F A S C I C U L I   M A T H E M A T I C I

Nr 42 2009

Alfred O. Bosede

NOOR ITERATIONS ASSOCIATED WITH ZAMFIRESCU MAPPINGS IN UNIFORMLY CONVEX BANACH SPACES

Abstract. In this paper, we establish some fixed point theorems for Noor iterations associated with Zamfirescu mappings in uniformly convex Banach spaces and deduce similar other results on Mann and Ishikawa iterations as special cases.
Our results improve a multitude of recent results in the fixed point theory especially the result of Ciric [5].

Key words: Zamfirescu mappings, uniformly convex Banach spaces, fixed point, Noor, Mann and Ishikawa iterations.

AMS Mathematics Subject Classification: 47H06, 47H10.

1. Introduction

Let K be a closed and convex subset of a Banach space E and let T be a selfmap of K. The set $F_T = \{x \in K : Tx = x\}$ is called the fixed point set of T in K.

A Banach space $(E, \|.\|)$ is called *uniformly convex* if, given $\epsilon > 0$, there exists $\delta_1 > 0$ such that for all $x, y \in E$ with $\|x\| \leq 1$, $\|y\| \leq 1$ and $\|x - y\| \geq \epsilon$, we have $\left\|\frac{1}{2}(x + y)\right\| < 1 - \delta_1$.

In Berinde [3], Zamfirescu proved the following result:

Theorem 1. *Let (E, d) be a complete metric space and $T : E \longrightarrow E$ be a mapping for which there exist real numbers α, β and γ saisfying $0 \leq \alpha < 1$, $0 \leq \beta$, $\gamma < 0.5$ such that, for each $x, y \in E$, at least one of the following is true:*

(Z_1) $d(Tx, Ty) \leq \alpha d(x, y)$;

(Z_2) $d(Tx, Ty) \leq \beta[d(x, Tx) + d(y, Ty)]$;

(Z_3) $d(Tx, Ty) \leq \gamma[d(x, Ty) + d(y, Tx)]$.

Then, T is a Picard mapping.

Remark 1. The proof of this Theorem is contained in Berinde [3].

Indeed, if

$$\delta = max\left\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\right\}, \tag{1}$$

in Theorem 1, we obtain

$$0 \leq \delta < 1. \tag{2}$$

Then, for all $x, y \in E$, and by using Z_2, it was proved in Berinde [3] that

$$d(Tx, Ty) \leq 2\delta d(x, Tx) + \delta d(x, y), \tag{3}$$

and by using Z_3, we obtain

$$d(Tx, Ty) \leq 2\delta d(x, Ty) + \delta d(x, y), \tag{4}$$

where $0 \leq \delta < 1$ is as defined by (1).

Remark 2. If $(E, \|.\|)$ is a normed linear space, then (3) becomes

$$\|Tx - Ty\| \leq 2\delta \|x - Tx\| + \delta \|x - y\|, \tag{5}$$

for all $x, y \in E$ and where $0 \leq \delta < 1$ is as defined by (1).

2. Preliminaries

Let K be a closed and convex subset of a Banach space E and $T : K \longrightarrow K$ a mapping which satisfies the condition

$$d(x, Tx) + d(y, Ty) \leq qd(x, y) \tag{6}$$

for all $x, y \in K$, where $2 \leq q < 4$.

Let x_0 in K be arbitrary and let a sequence $\{x_n\}_{n=0}^{\infty}$ be defined by

$$x_{n+1} = \frac{1}{2}(x_n + Tx_n), \qquad n = 0, 1, 2, ... \tag{7}$$

For arbitrary x_0 in K, we define Mann iteration $\{x_n\}_{n=0}^{\infty}$ by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n, \qquad n = 0, 1, 2, ... \tag{8}$$

with $\{\alpha_n\}_{n=o}^{\infty}$ being a sequence of real numbers in [0,1] satisfying conditions (i) $0 \leq \alpha_n \leq 1$ and (ii) $\sum_{n=0}^{\infty} \alpha_n(1-\alpha_n) = \infty$.

