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SLIGHTLY ω-CONTINUOUS FUNCTIONS

Abstract. A new weak form of both slightly continuous and ω-continuous, called slightly ω-continuous, is introduced and studied. Furthermore, basic properties and preservation theorems of slightly ω-continuous functions are investigated. Relationships between slightly ω-continuous functions and set ω-connected functions are investigated.

Key words: ω-open sets, slightly continuous, slightly ω-continuous, set ω-connected functions.

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1. Introduction

Throughout this paper (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. Let (X, τ) be a space and A be a subset of X. The closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. A point $x \in X$ is called a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is said to be ω-closed [7] if it contains all its condensation points. The complement of an ω-closed set is said to be ω-open. It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. The family of all ω-open subsets of a space (X, τ), denoted by τ_ω, forms a topology on X finer than τ. We set $\omega O(X, x)=\{U : x \in U \text{ and } U \in \tau_\omega\}$. The ω-closure and ω-interior, that can be defined in the same way to $Cl(A)$ and $Int(A)$, respectively, will be denoted by $Cl_\omega(A)$ and $Int_\omega(A)$, respectively. Several characterizations and properties of ω-closed subsets were provided in [1, 3, 7, 8].

Jain [9] introduced the notion of slightly continuous functions. He defined a function $f : X \to Y$ to be slightly continuous if, for every $x \in X$ and every clopen subset V of Y containing $f(x)$ there exists an open subset U of X with $x \in U$ and $f(U) \subseteq V$. Slight semi-continuity [13], slight β-continuity [11], slight precontinuity [5] and slight γ-continuity [6] are analogously defined as weak forms of slight continuity. In this paper, the notion of slightly ω-continuous functions is introduced and basic properties of slightly ω-continuous functions are investigated and obtained.

2. Slightly ω-continuous functions

In this section, the notion of slightly ω-continuous functions is introduced. If A is both ω-open and ω-closed, then it is said to be ω-clopen. The family of all ω-clopen (resp. clopen) sets of X is denoted by $\omega CO(X)$ (resp. $CO(X)$). The family of all ω-clopen (resp. clopen) sets of X containing $x \in X$ is denoted by $\omega CO(X, x)$ (resp. $CO(X, x)$). A set $U \subseteq X$ is said to be δ^*-open [14] if for each $x \in U$ there exists $V \in CO(X)$ such that $x \in V \subseteq U$. A set $F \subseteq X$ is called δ^*-closed if and only if $X - F$ is δ^*-open.

Definition 1. *A function $f : X \to Y$ is said to be slightly ω-continuous at a point $x \in X$ if for each clopen subset V in Y containing $f(x)$, there exists an ω-open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly ω-continuous if it has this property at each point of X.*

Theorem 1. *Let (X, τ) and (Y, σ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:*
(1) *f is slightly ω-continuous;*
(2) *for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is ω-open;*
(3) *for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is ω-closed;*
(4) *for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is ω-clopen;*
(5) *for every δ^*-open set $V \subseteq Y$, $f^{-1}(V)$ is ω-open;*
(6) *for every δ^*-closed set $V \subseteq Y$, $f^{-1}(V)$ is ω-closed.*

Proof. (1) $\Rightarrow$ (2): Let V be a clopen subset of Y and let $x \in f^{-1}(V)$. Since f is slightly ω-continuous, by (1) there exists an ω-open set U_x in X containing x such that $f(U_x) \subseteq V$; hence $U_x \subseteq f^{-1}(V)$. We obtain that $f^{-1}(V) = \cup\{U_x | x \in f^{-1}(V)\}$. Thus , $f^{-1}(V)$ is ω-open.

(2) $\Rightarrow$ (3): Let V be a clopen subset of Y. Then $Y \backslash V$ is clopen. By (2) $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is ω-open. Thus $f^{-1}(V)$ is ω-closed.

(3) $\Rightarrow$ (4): It can be shown easily.

