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COMMON FIXED POINT THEOREM FOR SIX MAPPINGS IN MENGER SPACE

Abstract: In this paper we prove common fixed point theorem for six mappings in Menger space under the condition of weak compatible mappings of type (α). Our theorem is an extension of some earlier results.

Key words: common fixed point, probabilistic metric space, compatible mappings of type (α), weak compatible mappings of type (α), complete Menger space.

1. Introduction

The concept of probabilistic metric space was first introduced and studied by Menger [15], which is a generalization of metric space and also the study of this space was expanded rapidly with the pioneering works of Schweizer and Sklar [19], [20]. The theory of probabilistic space is of fundamental importance in probabilistic functional analysis. The most interesting references in this direction are [9], [10], [19], [22], [23], [27] and [29]. Recently Bharucha-Reid [1], Bocsan [2], Chang [5], Ćirić [7], Hadzić [10-12], Hicks [13], Singh and Pant [24-26], Stojaković [28], [29], Cho et al. [6], Debeic and Sarapa [8], Radu [18], Cain and Kasriel [3], Chamola [4], Mishra [16], Walt [31], Sehgal [30] and many others have proved common fixed point theorems in probabilistic metric spaces and Menger spaces.

2. Preliminaries

Let R denote the set of reals and R^+ the non-negative reals. A mapping $F : R \to R^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf F = 0$ and $\sup F = 1$. We will denote by L the set of all distribution functions. A probabilistic metric space is a pair (X, F), where X is a non empty set and F is a mapping from $X \times X$ to L. For $(u, v) \in X \times X$, the distribution function $F(u, v)$ is denoted by Fu,v. The functions Fu,v are assumed to satisfy the following conditions:

(P_1) $Fu{,}v(x) = 1$ for every $x > 0$ if and only if $u = v$,
(P_2) $Fu{,}v(0) = 0$ for every $u, v \in X$,
(P_3) $Fu{,}v(x) = Fu{,}v(x)$ for every $u, v \in X$,
(P_4) if $Fu{,}v(x) = 1$ and $Fv{,}w(y) = 1$, then $Fu{,}w(x + y) = 1$ for all $u, v, w \in X$ and $x, y > 0$.

In a metric space (X, d), the metric d induces a mapping $F : X \times X \to L$ such that

$$F(u, v)(x) = Fu{,}v(x) = H(x - d(u, v)),$$

for every $u, v \in X$ and $x \in R$, where H is a distribution function defined by

$$H(x) = \begin{cases} 0\,, & x \leq 0\,, \\ 1\,, & x > 0\,. \end{cases}$$

Definition 2.1. *A Menger space is a triple (X, F, t), where (X, F) is a PM-space and t is T-norm with the following condition:*

$$(P_5) \qquad Fu{,}w(x + y) \geq t(Fu{,}v(x), Fv{,}w(y))$$

for every $u, v, w \in X$ and $x, y \in R^+$.

For topological preliminaries on a Menger space, Schweizer and Sklar [20] is an excellent reference.

Definition 2.2. [18] *Let (X, F, t) is a Menger space with the continuous T-norm t.*

(i) *A sequence $\{p_n\}$ in X is said to be convergent to a point $p \in X$ if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\varepsilon, \lambda)$ such that $p_n = \cup p(\varepsilon, \lambda)$ for all $n \geq N$, or equivalently, F_p, $p_n(\varepsilon) > 1 - \lambda$, for all $n \geq N$. We write $p_n \to p$ as $n \to \infty$ or $\lim_{n\to\infty} p_n = p$.*

(ii) *A sequence $\{p_n\}$ of points in X is said to be a Cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\varepsilon, \lambda) > 0$ such that F_{p_n}, $p_m(\varepsilon) > 1 - \lambda$, for all $n, m \geq N$.*

(iii) *The Menger space (X, F, t) is said to be complete if every Cauchy sequence in X is converges to a point in X.*

3. Weak compatible mappings of type (α)

In this section we give some definitions of compatible mappings and weak compatible mappings of type (α) on Menger space. The concept of compatible mappings and weak compatible mappings of type (α) are equivalent under some conditions in metric space and Menger space ([14] and [6]).