For any $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iteration defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n Ty_n \\ y_n &= (1-\beta_n)x_n + \beta_n Tx_n \end{aligned} \tag{9}$$

with $\{\alpha_n\}_{n=o}^{\infty}$, $\{\beta_n\}_{n=o}^{\infty}$ being sequences of real numbers in [0,1] satisfying conditions (i) $\sum_{n=0}^{\infty} \alpha_n = \infty$; (ii) $\sum_{n=0}^{\infty} \beta_n = \infty$ and (iii) $0 \leq \alpha_n,\ \beta_n \leq 1$.

For arbitrary $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the Noor iteration defined by

$$
\begin{aligned}
x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\
y_n &= (1-\beta_n)x_n + \beta_n T z_n \\
z_n &= (1-\gamma_n)x_n + \gamma_n T x_n
\end{aligned}
\tag{10}
$$

with $\{\alpha_n\}_{n=o}^{\infty}$, $\{\beta_n\}_{n=o}^{\infty}$, $\{\gamma_n\}_{n=o}^{\infty}$ being sequences of real numbers in [0,1] satisfying conditions (i) $\sum_{n=0}^{\infty} \alpha_n = \sum_{n=0}^{\infty} \beta_n = \sum_{n=0}^{\infty} \gamma_n = \infty$ and (ii) $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$.

By using the contractive definition (6) and iteration (7), Ciric [5] established the following result:

Theorem 2. *Let K be a closed and convex subset of a Banach space E with the norm $\|x\| = d(x, 0)$, $x \in K$ and $T : K \longrightarrow K$ a mapping which satisfies the contractive definition (6).*

Let x_0 in K be arbitrary and let a sequence $\{x_n\}_{n=0}^{\infty}$ be defined iteratively by (7).

Then, T has at least one fixed point.

Remark 3. The proof of Theorem 2.1 is contained in Ciric [5].

Our aim in this paper is to establish some fixed point theorems for Noor iterations associated with Zamfirescu mappings in uniformly convex Banach spaces by using the contractive definition (5). We shall also deduce similar other results on Mann and Ishikawa iterations as special cases.

We shall use the Noor iteration (10) instead of iteration (7) used by Ciric [5].

Remark 4. We observe that the contractive definition (5) is well-defined.

Also, iteration (10) used in our result is more general than iteration (7) used by Ciric [5] and many others in the following sense:

If $\gamma_n = 0, \ \forall\ n \in N$ in the Noor iteration (10), then we obtain the Ishikawa iteration (7).

Again, if $\beta_n = 0, \ \forall\ n \in N$ in the Ishikawa iteration (9), we obtain the Mann iteration (8).

Also, if $\alpha_n = \frac{1}{2}, \ \forall\ n \in N$ in the Mann iteration (8), we obtain iteration (7), which is the iteration used by Ciric [5] in Theorem 2.1 above.

The following result is a fixed point theorem for the Noor iterations (10) associated with Zamfirescu mappings in uniformly convex Banach spaces.

3. The main results

Theorem 3. *Let K be a closed and convex subset of a uniformly convex Banach space E and $T : K \longrightarrow K$ a mapping which satisfies the contractive definition (5).*

Let x_0 in K be arbitrary and let a sequence $\{x_n\}_{n=0}^{\infty}$ be defined iteratively by (10). Then, iteration (10) converges strongly to the fixed point of T.

Proof. Theorem 1 shows that T has a unique fixed point in K. Let us denote it by p.

