to X$ by $Sx = x$ if $x \in [0, \frac{1}{3})$, $S(x) = \frac{1}{3}$ if $x \geq \frac{1}{3}$ and $Tx = \frac{x}{1+x}$ for all $x \in [0, 2]$.

Consider the sequence $\{x_n = \frac{1}{2} + \frac{1}{n} : n \geq 1\} in X$.

Then $\lim_{n\to\infty} Sx_n = \frac{1}{3}$, $\lim_{n\to\infty} Tx_n = \frac{1}{3}$.

But $\lim_{n\to\infty} d(STx_n, TSx_n) = |\frac{1}{3} - \frac{1}{4}| \neq 0$.

Thus S and T are noncompatible. But S and T are commuting at their coincidence point $x = 0$, that is weakly compatible at $x = 0$.

Hence weakly compatible maps need not be compatible.

Lemma 1. *Let $a > 0$, $c > 0$ and $p \geq 1$. If $a + c < (3 - 3^{1-p})(3^p - 1) - 1$, then $a^{1/p} + c^{1/p} < 1$.*

Proof. Let $f(x) = x^p$ for all $x in (0, \infty)$ and $p \geq 1$. It follows from $p \geq 1$ that

$$f((1/3)(x + y)) \leq (1/3)f(x) + (1/3)f(y)$$

for all $x, y > 0$. Thus we have

$$\left((1/3)\left(a^{1/p} + c^{1/p}\right)\right)^p \leq (1/3)(a + c) < \left(1 - 3^{-p}\right)\left(3^p - 1\right)^{-1},$$

which implies that

$$a^{1/p} + c^{1/p} < (3^p(1 - 3^{-p})(3^p - 1)^{1/p} = 3^p \left\{(\frac{3^p - 1)}{3^p} \frac{1}{(3^p - 1)}\right\}^{1/p} = 1.$$

This completes the proof. ■

3. Main Results

Throughout this section, we assume that X is a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X.

Theorem 2. *Let A, B and S be three mappings of K into itself satisfying the following conditions:*

$$S \quad \text{and} \quad B \quad \text{are} \quad W-\text{affine}, \tag{1}$$

$$\text{the pairs } (S,A) \text{ and } (S,B) \text{ are weakly compatible}, \tag{2}$$

$$A(K) \subset S(K), \qquad B(K) \subset S(K), \tag{3}$$

$$\begin{aligned} d^p(Ax,By) \leq\ & ad^p(Sx,Sy) + b\max\{d^p(Ax,Sx), d^p(By,Sy)\} \\ & + c\max\{d^p(Sx,Sy), d^p(Ax,Sx), d^p(By,Sy)\} \end{aligned} \tag{4}$$

for all x, y in K, where $a,b,c > 0$, $p \geq 1$, $a+b+c = 1$, $(1-b)/2a < (3^p+1)^{-1}$ and $b + c < (3 - 3^{1-p})(3^p - 1)^{-1}$.

Then A, B and S have a unique common fixed point in K.

Proof. Let $x = x_0$ be an arbitrary point in K and choose four points x_1, x_2, x_3 and x_4 in K such that

$$Sx_1 = Ax, \quad Sx_2 = Bx_1, \quad Sx_3 = Ax_2, \quad Sx_4 = Bx_3.$$

In general

$$Sx_{2r+1} = Ax_2r, \quad Sx_{2r+2} = Bx_{2r+1}, \quad \text{for} \quad r = 0, 1.$$

This can be done since $A(K) \subset S(K)$, $B(K) \subset S(K)$. For r = 1 (4) leads to

$$\begin{aligned} d^p(Ax_{2r}, Bx_{2r-1}) \leq\ & ad^p(Sx_{2r}, Sx_{2r-1}) + b\max\{d^p(Ax_{2r}, Sx_{2r}), \\ & d^p(Bx_{2r-1}, Sx_{2r-1})\} + c\max\{d^p(Sx_{2r}, Sx_{2r-1}), \\ & d^p(Ax_{2r}, Sx_{2r}), d^p(Bx_{2r-1}, Sx_{2r-1})\}. \end{aligned}$$