Definition 3.1. *Let (X, F, t) be a Menger space such that the T-norm t is continuous and A, S be mappings from X into itself. A and S are said to be compatible if*

$$\lim_{n\to\infty} FASx_n, SAx_n(x) = 1$$

for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$$

for some $z \in X$.

Definition 3.2. *Let (X, F, t) be a Menger space such that the T-norm t is continuous and A, S be mappings from X into itself. A and S are said to be compatible mappings of type (α) if*

$$\lim_{n\to\infty} FSAx_n, AAx_n(x) = 1 \quad \text{and} \quad \lim_{n\to\infty} FASx_n, SSx_n(x) = 1,$$

for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$$

for some $z \in X$.

Definition 3.3. *Let (X, F, t) be a Menger space such that the T-norm t is continuous and A, S be mappings from X into itself. A and S are said to be weak compatible mappings of type (α) if*

$$\lim_{n\to\infty} FASx_n, SSx_n(x) \geq \lim_{n\to\infty} FSAx_n, AAx_n(x)$$

and

$$\lim_{n\to\infty} FSAx_n, SSx_n(x) \geq \lim_{n\to\infty} FASx_n, AAx_n(x)$$

for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$$

for some $z \in X$.

Proposition 3.1. [14] *Let (X, F, t) be a Menger space such that the T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$, and $A, S : X \to X$ be mappings. If A and S are weak compatible mappings of type (α) and $Az = Sz$ for some $z \in X$, then $AAz = ASz = SAz = SSz$.*

Proposition 3.2. [14] *Let (X, F, t) be a Meneger space such that the T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$, and $A, S : X \to X$*

be mappings. Let A and S be weak compatible mappings of type (α) and $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Then we have

(i) $\lim_{n\to\infty} SAx_n = Az$, if A is continuous,

(ii) $\lim_{n\to\infty} ASx_n = Sz$, if S is continuous,

(ii) $ASz = SAz$ and $Az = Sz$, if A and S are continuous.

We need the following lemmas due to Schweizer and Skalr [20] and Singh and Pant [25], in the proof of the theorems.

Lemma 3.1. *Let $\{x_n\}$ be a sequence in a Menger space (X, F, t), where t is a continuous T-norm and $t(x, x) \geq x$ for all $x \in [0, 1]$. If there exists a constant $k \in (0, 1)$ such that*

$$Fx_n, x_{n+1}(kx) \geq Fx_{n-1}, x_n(x)$$

for all $x > 0$ and $n \in N$, the $\{x_n\}$ is a Chauchy sequence in X.

Remark 3.1. [17] The condition "the T-norm t is continuous and $(x, x) \geq x$ for all $x \in [0, 1]$" can be replaced by "$t(x, y) = \min\{x, y\}$ for all $x, y \in [0, 1]$". In fact, since $t(a, 1) = a$ and $t(1, b) = b$ for all $a, b \in [0, 1]$, we have

$$t(a, b) \leq \min\{t(a, 1), t(1, b)\} = \min\{a, b\}$$

for all $a, b \in [0, 1]$. On the other hand, we have

$$t(a, b) \leq t(\min(a, b), \min\{a, b\}) = \min\{a, b\}$$

for all $a, b \in [0, 1]$, which implies $t(a, b) = \min\{a, b\}$.

4. Main result

Theorem 4.1. *Let (X, F, t) be a complete Menger space with $t(x, y) = \min\{x, y\}$ for all $x, y \in [0, 1]$ and A, B, S, T, P and Q be mappings from X into itself such that*

(4.1) $P(X) \subset ST(X)$ *and* $Q(X) \subset AB(X)$,

(4.2) $PA = AP$, $PB = BP$, $AB = BA$, $ST = TS$ *and* $QS = SQ$,

(4.3) *the pairs* $\{P, AB\}$ *and* $\{Q, ST\}$ *are weak compatible of type* (α),

(4.4) P *is continuous*

(4.5) $[FPu, Qv(kx)]^2 \geq \min\Big\{[FABu, STv(x)]^2, FABu, Pu(x).$

$$FSTv, Qv(x),\ FABu, STv(x). FABu, Pu(x),\ FABu, STv(x).$$
$$FSTv, Qv(x),\ FABu, STv(x). FABu, Qv(2x),\ FABu, STv(x).$$
$$FSTv, Pu(x),\ FABu, Qv(2x). FSTv, Pu(x),\ FABu, Pu(x).$$
$$FSTv, Pu(x),\ FABu, Qv(2x). FSTv, Qv(x)\}$$

for all $u, v \in X$ and $x \geq 0$, where $k \in (0,1)$. Then A, B, S, T, P and Q have a unique common fixed point.