For arbitrary x_0 in K and by using iteration (10), we get

$$\begin{aligned} x_{n+1} - p &= (1-\alpha_n)x_n + \alpha_n T y_n - p \\ &= (1-\alpha_n)x_n + \alpha_n T y_n - \alpha_n p - (1-\alpha_n)p \\ &= (1-\alpha_n)(x_n - p) + \alpha_n (T y_n - p) \end{aligned}$$

and hence,

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1-\alpha_n)(x_n - p) + \alpha_n (T y_n - p)\| \\ &\le (1-\alpha_n)\|x_n - p\| + \alpha_n \|T y_n - p\| \\ &= (1-\alpha_n)\|x_n - p\| + \alpha_n \|T y_n - Tp\| \\ &= (1-\alpha_n)\|x_n - p\| + \alpha_n \|Tp - T y_n\|. \end{aligned}$$

By using the contractive definition (5), we obtain

$$\begin{aligned} \|x_{n+1} - p\| &\le (1-\alpha_n)\|x_n - p\| + \alpha_n (2\delta \|p - Tp\| + \delta \|p - y_n\|) \\ &= (1-\alpha_n)\|x_n - p\| + \alpha_n (2\delta \|p - p\| + \delta \|y_n - p\|) \\ &= (1-\alpha_n)\|x_n - p\| + \alpha_n (2\delta(0) + \delta \|y_n - p\|) \\ &= (1-\alpha_n)\|x_n - p\| + \alpha_n (0 + \delta \|y_n - p\|), \quad 0 \le \delta < 1 \end{aligned}$$

which implies that,

$$\|x_{n+1} - p\| \le (1-\alpha_n)\|x_n - p\| + \alpha_n \delta \|y_n - p\|. \tag{11}$$

But, $y_n = (1-\beta_n)x_n + \beta_n T z_n$, therefore,

$$\begin{aligned} \|y_n - p\| &\le \|(1-\beta_n)x_n + \beta_n T z_n - p\| \\ &= \|(1-\beta_n)(x_n - p) + \beta_n (T z_n - p)\| \\ &\le (1-\beta_n)\|x_n - p\| + \beta_n \|T z_n - p\| \\ &= (1-\beta_n)\|x_n - p\| + \beta_n \|T z_n - Tp\| \\ &= (1-\beta_n)\|x_n - p\| + \beta_n \|Tp - T z_n\|. \end{aligned}$$

Again, by using the contractive definition (5), we obtain

$$\begin{aligned}\|y_n - p\| &\le (1-\beta_n)\|x_n - p\| + \beta_n(2\delta\|p - Tp\| + \delta\|p - z_n\|) \\ &= (1-\beta_n)\|x_n - p\| + \beta_n(2\delta\|p - p\| + \delta\|z_n - p\|) \\ &= (1-\beta_n)\|x_n - p\| + \beta_n(2\delta(0) + \delta\|z_n - p\|) \\ &= (1-\beta_n)\|x_n - p\| + \beta_n(0 + \delta\|z_n - p\|), \quad 0 \le \delta < 1\end{aligned}$$

and hence,

$$\|y_n - p\| \le (1-\beta_n)\|x_n - p\| + \beta_n\delta\|z_n - p\|. \tag{12}$$

But, $z_n = (1-\gamma_n)x_n + \gamma_n Tx_n$, hence,

$$\begin{aligned}\|z_n - p\| &\le \|(1-\gamma_n)x_n + \gamma_n Tx_n - p\| \\ &= \|(1-\gamma_n)(x_n - p) + \gamma_n(Tx_n - p)\| \\ &\le (1-\gamma_n)\|x_n - p\| + \gamma_n\|Tx_n - p\| \\ &= (1-\gamma_n)\|x_n - p\| + \gamma_n\|Tx_n - Tp\| \\ &= (1-\gamma_n)\|x_n - p\| + \gamma_n\|Tp - Tx_n\|.\end{aligned}$$