(4) $\Rightarrow$ (5): Let H be a δ^*-open set in Y and let $x \in f^{-1}(H)$. Then $f(x) \in H$. Since H is δ^*-open, there exists a $V \in CO(Y)$ such that $f(x) \in V \subseteq H$. This implies that $x \in f^{-1}(V) \subseteq f^{-1}(H)$. By (4) $f^{-1}(V)$ is ω-clopen. Hence $f^{-1}(H)$ is an ω-neighbourhood of each of its points. Consequently, $f^{-1}(H) \in \omega O(X)$.

(5) $\Leftrightarrow$ (6): It is obvious from the fact that the complement of a δ^*-closed set is δ^*-open.

(5) $\Rightarrow$ (1): This easily follows from the fact that every clopen set is δ^*-open. ∎

Corollary 1. *Let (X,τ) and (Y,σ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:*
(1) *f is slightly ω-continuous;*
(2) *for every $x \in X$ and each clopen set $V \subseteq Y$ containing $f(x)$, there exists $U \in \omega O(X,x)$ such that $f(U) \subseteq V$.*

Theorem 2. *If $f : X \to Y$ is slightly ω-continuous and $A \in \omega O(X)$, then the restriction $f|_A : A \to Y$ is slightly ω-continuous.*

Proof. Let V be a clopen subset of Y. We have $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ and A are ω-open, therefore $(f|_A)^{-1}(V)$ is ω-open in the relative topology of A. Thus $f|_A$ is slightly ω-continuous. ■

Definition 2. *A function $f : X \to Y$ is said to be*
(1) *ω-irresolute [4] if for every ω-open subset U of Y, $f^{-1}(U)$ is ω-open in X,*
(2) *ω-open [7] if for every ω-open subset A of X, $f(A)$ is ω-open in Y,*
(3) *weakly ω-continuous [7] if for each point $x \in X$ and each open subset V in Y containing $f(x)$, there exists an ω-open subset U in X containing x such that $f(U) \subseteq Cl(V)$,*
(4) *contra ω-continuous if $f^{-1}(F)$ is ω-open for each closed set F of Y.*

Theorem 3. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*
(1) *If f is ω-irresolute and g is slightly ω-continuous, then $g \circ f$ is slightly ω-continuous.*
(2) *If f is slightly ω-continuous and g is slightly continuous, then $g \circ f$ is slightly ω-continuous.*

Proof. (1) Let V be any clopen set in Z. By the slight ω-continuity of g, $g^{-1}(V)$ is ω-open. Since f is ω-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is ω-open. Therefore, $g \circ f$ is slightly ω-continuous.

(2) Let V be any clopen set in Z. By the slight-continuity of g, $g^{-1}(V)$ is clopen . Since f is slightly ω-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is ω-open. Therefore, $g \circ f$ is slightly ω-continuous. ■

Corollary 2. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*
(1) *If f is ω-irresolute and g is ω-continuous, then $g \circ f$ is slightly ω-continuous;*
(2) *If f is ω-irresolute and g is slightly continuous, then $g \circ f$ is slightly ω-continuous;*
(3) *If f is ω-continuous and g is slightly continuous, then $g \circ f$ is slightly ω-continuous;*

(4) *If f is slightly ω-continuous and g is continuous, then $g \circ f$ is slightly ω-continuous.*

Corollary 3. *Let $f : X \to Y$ be an ω-irresolute, ω-open surjection and $g : Y \to Z$ be a function. Then g is slightly ω-continuous if and only if $g \circ f$ is slightly ω-continuous.*

Proof. If g be slightly ω-continuous. Then by Theorem 3, $g \circ f$ is slightly ω-continuous. Conversely, let $g \circ f$ be slightly ω-continuous and V be clopen set in Z. Then $(g \circ f)^{-1}(V)$ is ω-open. Since f is an ω-open surjection, then $f((g \circ f)^{-1}(V)) = g^{-1}(V)$ is ω-open in Y. This shows that g is slightly ω-continuous. ■

Recall that a space X is called ω-connected [2] provided that X is not the union of two disjoint nonempty ω-open sets.