Proof. For any point x_0 in X, there exists a point $x_1 \in X$, such that $Px_0 = STx_1$. For this point x_1, we can choose a point x_2 in X, such that $Qx_1 = ABx_2$ and so on, in this manner we can define a sequence $\{y_n\}$ in X such that $y_{2n} = Px_{2n} = STx_{2n+1}$ and $y_{2n+1} = Qx_{2n+1} = ABx_{2n+2}$, for $n = 0, 1, 2, \ldots$.

Now we shall prove $Fy_{2n}, y_{2n+1}(kx) \geq Fy_{2n-1}, y_{2n}(x)$ for all $x > 0$, where $k \in (0,1)$. Suppose that $Fy_{2n}, y_{2n+1}(kx) < Fy_{2n-1}, y_{2n}(x)$. Then by using (4.5) and $Fy_{2n}, y_{2n+1}(kx) \leq Fy_{2n}, y_{2n+1}(x)$, we have

$$\begin{aligned}
[Fy_{2n}, y_{2n+1}(kx)]^2 &= [FPx_{2n}, Qx_{2n+1}(kx)]^2 \\
&\geq \min\Big\{[Fy_{2n-1}, y_{2n}(x)]^2, Fy_{2n-1}, y_{2n}(x).Fy_{2n}, y_{2n+1}(x), \\
&Fy_{2n-1}, y_{2n}(x).Fy_{2n-1}, y_{2n}(x), Fy_{2n-1}, y_{2n}(x)Fy_{2n}, y_{2n+1}(x), \\
&Fy_{2n-1}, y_{2n}(x).Fy_{2n-1}, y_{2n+1}(2x), Fy_{2n-1}, y_{2n}(x).Fy_{2n}, y_{2n}(x), \\
&Fy_{2n-1}, y_{2n+1}(2x).Fy_{2n}, y_{2n}(x), Fy_{2n-1}, y_{2n}(x).Fy_{2n}, y_{2n}(x), \\
&Fy_{2n-1}, y_{2n+1}(2x).Fy_{2n}, y_{2n+1}(x)\}
\end{aligned}$$

$$\begin{aligned}
\geq \min\Big\{&[Fy_{2n-1}, y_{2n}(x)]^2, Fy_{2n-1}, y_{2n}(x).Fy_{2n}, y_{2n+1}(x), \\
&[Fy_{2n-1}, y_{2n}(x)]^2, Fy_{2n-1}, y_{2n}(x).Fy_{2n}, y_{2n+1}(x), \\
&Fy_{2n-1}, y_{2n}(x).t\left(Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x)\right), Fy_{2n-1}, y_{2n}(x), \\
&t\left(Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x)\right), Fy_{2n-1}, y_{2n}(x), \\
&t\left(Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x)\right), Fy_{2n}, y_{2n+1}(x)\}
\end{aligned}$$

$$\begin{aligned}
\geq \min\Big\{&[Fy_{2n}, y_{2n+1}(kx)]^2, [Fy_{2n}, y_{2n+1}(kx)]^2, [Fy_{2n}, y_{2n+1}(kx)]^2, \\
&[Fy_{2n}, y_{2n+1}(kx)]^2, [Fy_{2n}, y_{2n+1}(kx)]^2, Fy_{2n}, y_{2n+1}(kx), \\
&Fy_{2n}, y_{2n+1}(kx), Fy_{2n}, y_{2n+1}(kx), [Fy_{2n}, y_{2n+1}(kx)]^2 \\
= [Fy_{2n}, y_{2n+1}(kx)]^2\Big\}&
\end{aligned}$$

which is a contradiction. Thus we have

$$Fy_{2n}, y_{2n+1}(kx) \geq Fy_{2n-1}, y_{2n}(x).$$

Similarly we can have $Fy_{2n+1}, y_{2n+2}(kx) \geq Fy_{2n}, y_{2n+1}(x)$. Therefore, for every $n \in N$,

$$Fy_n, y_{n+1}(kx) \geq Fy_{n-1}, y_n(x).$$

Therefore by Lemma 3.1, $\{y_n\}$ is a Cauchy sequence in X. Since the Menger space (X, F, t) is complete, $\{y_n\}$ converges to a point z in X, and the subsequences $\{Px_{2n}\}$, $\{Qx_{2n+1}\}$, $\{ABx_{2n}\}$ and $\{STx_{2n+1}\}$ of $\{y_{2n}\}$ also converges to z.