By using the contractive definition (5), we get

$$\begin{aligned}\|z_n - p\| &\le (1-\gamma_n)\|x_n - p\| + \gamma_n(2\delta\|p - Tp\| + \delta\|p - x_n\|) \\ &= (1-\gamma_n)\|x_n - p\| + \gamma_n(2\delta\|p - p\| + \delta\|x_n - p\|) \\ &= (1-\gamma_n)\|x_n - p\| + \gamma_n(2\delta(0) + \delta\|x_n - p\|) \\ &= (1-\gamma_n)\|x_n - p\| + \gamma_n(0 + \delta\|x_n - p\|) \\ &= (1-\gamma_n)\|x_n - p\| + \gamma_n\delta\|x_n - p\|, \quad 0 \le \delta < 1\end{aligned}$$

which implies that,

$$\|z_n - p\| \le (1 - \gamma_n + \gamma_n\delta)\|x_n - p\|.$$

By observing that $0 \le \gamma_n \le 1$, $0 \le \delta < 1$ and since $0 \le (1 - \gamma_n + \gamma_n\delta) < 1$, we get

$$\|z_n - p\| \le \|x_n - p\|. \tag{13}$$

Substitute (13) into (12) yields

$$\begin{aligned}\|y_n - p\| &= (1-\beta_n)\|x_n - p\| + \beta_n\delta\|x_n - p\| \\ &\le (1 - \beta_n + \beta_n\delta)\|x_n - p\|.\end{aligned}$$

Again, by observing that $0 \le \beta_n \le 1$, $0 \le \delta < 1$ and since $0 \le (1 - \beta_n + \beta_n\delta) < 1$, we obtain

$$\|y_n - p\| \le \|x_n - p\|. \tag{14}$$

Substitute (14) into (11) yields

$$x_{n+1} - p \leq (1 - \alpha_n + \alpha_n \delta) \|x_n - p\| .$$

By observing that $0 \leq \alpha_n \leq 1$, $0 \leq \delta < 1$ and since $0 \leq (1 - \alpha_n + \alpha_n \delta) < 1$, we obtain

$$\|x_{n+1} - p\| \leq \|x_n - p\| \tag{15}$$

which shows that the sequence $\{\|x_n - p\|\}$ is monotone decreasing.

We also have

$$\begin{aligned}
\|x_n - Tx_n\| &= \|(x_n - p) - (Tx_n - p)\| \\
&\leq \|x_n - p\| + \|Tx_n - p\| \\
&= \|x_n - p\| + \|Tx_n - Tp\| \\
&= \|x_n - p\| + \|Tp - Tx_n\| \\
&\leq \|x_n - p\| + 2\delta \|p - Tp\| + \delta \|p - x_n\| \\
&= \|x_n - p\| + 2\delta \|p - p\| + \delta \|x_n - p\| \\
&= \|x_n - p\| + 2\delta(0) + \delta \|x_n - p\| \\
&= \|x_n - p\| + 0 + \delta \|x_n - p\| \\
&= (1 + \delta) \|x_n - p\| , \quad 0 \leq \delta < 1.
\end{aligned}$$

Now, let us assume that there exists a real number $a > 0$ such that $\|x_n - p\| \geq a$ for all n.

Suppose $\{\|x_n - Tx_n\|\}$ does not converge to zero. Then, there are two possibilities: Either there exists an $\epsilon > 0$ such that $\|x_n - Tx_n\| \geq \epsilon$ for all n or $\liminf \|x_n - Tx_n\| = 0$.

In the first case and as in Berinde [3], putting $b = 2\delta(\frac{\epsilon}{\|x_0 - p\|})$, we get

$$\begin{aligned}
\|x_{n+1} - p\| &\leq (1 - \alpha_n(1 - \alpha_n)b) \|x_n - p\| \\
&\leq \|x_{n-1} - p\| - \alpha_{n-1}(1 - \alpha_{n-1})b \|x_n - p\| - b\alpha_n(1 - \alpha_n) \|x_n - p\| \\
&\leq \|x_{n-1} - p\| - b[\alpha_{n-1}(1 - \alpha_{n-1}) + \alpha_n(1 - \alpha_n)] \|x_n - p\| .
\end{aligned}$$

By induction, we obtain

$$a \leq \|x_{n+1} - p\| \leq \|x_0 - p\| - b \sum_{k=0}^{n} \alpha_k(1 - \alpha_k) \|x_n - p\| .$$

Therefore,

$$a + b \sum_{k=0}^{n} \alpha_k(1 - \alpha_k) \|x_n - p\| \leq \|x_0 - p\| .$$

It follows that $\sum_{k=0}^{n} \alpha_k(1-\alpha_k)$ is bounded which contradicts condition $(8)(ii)$.