Theorem 4. *If $f : X \to Y$ is a slightly ω-continuous surjection and X is ω-connected, then Y is connected.*

Proof. Suppose that Y is a disconnected space. Then there exist nonempty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since f is slightly ω-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are ω-clopen in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. Since f is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty. Therefore, X is not ω-connected. This is a contradiction and hence Y is connected. ■

Corollary 4. *The inverse image of a disconnected space under a surjection slightly ω-continuous is ω-disconnected.*

It is easy to show that weak ω-continuity, contra ω-continuity and slight continuity imply slight ω-continuity. None of these implications is reversible.

$$\begin{array}{ccccc} & & \omega\text{-continuous} & \Rightarrow & \text{weakly } \omega\text{-continuous} \\ & & & & \Downarrow \\ \text{contra } \omega\text{-continuous} & \Rightarrow & \text{slightly } \omega\text{-continuous} & \Leftarrow & \text{slightly continuous.} \end{array}$$

Example 1. Let $X = \mathbb{R}$ with the usual topology τ. And let $Y = \{a, b, c\}$ with the topology $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Then the function $f : (X, \tau) \to (Y, \sigma)$ defined by

$$f(x) = \begin{cases} a, & \text{if} \quad x \in \mathbb{Q}; \\ b, & \text{if} \quad x \notin \mathbb{Q}. \end{cases}$$

is slightly ω-continuous but not weakly ω-continuous.

Example 2. Let $X = \mathbb{R}$ with the usual topology τ and $Y = \{a, b, c, d\}$ with the topology $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the function $f : (X, \tau) \to (Y, \sigma)$ defined by

$$f(x) = \begin{cases} c, & \text{if} \quad x \in \mathbb{Q}; \\ a, & \text{if} \quad x \notin \mathbb{Q}. \end{cases}$$

is slightly ω-continuous but not contra ω-continuous since $f^{-1}(\{b, c, d\})$ is not an ω-open set.

Example 3. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{X, \phi, \{c\}\}$. Let $f : (X, \sigma) \to (X, \tau)$ be the identity function. Then f is slightly ω-continuous. But it is not slightly continuous since $f^{-1}(\{a\}) = \{a\}$ is not open set in X.

Corollary 5. *If $f : X \to Y$ is a weakly ω-continuous, contra ω-continuous, or slightly continuous surjection and X is ω-connected, then Y is connected.*

Recall that a space X is said to be (1) extremally disconnected if the closure of every open set of X is open, (2) locally indiscrete if every open set of X is closed in X, (3) 0-dimensional if its topology has a base consisting of clopen sets.

Theorem 5. *If $f : X \to Y$ is slightly ω-continuous and Y is extremally disconnected, then f is weakly ω-continuous.*

Proof. Let $x \in X$ and let V be an open subset of Y containing $f(x)$. Then $Cl(V)$ is open and hence clopen. Therefore there exists an ω-open set $U \subseteq X$ with $x \in U$ and $f(U) \subseteq Cl(V)$. Thus f is weakly ω-continuous. ■

Theorem 6. *If $f : X \to Y$ is slightly ω-continuous and Y is locally indiscrete, then f is ω-continuous and contra ω-continuous.*

Proof. Let V be any open set of Y. Since Y is locally indiscrete, V is clopen and hence $f^{-1}(V)$ is ω-open and ω-closed in X. Therefore, f is ω-continuous and contra ω-continuous. ■

Theorem 7. *If $f : X \to Y$ is slightly ω-continuous and Y is 0-dimensional, then f is ω-continuous.*

Proof. Let $x \in X$ and $V \subseteq Y$ be any open set containing $f(x)$. Since Y is 0-dimensional, there exists a clopen set U containing $f(x)$ such that $U \subseteq V$. But f is slightly ω-continuous and there exists $G \in \omega O(X, x)$ such that $f(x) \in f(G) \subseteq U \subseteq V$. Hence f is ω-continuous. ■

Recall that a space X is anti locally countable [4] if every non empty open subset is uncountable.