Now suppose that P is continuous, since P and AB are weak compatible of type (α), it follows from

$$(AB)Px_{2n} \to Pz \quad \text{and} \quad PPx_{2n} \to Pz \text{ as } n \to \infty.$$

Now putting $u = Px_{2n}$ and $v = x_{2n+1}$ in the equation (4.5), we have

$$\begin{aligned}
[FPPx_{2n}, Qx_{2n+1}(kx)]^2 \geq \min\big\{&[F(AB)Px_{2n}, STx_{2n+1}(x)]^2,\\
&F(AB)Px_{2n}, PPx_{2n}(x).FSTx_{2n+1}, Qx_{2n+1}(x),\\
&F(AB)Px_{2n}, STx_{2n+1}(x).\, F(AB)Px_{2n}, PPx_{2n}(x),\\
&F(AB)Px_{2n}, STx_{2n+1}(x).\, FSTPx_{2n+1}, Qx_{2n+1}(x),\\
&F(AB)Px_{2n}, STx_{2n+1}(x).\, F(AB)Px_{2n}, Qx_{2n+1}(2x),\\
&F(AB)Px_{2n}, STx_{2n+1}(x).\, FSTx_{2n+1}, PPx_{2n}(x),\\
&F(AB)Px_{2n}, Qx_{2n+1}(2x).\, FSTx_{2n+1}, PPx_{2n}(x),\\
&F(AB)Px_{2n}, PPx_{2n}(x).\, FSTx_{2n+1}, PPx_{2n}(x),\\
&F(AB)Px_{2n}, Qx_{2n+1}(2x).\, FSTx_{2n+1}, Qx_{2n+1}(x)\big\}.
\end{aligned}$$

Taking the limit $n \to \infty$, we have

$$\begin{aligned}
[FPz,z(kx)]^2 \quad \geq \quad \min\big\{&[FPz,z(x)]^2,\ FPz,Pz(x)]^2.\ Fz,z(x),\\
&FPz,z(x).\ FPz,Pz(x),\ FPz,z(x).\ Fz,z(x),\\
&FPz,z(x).\ FPz,z(2x),\ FPz,z(x).\ Fz,Pz(x),\\
&FPz,z(2x).\ Fz,Pz(x),\ FPz,Pz(x).\ Fz,Pz(x),\\
&FPz,z(2x).\ Fz,z(x)\big\} = [FPz,z(x)]^2
\end{aligned}$$

which is a contradiction. Thus we have $Pz = z$. Since $P(X) \subset ST(X)$, there exists a point $u \in X$ such that $z = Pz = STp$. Again putting $u = Px_{2n}$

and $v = p$ in (4.5), we have

$$\begin{aligned}[FPPx_{2n}, Qp(kx)]^2 \ \geq \ \min\big\{&[F(AB)Px_{2n}, STp(x)]^2,\\ &F(AB)Px_{2n}, PPx_{2n}(x).\\ &FSTp, Qp(x),\ F(AB)Px_{2n}, STp(x).\ F(AB)Px_{2n}, PPx_{2n}(x),\\ &F(AB)Px_{2n}, STp(x).\ FSTp, Qp(x),\ F(AB)Px_{2n}, STp(x).\\ &F(AB)Px_{2n}, Qp(2x),\ F(AB)Px_{2n}, STp(x).\ FSTp, PPx_{2n}(x),\\ &F(AB)Px_{2n}, Qp(2x).\ FSTp, PPx_{2n}(x),\ F(AB)Px_{2n}, PPx_{2n}(x).\\ &FSTp, PPx_{2n},\ F(AB)Px_{2n}, Qp(2x).\ FSTp, Qp(x)\big\}.\end{aligned}$$