Hence, there is no real number $a > 0$ such that $\|x_n - p\| \geq a$ for all n, which implies that $\{\|x_n - Tx_n\|\}$ converges to zero.

In the second case, there exists a subsequence $\{x_{n_k}\}$ such that

$$\lim_{k \longrightarrow \infty} \|x_{n_k} - Tx_{n_k}\| = 0. \tag{16}$$

If $\{x_{n_k}\}$, $\{x_{n_i}\}$ satisfy the contractive definition (5), then

$$\begin{aligned} \|Tx_{n_k} - Tx_{n_i}\| &\leq 2\delta \|x_{n_k} - Tx_{n_k}\| + \delta \|x_{n_k} - x_{n_i}\| \\ &\leq 2\delta \|x_{n_k} - Tx_{n_k}\| + \delta(\|x_{n_k} - Tx_{n_k}\| \\ &\quad + \|Tx_{n_k} - Tx_{n_i}\| + \|Tx_{n_i} - x_{n_i}\|) \\ &= 3\delta \|x_{n_k} - Tx_{n_k}\| + \delta \|Tx_{n_k} - Tx_{n_i}\| + \delta \|x_{n_i} - Tx_{n_i}\| \end{aligned}$$

which implies that

$$(1-\delta) \|Tx_{n_k} - Tx_{n_i}\| \leq 3\delta \|x_{n_k} - Tx_{n_k}\| + \delta \|x_{n_i} - Tx_{n_i}\|.$$

Thus,

$$\|Tx_{n_k} - Tx_{n_i}\| \leq \frac{3\delta}{1-\delta} \|x_{n_k} - Tx_{n_k}\| + \frac{\delta}{1-\delta} \|x_{n_i} - Tx_{n_i}\|.$$

But, $0 \leq \delta < 1$, therefore, $\{Tx_{n_k}\}$ is a Cauchy sequence, and hence convergent.

Let u be its limit. From (16), it results that

$$\lim_{k \longrightarrow \infty} x_{n_k} = \lim_{k \longrightarrow \infty} Tx_{n_k} = u. \tag{17}$$

We will show that $u = Tu$, that is, u is a fixed point of T. Indeed,

$$\|u - Tu\| \leq \|u - x_{n_k}\| + \|x_{n_k} - Tx_{n_k}\| + \|Tx_{n_k} - Tu\|. \tag{18}$$

If $\{x_{n_k}\}$, u satisfy the contractive definition (5), then

$$\|Tx_{n_k} - Tu\| \leq 2\delta \|x_{n_k} - Tx_{n_k}\| + \delta \|x_{n_k} - u\|. \tag{19}$$

Substitute (19) into (18) gives

$$\begin{aligned} \|u - Tu\| \leq & \; \|u - x_{n_k}\| + \|x_{n_k} - Tx_{n_k}\| \\ + & \; 2\delta \|x_{n_k} - Tx_{n_k}\| + \delta \|x_{n_k} - u\|. \end{aligned} \tag{20}$$

By using (17) and letting $k \longrightarrow \infty$ in (20), we get

$$\|u - Tu\| = 0.$$

Hence, $u = Tu$. Now, since p is the unique fixed point of T, it follows that $p = u$. Therefore, by condition (17) and the fact that $\{\|x_n - p\|\}$ is monotone decreasing with respect to n, we obtain

$$\lim_{n \longrightarrow \infty} x_n = u = p.$$

This completes the proof. ■

Now, we shall use the Ishikawa iteration (9) instead of iteration (7) used by Ciric [5].