Therefore, we have

$$d^p(Ax_{2r}, Bx_{2r-1}) \leq d^p(Bx_{2r-1}, Sx_{2r-1}). \tag{5}$$

Letting $z = W(x_2, x_4, 1/2)$ then $z \in K$ and since S is W-affine, we have

$$Sz = W(Sx_2, Sx_4, 1/2) = W(Bx_1, Bx_3, 1/2). \tag{6}$$

It follows from (6) and $p \geq 1$ that

$$(7)\qquad \begin{aligned} d^p(Sz, Sx_1) &= d^p(Sx_1, W(Sx_2, Sx_4, \frac{1}{2} \\ &\leq (1/2)d^p(Sx_1, Sx_2) + (1/2)d^p(Sx_1, Sx_4) \\ &\leq (1/2)d^p(Ax, Sx) + (1/2)3^p d^p(Ax, Sx) \\ &= (1/2)(3^p + 1)d^p(Ax, Sx) \end{aligned}$$

and

$$(8)\qquad \begin{aligned} d^p(Sz, Sx_3) &= d^p(Sx_3, W(Sx_2, Sx_4, 1/2)) \\ &\leq ((1/2)d(Sx_3, Sx_2) + (1/2)d(Sx_3, Sx_4))^p \\ &\leq d^p(Ax, Sx). \end{aligned}$$

By (4), (5), (7), (8) and $p \geq 1$, we have

$$(9)\qquad \begin{aligned} & d^p(Az, Sz) = d^p(Az, W(Bx_1, Bx_3, 1/2)) \\ & \leq (1/2)d^p(Az, Bx_1) + (1/2)d^p(Az, Bx_3) \\ & \leq (1/2)(a/4)(3^p + 1)d^p(Ax, Sx) + b \max\{d^p(Az, Sz), d^p(Ax, Sx)\} \\ & \quad + (c/4)(3^p + 1) \\ & \quad + (c/2) \max\{1/2(3^p + 1)d^p(Ax, Sx), d^p(Az, Sz), d^p(Ax, Sx)\} \\ & \quad + (a/2)d^p(Ax, Sx) + (b/2) \max\{d^p(Az, Sz), d^p(Ax, Sx)\} \\ & \quad + (c/2) \max\{d^p(Ax, Sx), d^p(Az, Sz), d^p(Ax, Sx)\} \\ & \leq (a/4)(3^p + 1) + (a/2)d^p(Ax, Sx) + b \max\{d^p(Az, Sz), d^p(Ax, Sx)\} \\ & \quad + (c/4)(3^p + 1) + (c/2) \max\{d^p(Az, Sz), d^p(Ax, Sx)\} \\ & \leq \lambda \max\{d^p(Ax, Sx), d^p(Ax, Sx)\}, \end{aligned}$$

where $\lambda = (a/4)(3^p + 1) + (a/2) + b + (c/4)(3^p + 1) + c$. It is easy to see that $0 < \lambda < 1$ since

$$(1 - b)/2a < (3^p + 1)^{-1}$$

and $\lambda = ((1 - b)/4)(3^p + 1) + 1 - a/2$. Hence (3.9) implies

$$(10)\qquad d^p(Az, Sz) \leq \lambda d^p(Ax, Sx).$$

Since x is an arbitrary point in K from (3.10) it follows that there exists a sequence $\{z_n\}$ in K such that

$$d^p(Az_0, Sz_0) \leq \lambda d^p(Ax_0, Sx_0),$$

$$d^p(Az_1, Sz_1) \leq \lambda d^p(Az_0, Sz_0),$$

$$d^p(Az_n, Sz_n) \leq \lambda d^p(Az_{n-1}, Sz_{n-1})$$

which yield that $d^p(Az_n, Sz_n) \leq \lambda^{n+1} d^p(Ax_0, Sx_0)$, and so we have

$$\lim_{n\to\infty} d(Az_n, Sz_n) = 0\,. \tag{11}$$

Setting $K_{n_1} = \{x \in K : d(Ax, Sx) \leq 1/n_1\}$ for $n_1 = 1, 2, ...$ then(3.11) shows that $K_{n_1} \neq \phi$ for $n_1 = 1, 2, ...$ and $K_1 \supset K_2 \supset K_3 \supset ...$ Obviously, we have $\overline{AK}n_1 \neq \phi$ and $\overline{AK}_{n_1+1} \supset \overline{AK}_{n_1+1}$ for $n_1 = 1, 2, ...$ If we take $u = W(x_1, x_3, 1/2)$ and since B is W-affine we can see that

$$d^p(Au, Bu) \leq \lambda \max\{d^p(Ax, Sx), d^p(Ax, Sx)\},$$

where the value of λ is same as given above. By the same way as shown above we can find

$$\lim_{n\to\infty} d(Au_n, Bu_n) = 0. \tag{12}$$

Setting $K_{n_2} = \{x \in K : d(Ax, Bx) \leq 1/n_2\}$ for $n_2 = 1, 2, ...$ then $\overline{AK}_{n_2} \neq \phi$ and $\overline{AK}_{n_2} \supset \overline{AK}_{n_2+1}$ for $n_2 = 1, 2, ...$