Taking the limit $n \to \infty$, we have

$$[Fz, Qp(kx)]^2 \geq [Fz, Qp(x)]^2$$

which is a contradiction, therefore $z = Qp$. Since Q and ST are weak compatible of type (α) and $STp = Qp = z$, by Proposition 3.1, $(ST)Qp = Q(ST)p$ and hence $STz = (ST)Qp = Q(ST)p = Qz$. Again by putting $u = x_{2n}$ and $v = z$ in (4.5), we have

$$\begin{aligned}[FPx_{2n}, Qz(kx)]^2 \ \geq \ \min\big\{&[FABx_{2n}, STz(x)]^2,\ FABx_{2n}, Px_{2n}(x).\\ &FSTz, Qz(x),\ FABx_{2n}, STz(x).\ FABx_{2n}, Px_{2n}(x),\\ &FABx_{2n}, STz(x).\ FSTz, Qz(x),\ FABx_{2n}, STz(x).\\ &FABx_{2n}, Qz(2x),\ FABx_{2n}, STz(x).\ FSTz, Px_{2n}(x),\\ &FABx_{2n}, Qz(2x).\ FSTz, Px_{2n}(x),\ FABx_{2n}, Px_{2n}(x).\\ &FSTz, Px_{2n},\ FABx_{2n}, Qz(2x).\ FSTz, Qz(x)\big\}.\end{aligned}$$

Letting $n \to \infty$, we have

$$[Fz, Qz(kx)]^2 \geq [Fz, Qz(x)]^2$$

which is a contradiction, therefore we have $Qz = z$. Thus $Qz = STz = z$. Similarly since P and AB are weak compatible of type (α) and by Proposition 3.1, we have $ABz = Pz = z$. Now we prove $Az = z$. Suppose that $Az \neq z$ then by putting $u = Az$ and $v = z$ in (4.5), we have

$$\begin{aligned}[FPAz, Qz(kx)]^2 \ \geq \ \min\big\{&[F(AB)Az, STz(x)]^2,\ F(AB)Az, PAz(x).\\ &FSTz, Qz(x),\ F(AB)Az, STz(x).\ F(AB)Az, PAz(x),\\ &F(AB)Az, STz(x).\ FSTz, Qz(x),\ F(AB)Az, STz(x).\\ &F(AB)Az, Qz(2x),\ F(AB)Az, STz(x).\ FSTz, PAz(x),\\ &F(AB)Az, Qz(2x).\ FSTz, PAz(x),\ F(AB)Az, PAz(x).\\ &FSTz, PAz(x),\ F(AB)Az, Qz(2x).\ FSTz, Qz(x)\big\}\end{aligned}$$

which yields

$$[FAz,z(kx)]^2 \geq [FAz,z(x)]^2$$

which is a cotradication, there fore we have $Az = z$. Similarly if we put $u = Bz$ and $y = z$ in (4.5), we have

$$\begin{aligned}[FPBz, Qz(kx)]^2 \ \geq \ \min\{&[F(AB)Bz, STz(x)]^2,\ F(AB)Bz, PBz(x).\\ &FSTz, Qz(x),\ F(AB)Bz, STz(x).\ F(AB)Bz, PBz(x),\\ &F(AB)Bz, STz(x).\ FSTz, Qz(x),\ F(AB)Bz, STz(x).\\ &F(AB)Bz, Qz(2x),\ F(AB)Bz, STz(x).\ FSTz, PBz(x),\\ &F(AB)Bz, Qz(2x).\ FSTz, PBz(x),\ F(AB)Bz, PBz(x).\\ &FSTz, PBz(x),\ F(AB)Bz, Qz(2x).\ FSTz, Qz(x)\}\end{aligned}$$

which gives

$$[FBz,z(kx)]^2 \geq [FBz,z(x)]^2$$

which is a cotradication, therefore we have $Bz = z$. So $Az = Bz = z$. Finally we show that $Sz = z$. By using (4.5), we have