Lemma 1 ([4]). *If (X, τ) is an anti locally countable space, then $Int_\omega(A) = Int(A)$ for every ω-closed subset A of X and $Cl_\omega(A) = Cl(A)$ for every ω-open subset A of X.*

Corollary 6. *Let X be anti locally countable, then $U \in \omega CO(X)$ if and only if $U \in CO(X)$.*

Proof. This follows immediately from Lemma 1. ■

Theorem 8. *If $f : X \to Y$ is slightly ω-continuous and X is anti locally countable, then f is slightly-continuous.*

Proof. Let $x \in X$ and $V \in CO(Y, f(x))$. Since f is slightly ω-continuous, by Corollary 1 there exists $U \in \omega CO(X, x)$ such that $f(U) \subseteq V$. Since X is anti locally countable, then by Corollary 6 $U \in CO(X)$ and hence f is slightly-continuous. ■

3. Separation axioms related to ω-open sets

Theorem 9. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. Then g is slightly ω-continuous if and only if f is slightly ω-continuous.*

Proof. Let $V \in CO(Y)$, then $X \times V$ is clopen in $X \times Y$. Since g is slightly ω-continuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \omega O(X)$. Thus, f is slightly ω-continuous. Conversely, let $x \in X$ and F be a clopen subset of $X \times Y$ containing $g(x)$. Then $F \cap (\{x\} \times Y)$ is clopen in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y | (x, y) \in F\}$ is a clopen subset of Y. Since f is slightly ω-continuous, $\cup\{f^{-1}(y) | (x, y) \in F\}$ is an ω-open set of X. Further $x \in \cup\{f^{-1}(y) | (x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is ω-open. Then g is slightly ω-continuous. ■

A subset A of a topological space X is said to be ω^*-closed if for each $x \in X - A$ there exists a ω-clopen set U containing x such that $U \cap A = \phi$. A topological space X is said to be ultra Hausdorff [15] if every two distinct points of X can be separated by disjoint clopen sets.

Theorem 10. *If $f : X \to Y$ is slightly ω-continuous and Y is ultra Hausdorff and the product of two ω-open sets is ω-open, then*

(1) *The graph $G(f)$ of f is ω^*-closed in the product space $X \times Y$,*
(2) *The set $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is ω^*-closed in the product space $X \times X$.*

Proof. (1) Let $(x, y) \in (X \times Y) - G(f)$. Then $f(x) \neq y$. Since Y is ultra Hausdorff, there exist clopen sets V and W such that $y \in V$ and $f(x) \in W$

and $V \cap W = \phi$. Since f is slightly ω-continuous , there exists an ω-clopen set U containing x such that $f(U) \subseteq W$. Therefore , we obtain $V \cap f(U) = \phi$ and hence $(U \times V) \cap G(f) = \phi$ and $U \times V$ is an ω-clopen set of $X \times Y$. This shows that $G(f)$ is ω^*-closed in $X \times Y$.

(2) Set $A = \{(x_1, x_2)|f(x_1) = f(x_2)\}$. Let $(x_1, x_2) \notin A$ then $f(x_1) \neq f(x_2)$. Since Y is ultra Hausdorff, there exist $V_1, V_2 \in CO(Y)$ containing $f(x_1), f(x_2)$ respectively, such that $V_1 \cap V_2 = \phi$. Since f slightly ω-continuous, there exist ω-clopen sets U_1, U_2 of X such that $x_1 \in U_1, f(U_1) \subseteq V_1$ and $x_2 \in U_2, f(U_2) \subseteq V_2$; hence $f(U_1) \cap f(U_1) = \phi$. Thus $(x_1, x_2) \in U_1 \times U_2$ and $(U_1 \times U_2) \cap A = \phi$. Moreover $U_1 \times U_2$ is ω-clopen in $X \times X$ and A is ω^*-closed in the product space $X \times X$. ■