The next result is a fixed point theorem for the Ishikawa iteration (9) associated with Zamfirescu mappings in uniformly convex Banach spaces and is our second main result in this work.

Theorem 4. *Let K and the mapping $T : K \longrightarrow K$ be as defined in Theorem 3 above. Let x_0 in K be arbitrary and let a sequence $\{x_n\}_{n=0}^{\infty}$ be defined iteratively by (9). Then, iteration (9) converges strongly to the fixed point of T.*

Proof. Just as in the proof of Theorem 3, we know that T has a unique fixed point in K. Call it p, that is, $p = Tp$, for $p \in K$. It suffices to show that the sequence $\{\|x_n - p\|\}$ is monotone decreasing as the rest of the proof is exactly the same as that of Theorem 3 above.

Indeed, for arbitrary $x_0 \in K$, by using (3.1) and Ishikawa iteration (9), we get

$$\|x_{n+1} - p\| \leq (1 - \alpha_n) \|x_n - p\| + \alpha_n \delta \|y_n - p\| .$$

Again, from Ishikawa iteration (9) and by using the contractive definition (5), we obtain

$$\|y_n - p\| \leq (1 - \beta_n + \beta_n \delta) \|x_n - p\|$$

and hence, for $0 \leq \beta_n \leq 1,\ 0 \leq \delta < 1$ and since $0 \leq (1 - \beta_n + \beta_n \delta) < 1$, we get

$$\|y_n - p\| \leq \|x_n - p\| . \tag{21}$$

Substitute (21) into (11) gives

$$\|x_{n+1} - p\| \leq (1 - \alpha_n + \alpha_n \delta) \|x_n - p\| .$$

For $0 \leq \alpha_n \leq 1$, $0 \leq \delta < 1$ and since $0 \leq (1 - \alpha_n + \alpha_n \delta) < 1$, we get

$$\|x_{n+1} - p\| \leq \|x_n - p\|$$

which shows that the sequence $\{\|x_n - p\|\}$ is monotone decreasing.

This completes the proof. ■

The following result is a fixed point theorem for the Mann iteration (8) associated with Zamfirescu mappings in uniformly convex Banach spaces.

Theorem 5. *Let K and the mapping $T : K \longrightarrow K$ be as defined in Theorem 3 above. Let x_0 in K be arbitrary and let a sequence $\{x_n\}_{n=0}^{\infty}$ be defined iteratively by (8). Then, the Mann iteration (8) converges strongly to the fixed point of T.*

Proof. The proof follows the same standard method as in Theorem 3, Theorem 4 and it is therefore omitted. ■

Acknowledgement. The author thanks Professors V. Berinde and B.E. Rhoades for supplying the preprints of some of their works which were very useful in the preparation of this paper.

References

[1] AGARWAL R.P., MEEHAN M., O′REGAN D., Fixed Point Theory and Applications, *Cambridge University Press,* 2001.

[2] BERINDE V., Iterative approximation of fixed points for pseudo-contractive operators, *Seminar on Fixed Point Theory,* 3(2001), 210-216.

[3] BERINDE V., *Iterative Approximation of Fixed Points,* Editura Efemeride. 2002

[4] CIRIC L.B., Quasi-contractions in Banach spaces, *Publ. Inst. Math.,* (Beograd) (N.S), 21(35)(1977), 11-18.

[5] CIRIC L.B., Fixed point theorems in Banach spaces, *Publ. Inst. Math., ,* (Beograd), 47(61)(1990), 85-87.

[6] CIRIC L.B., Fixed Point Theory. Contraction Mapping Principle, *FME Press,* Beograd 2003.

[7] EDELSTEIN M., A remark on a theorem of M.A Krasnoselkij, *Amer. Math.,* Monthly 73(1966), 509-510.