Using (4) and Minkowski's inequality, we have

$$\begin{aligned} d(Ax, By) \leq\ & a^{1/p} d(Sx, Sy) + b^{1/p} \max\{d(Ax, Sx), d(By, Sy)\} \\ & + c^{1/p} \max\{d(Sx, Sy), d(Ax, Sx), d(By, Sy)\} \end{aligned} \tag{13}$$

for all $x, yinK$. For any $x, y \in K_{n_1} \cap K_{n_2}$, by (3.13), we have

$$\begin{aligned} d(Ax, By) &\leq a^{1/p} d(Sx, Sy) + (n_1^{-1} + n_2^{-1}) b^{1/p} \\ &\quad + c^{1/p} \max\{d(Sx, Sy), (n_1^{-1} + n_2^{-1})\} \\ &\leq a^{1/p}(2n_1^{-1} + d(Ax, Ay)) + (n_1^{-1} + n_2^{-1}) b^{1/p} \\ &\quad + c^{1/p} \max\{2n_1^{-1} + d(Ax, Ay), (n_1^{-1} + n_2^{-1})\}. \end{aligned}$$

Let $n = \min(n_1, n_2)$, then

$$\begin{aligned} d(Ax, By) &\leq a^{1/p}(2n^{-1} + d(Ax, Ay)) + 2n^{-1} b^{1/p} \\ &\quad + c^{1/p} \max\{2n^{-1} + d(Ax, Ay)\}. \end{aligned} \tag{14}$$

Since $(a/4)\{(a + c)3^p + b + 3\} + (C/2) < \lambda < 1$, we have

$$(1/4)((a + c)3^p + b + 3^{1-p}) < 1$$

and hence $a + c < (3 - 3^{1-p})(3^p - 1)^{-1}$. It follows from (13) and Lemma 1, that

$$d(Ax, By) \leq 2\,n^{-1}(a^{1/p} + b^{1/p} + c^{1/p})(1 - a^{1/p} - c^{1/p})^{-1}.$$

Therefore, we have

$$\begin{aligned} d(Ax, Ay) &\leq d(Ax, By) + d(By, Ay) \\ &\leq 2n^{-1}(a^{1/p} + b^{1/p} + c^{1/p})(1 - a^{1/p} - c^{1/p})^{-1} + n^{-1} \end{aligned}$$

It follows that

$$\lim_{n\to\infty} \operatorname{diam}(\overline{AK}_{n_1}) = \lim_{n\to\infty} \operatorname{diam}(AK_{n_1}) = 0.$$

Also $\lim_{n\to\infty} \operatorname{diam}(\overline{AK}_{n_2}) = \lim_{n\to\infty} \operatorname{diam}(AK_{n_2}) = 0$. By Cantor's theorem, there exists a point v_1 in K such that

$$\bigcap_{n_1=1}^{\infty} \overline{AK}_{n_1} = \{v_1\}$$

Similarly there exists a point v_2 in K such that

$$\bigcap_{n_2=1}^{\infty} \overline{AK}_{n_2} = \{v_2\}$$

So there exists a point v in K such that

$$\bigcap_{n=1}^{\infty} \overline{AK}_{n} = \{v\}.$$

Since $v \in K$ for each $n = 1, 2, ...,$ there exists a point $y_n \in AK_n$ such that $d(y_n, v) < n^{-1}$. Then there exists a point x_n in K_n such that $d(v, Ax_n) < n^{-1}$ and so $Ax_n \to v$ as $n \to \infty$. Since $x_n \in K_n$ we also have $d(Ax_n, Sx_n) < n^{-1}$ and $d(Ax_n, Bx_n) < n^{-1}$. So $Sx_n \to v$ and $Bx_n \to v$ as $n \to \infty$. Since $A(K) \subset S(K)$, there exists a point $u \in K$ such that $v = Su$. Then using (4), we have

$$\begin{aligned} d(Au, v) &\leq d(Au, Bx_n) + d(Bx_n, v) \\ &\leq a^{1/p} d(Su, Sx_n) + b^{1/p} \max\{d(Au, Su), d(Bx_n, Sx_n)\} \\ &\quad + c^{1/p} \max\{d(Su, Sx_n), d(Au, Su), d(Bx_n, Sx_n)\} + d(Bx_n, v). \end{aligned}$$