$$\begin{aligned}[Fz, QSz(kx)]^2 \ \geq \ \min\{&[Fz, (ST)Sz(x)]^2,\ Fz,z(x).\\ &F(ST)Sz, QSz(x),\ Fz, (ST)Sz(x).\ Fz,z(x),\ Fz, (ST)Sz(x).\\ &F(ST)Sz, QSz(x),\ Fz, (ST)Sz(x).\ Fz, QSz(2x),\\ &Fz, (ST)Sz(x).\ F(ST)Sz,z(x),\ Fz, QSz(2x).\\ &F(ST)Sz,z(x),\ Fz,z(x).\ F(ST)Sz,z(x),\\ &Fz, QSz(2x).\ F(ST)Sz, QSz(x)\}\end{aligned}$$

which gives

$$[Fz, Sz(kx)]^2 \geq [Fz, Sz(x)]^2$$

which is a contradiction, therefore we have $Sz = z$. So $Sz = Tz = z$. Thus combining the results, we have $Pz = Qz = Az = Bz = Sz = Tz = z$. Thus z is a common fixed point of A, B, S, T, P and Q.

For uniqueness let w ($z \neq w$) be another common fixed point of A, B, S, T, P and Q, then by (4.5), we have

$$\begin{aligned}[Fz,w(kx)]^2 \ = \ [FPz, Qw(kx)]^2 \geq \min\{&[Fz,w(x)]^2,\ Fz,z(x).\ Fw,w(x),\\ &Fz,w(x).\ Fz,z(x),\ Fz,w(x).\ Fw,w(x),\\ &Fz,w(x).\ Fz,w(2x),\ Fz,w(x).\ Fw,z(x),\\ &Fz,w(2x).\ Fw,z(x),\ Fz,z(x).\ Fw,z(x),\\ &Fz,w(2x).\ Fw,w(x)\} = [Fz,w(x)]^2\end{aligned}$$

which is a contradiction, therefore $z = w$. Hence z is a unique common fixed point of A, B, S, T, P and Q. ■

If we put $B = T = I$ (I is identity mapping on X) in Theorem 4.1., we obtain the following result due to Pathak et al. [17].

Corollary 4.1. *Let (X, F, t) be a complete Menger space with $t(x, y) = \min\{x, y\}$ for all $x, y \in [0, 1]$ and P, Q, A and S be mappings from X into itself such that*

(4.6) $P(X) \subset S(X)$ *and* $Q(X) \subset A(X)$,

(4.7) *the pairs* $\{P, A\}$ *and* $\{Q, S\}$ *are weak compatible of type* (α)

(4.8) P *is continuous,*

(4.9)
$$[FPu, Qv(kx)]^2 \geq \min\{FAu, Sv(x)]^2, FAu, Pu(x).\ FSv, Qv(x),$$
$$FAu, Sv(x).\ FAu, Pu(x),\ FAu, Sv(x).$$
$$FSv, Qv(x),\ FAu, Sv(x).\ FAu, Qv(2x),\ FAu, Sv(x).$$
$$FSv, Pu(x),\ FAu, Qv(2x).\ FSv, Pu(x),\ FAu, Pu(x).$$
$$FSv, Pu(x),\ FAu, Qv(2x).\ FSv, Qv(x)\}$$

for all $u, v \in X$ and $x \geq 0$, where $k \in (0, 1)$. Then P, Q, A and S have a unique common fixed point.

If we put $A = B = S = T = I$ (I is identity mapping on X) in Theorem 4.1, we have the following:

Corollary 4.2. *Let (X, F, t) be a complete Menger space with $t(x, y) = \min\{x, y\}$ for all $x, y \in [0, 1]$ and P and Q be mappings from X into itself such that*

(4.10) $P(X) \subset Q(X)$,

(4.11) P *is continuous,*

(4.12)
$$[FPu, Qv(kx)]^2 \geq \min\{Fu, v(x)]^2, Fu, Pu(x).\ Fv, Qv(x),$$
$$Fu, v(x).\ Fu, Pu(x),\ Fu, v(x).\ Fv, Qv(x)$$
$$Fu, v(x).\ Fu, Qv(2x),\ Fu, v(x).\ Fv, Pu(x),$$
$$Fu, Qv(2x).\ Fv, Pu(x),\ Fu, Pu(x).\ Fv, Pu(x),$$
$$Fu, Qv(2x),\ Fv, Qv(x)\}$$

for all $u, v \in X$ and $x \geq 0$, where $k \in (0, 1)$. Then P and Q have a unique common fixed point.

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