Definition 3. *[4] A space X is said to be*

(1) *ω-T_0 (resp. ω-T_1) if for each $x, y \in X$ such that $x \neq y$, there exists an ω-open set containing x but not y, or (resp. and) an ω-open set containing y but not x;*

(2) *ω-T_2 if for each $x, y \in X$ such that $x \neq y$, there exist disjoint ω-open sets U and V such that $x \in U$ and $y \in V$;*

(3) *ω-regular if for each closed set F of X and each point $x \notin F$, there exist disjoint ω-open sets U and V such that $F \subseteq U$ and $x \in V$;*

(4) *ω-normal if for every pair of disjoint closed sets F_1 and F_2 of X there exist disjoint ω-open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.*

Theorem 11. *Let Y be a 0-dimensional space and $f : X \to Y$ be a slightly ω-continuous injection. Then the following properties hold:*

(1) *If Y is T_1 (resp. T_2), then X is ω-T_1 (resp. ω-T_2).*

(2) *If f is either open or closed, then X is ω-regular.*

(3) *If f is closed and Y is normal, then X is ω-normal.*

Proof. (1) We prove only the second statement, the proof for the first being analogous. Let Y be T_2. Since f is injective, for any pair of distinct points $x, y \in X$, $f(x) \neq f(y)$. Since Y is T_2, there exist open sets V_1, V_2 in Y such that $f(x) \in V_1$, $f(y) \in V_2$ and $V_1 \cap V_2 = \phi$. Since Y is a 0-dimensional space, there exist $U_1, U_2 \in CO(Y)$ such that $f(x) \in U_1 \subseteq V_1$ and $f(y) \in U_2 \subseteq V_2$. Consequently $x \in f^{-1}(U_1) \subseteq f^{-1}(V_1)$, $y \in f^{-1}(U_2) \subseteq f^{-1}(V_2)$, and $f^{-1}(U_1) \cap f^{-1}(U_2) = \phi$. Since f is slightly ω-continuous, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are ω-open sets and this implies that X is ω-T_2.

(2) First suppose f is open . Let $x \in X$ and U be an open set containing x. Then, $f(x) \in f(U)$ which is open in Y because of the openness of f. On the other hand, 0-dimensionality of Y gives the existence of a $V \in CO(Y)$ such that $f(x) \in V \subseteq f(U)$. So, $x \in f^{-1}(V) \subseteq U$ (since f is injective). Again f is slightly ω-continuous and $f^{-1}(V)$ is an ω-clopen set in X by Theorem 1 and hence $x \in f^{-1}(V) = Cl_\omega(f^{-1}(V) \subseteq U$. This implies that X is ω-regular.

Now suppose f is closed. Let $x \in X$ and F be a closed set of X such that $x \notin F$. Then, $f(x) \notin f(F)$ and $f(x) \in Y - f(F)$ which is an open set in Y since f is closed. But Y is 0-dimensional and there exists a clopen set V in Y such that $f(x) \in V \subseteq Y - f(F)$. Since f is slightly ω-continuous, we have $x \in f^{-1}(V) \in \omega CO(X)$ and $F \subseteq X - f^{-1}(V) \in \omega CO(X)$. Therefore X is ω-regular.