[8] GOEBEL K., KIRK W., SHIMI T., A fixed point theorem in uniformly convex spaces, *Bull. U.M.E,* 4(7)(1973), 67-75.

[9] IMORU C.O., AKINBO G., BOSEDE A.O., On the fixed points for weak compatible type and parametrically $\varphi(\epsilon, \delta; a)$-contraction mappings, *Math. Sci. Res. Journal,* 10(10)(2006), 259-267.

[10] ISHIKAWA S., Fixed points by a new iteration method, *Proc. Amer. Math. Soc.,* 44(1974), 147-150.

[11] MANN W.R., Mean value iteration, *Proc. Amer. Math. Soc.,* 4(1953), 506-510.

[12] MUTANGADURA S.A., CHIDUME C.E., An example on the Mann iteration method for Lipschitz pseudocontraction, *Proc. Amer. Math. Soc.,* 129(2001), 2359-2363.

[13] NOOR M.A., General variational inequalities, *Appl. Math. Letters,* 1(1988), 119-121.

[14] NOOR M.A., New approximations schemes for general variational inequalities, *Math. Anal. Appl.,* 251(2000), 217-299.

[15] NOOR M.A., Some new developments in general variational inequalities, *Appl. Math. Computation,* 152(2004), 199-277.

[16] NOOR M.A., NOOR K.I., RASSIAS T.M., Some aspects of variational inequalities, *J. Comput. Appl. Math.,* 47(1993), 493-512.
[17] OSILIKE M.O., Some stability results for fixed point iteration procedures, *J. Nigerian Math. Soc.,* 14/15(1995), 17-29.
[18] OWOJORI O.O., IMORU C.O., On a general Ishikawa fixed point iteration process for continuous hemicontractive maps in Hilbert spaces, *Advanced Studies in Contemporary Mathematics,* 4(1)(2001), 1-15.
[19] OWOJORI O.O., On generalized fixed point iterations for asymptotically nonexpansive maps in arbitrary Banach spaces, *Proceedings Jangjeon Mathematical Society,* 6(1)(2003), 49-58.
[20] PARK J.A., Mann-iteration for strictly pseudocontractive maps, *J. Korean Math. Soc.,* 31(1994), 333-337.
[21] RHOADES B.E., Fixed point theorems and stability results for fixed point iteration procedures, *Indian J. Pure Appl. Math.,* 21(1)(1990), 1-9.
[22] RHOADES B.E., Some fixed point iteration procedures, *Internat. J. Math. and Math. Sci.,* 14(1)(1991), 1-16.
[23] RUS I.A., Basic problems of the metric fixed point theory revisited (11), *Stud. Univ. Babes-Bolyal,* 36(1991), 81-99.
[24] RUS I.A., *Generalized Contractions and Applications,* Cluj-Napoca 2001.
[25] RUS I.A., PETRUSEL A., PETRUSEL G., *Fixed Point Theory, 1950-2000, Romanian Contributions,* House of the Book of Science, Cluj-Napoca 2002.
[26] WENG X.L., Fixed point iteration for local strictly pseudocontractive mapping, *Proc. Amer. Math. Soc.,* 113(1991), 727-731.
[27] XU B., NOOR M.A., Fixed point iterations for asymptotically nonexpansive mappings in Banach spaces, *Journal of Mathematical Analysis and Applications,* 267(2002), 444-453.
[28] ZAMFIRESCU T., Fix point theorems in metric spaces, *Arch. Math.,* 23(1972), 292-298.
[29] ZEIDLER E., *Nonlinear Functional Analysis and its Applications:Fixed Point Theorems,* Springer Verlag, New York, Inc. 1986.

ALFRED OLUFEMI BOSEDE
DEPARTMENT OF MATHEMATICS
LAGOS STATE UNIVERSITY, OJO, NIGERIA
e-mail: aolubosede@yahoo.co.uk

Received on 20.06.2008 and, in revised form, on 18.08.2009.