Taking the limit as $n \to \infty$ yields

$$d(Au, v) \leq (b^{1/p} + c^{1/p}) d(Au, v).$$

By Lemma 1, we have $Au = v$. Therefore $Au = Su = v$. Since $B(K) \subset S(K)$, there exists a point $w \in X$ such that $v = w$.

$$\begin{aligned} d(v, Bw) &= d(Ax_n, Bw) + d(Ax_n, v) \\ &\leq a^{1/p} d(Sx_n, Sw) + b^{1/p} \max\{d(Ax_n, Sx_n), d(Bw, Sw)\} \\ &\quad + c^{1/p} \max\{d(Sx_n, Sw) d(Ax_n, Sx_n), d(Bw, Sw)\} + d(Ax_n, v) \end{aligned}$$

Taking the limit as $n \to \infty$ yields

$$d(v, Bw) \leq (b^{1/p} + c^{1/p}) d(Bw, v).$$

By Lemma 1, we have $Bw = v$. Therefore $Bw = Sw = v$. Since S and A are weakly compatible maps then $ASu = SAu$ i. e. $Av = Sv$. Let $z = Av = Sv$. Again weak compatibility of A and S imply $ASv = SAv$ i.e. $Az = Sz$.

Similarly S and B are weakly compatible mappings then $BSw = SBw$ i.e. $Bv = Sv$. Therefore $z = Av = Sv = Bv$. Again weak compatibility of S and B imply $BSv = SBv$ i. e. $Bz = Sz$.

Now, we show that z is a fixed point of A. From (4), we have

$$\begin{aligned} d^p(Az, z) &= d^p(Az, Bv) \\ &\leq ad^p(Sz, Sv) + b \max\{d^p(Az, Sz), d^p(Bv, Sv)\} \\ &\quad + c \max\{d^p(Sz, Sv), d^p(Az, Sz), d^p(Bv, Sv)\} \\ &= (a + c) d^p(Az, z). \end{aligned}$$

Thus $Az = z$. Since $a + c < 1$. Therefore $Az = Sz = z$. Again from (4), we have

$$\begin{aligned} d^p(Az, Bz) &= d^p(Av, Bz) \\ &\leq a\, d^p(Sv, Sz) + b \max\{d^p(Av, Sv), d^p(Bz, Sz)\} \\ &\quad + c \max\{d^p(Sv, Sz), d^p(Av, Sv), d^p(Bz, Sz)\} \\ &= (a + c) d^p(z, Bz). \end{aligned}$$

Thus $Bz = z$, since $a + c < 1$. Therefore $Az = Bz = Sz = z$.

Finally, in order to prove the uniqueness of z suppose that z and z_1 are two common fixed points of A, B and S, where z is not equal to z_1. Then by (4), we obtain

$$\begin{aligned} d^p(z, z_1) &= d^p(Az, Bz_1) \\ &\leq a\, d^p(Sz, Sz_1) + b \max\{d^p(Az, Sz), d^p(Bz_1, Sz_1)\} \\ &\quad + c \max\{d^p(Sz, Sz_1), d^p(Az, Sz), d^p(Bz_1, Sz_1)\} \\ &= (a + c) d^p(z, z_1). \end{aligned}$$

Thus $z = z_1$. Since $a + c < 1$. This completes the proof. ∎

From Theorem 2, the following corollaries can be obtained:

Corollary 1. *Let A, B and S be three mappings of K into itself satisfying the conditions* (1) *to* (3) *of Theorem 2 and*

$$\begin{aligned} d(Ax, By) &\leq a\, d(Sx, Sy) + b \max\{d(Ax, Sx), d(By, Sy)\} \\ &\quad + c \max\{d(Sx, Sy), d(Ax, Sx), d(By, Sy)\} \end{aligned} \tag{15}$$

for all x, y in K, where $a, b, c > 0$, $a + b + c = 1$ and $a + c < a^{1/2}$. Then A, B and S have a unique common fixed point in K.

Corollary 2. *Let A, B and S be three mappings of K into itself satisfying the conditions* (1) *to* (3) *of Theorem 2 and*

$$d(Ax, By) \leq a\, d(Sx, Sy) + (1 - a) \max\{d(Ax, Sx), d(By, Sy)\} \tag{16}$$

for all x, y in K, where $0 < a < 1$. Then A and S have a unique common fixed point in K.

Remark. (1) Corollary 2 is an extension of the Gregus fixed point theorem [9] in convex metric spaces.

(2) Theorem 2, Corollary 1 and Corollary 2 are improvement and generalization of some main results in [4], [6]-[9], [14], [18], [19], [23] and [25].

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