(3) Let F_1 and F_2 be any two closed sets in X such that $F_1 \cap F_2 = \phi$. Since f is closed and injective, we have $f(F_1)$ and $f(F_2)$ are two closed sets in Y with $f(F_1) \cap f(F_2) = \phi$. By normality of Y, there exist two open sets U and V in Y such that $f(F_1) \subseteq U$, $f(F_2) \subseteq V$ and $U \cap V = \phi$. Let $y \in f(F_1)$, then $y \in U$. Since Y is 0-dimensional and U is open in Y, there exists a clopen set U_y such that $y \in U_y \subseteq U$. Then $f(F_1) \subseteq \cup\{U_y | U_y \in CO(Y), y \in f(F_1)\} \subseteq U$, and thus $F_1 \subseteq \cup\{f^{-1}(U_y) | U_y \in CO(Y), y \in f(F_1)\} \subseteq f^{-1}(U)$. Since f is slightly ω-continuous, $f^{-1}(U_y)$ is ω-open for each $U_y \in CO(Y)$ so that $G = \cup\{f^{-1}(U_y) | y \in f(F_1)\}$ is ω-open in X and $F_1 \subseteq G \subseteq f^{-1}(U)$. Similarly, there exists an ω-open set H in X such that $F_2 \subseteq H \subseteq f^{-1}(V)$ and $G \cap H \subseteq f^{-1}(U \cap V) = \phi$. This shows that X is ω-normal. ■

Next we use the concept of an ω-open set to define an analogue of the notion of a set connected function.

Definition 4. *A space X is said to be ω-connected between subsets A and B provided there is no ω-clopen set F for which $A \subseteq F$ and $F \cap B = \phi$.*

Definition 5. *A function $f : X \to Y$ is said to be set ω-connected if whenever X is ω-connected between A and B, then $f(X)$ is connected between $f(A)$ and $f(B)$ with respect to the relative topology on $f(X)$.*

The next result is analogous to the characterization of set connected functions obtained by Kwak [10].

Theorem 12. *A function $f : X \to Y$ is set ω-connected if and only if $f^{-1}(F)$ is ω-clopen for every clopen subset F of $f(X)$ (with respect to the relative topology on $f(X)$).*

Proof. *Necessity.* Assume that F is a clopen subset of $f(X)$ with respect to the relative topology on $f(X)$. Suppose that $f^{-1}(F)$ is not ω-closed in X. Then there exists $x \in X - f^{-1}(F)$ such that for every ω-open set U with $x \in U$ and $U \cap f^{-1}(F) \neq \phi$. We claim that the space X is set ω-connected between x and $f^{-1}(F)$. Suppose there exists an ω-clopen set A such that $f^{-1}(F) \subseteq A$ and $x \notin A$. Then $x \in X - A \subseteq X - f^{-1}(F)$ and evidently $X - A$ is an ω–open set containing x and disjoint from $f^{-1}(F)$ this contradiction implies that X is set ω-connected between x and $f^{-1}(F)$. Since f is set ω-connected, $f(X)$ is connected between $f(x)$ and $f(f^{-1}(F))$. But $f(f^{-1}(F)) \subseteq F$ which is clopen in $f(X)$ and $f(x) \notin F$, which is a contradiction. Therefore $f^{-1}(F)$ is ω-closed in X and an argument using complements will show that $f^{-1}(F)$ is also ω-open.

Sufficiency. Suppose X is ω-connected between A and B and also $f(X)$ is not connected between $f(A)$ and $f(B)$ (in the relative topology on $f(X)$). Thus there is a set $F \subseteq f(X)$ that is clopen in the relative topology on $f(X)$ such that $f(A) \subseteq F$ and $F \cap f(B) = \phi$. Then $A \subseteq f^{-1}(F)$, $B \cap f^{-1}(F) = \phi$ and $f^{-1}(F)$ is ω-clopen, which implies that X is not ω-connected between A and B. It follows that f is set ω-connected. ■

Corollary 7. *Every slightly ω-continuous surjection is set ω-connected.*

Theorem 13. *Every set ω-connected function is slightly ω-continuous*

Proof. Assume $f : X \to Y$ is set ω-connected. Let F be a clopen subset of Y. Then $F \cap f(X)$ is clopen in the relative topology on $f(X)$. Since f is set ω-connected, by Theorem 12, $f^{-1}(F) = f^{-1}(F \cap f(X))$ is ω-clopen in X. ■

Corollary 8. *A surjective function is slightly ω-continuous if and only if it is set ω-connected.*

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