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THE GENERALIZED DOUBLE DIFFERENCE OF GAI SEQUENCE SPACES

Abstract. In this paper, we define some new sequence spaces and give some topological properties of the sequence spaces $\chi^2\left(\Delta_v^m, s, p\right)$ and $\Lambda^2\left(\Delta_v^m, s, p\right)$ and investigate some inclusion relations.

Key words: double difference sequence spaces, gai sequence, analytic sequence, paranorm.

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1. Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences respectively.

We write w^2 for the set of all complex sequences (x_{mn}), where $m, n \in \aleph$ the set of positive integers. Then w^2 is a linear space under the coordinate wise addition and scalar multiplication.

Some initial works on double sequence spaces is found in Bromwich [3]. Later on it was investigated by Hardy [5], Moricz [9], Moricz and Rhoades [10], Basarir and Solankan [2], Tripathy [11], Colak and Turkmenoglu [4], Turkmenoglu [12], and many others.

We need the following inequality in the sequel of the paper. For $a, b \geq 0$ and $0 < p < 1$, we have

$$(a+b)^p \leq a^p + b^p \tag{1}$$

The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence. (s_{mn}) is called convergent, where $s_{mn} = \sum_{i,j=1}^{m,n} x_{ij}$ $(m, n = 1, 2, 3, ...)$ (see[9]). A sequence $x = (x_{mn})$ is said to be double analytic if $\sup_{mn} |x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences will be denoted by Λ^2. A sequence $x = (x_{mn})$ is called double gai sequence if $\left((m+n)! \, |x_{mn}|\right)^{1/m+n} \to 0$ as $m, n \to \infty$. The double gai sequences will be denoted by χ^2. Let $\phi = \{allfinitesequences\}$. Consider a double sequence $x = (x_{ij})$. The $(m,n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\Im_{ij}$ for all $m, n \in N$,

$$\Im_{mn} = \begin{pmatrix} 0, & 0, & ...0, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \\ . \\ . \\ . \\ 0, & 0, & ...1, & -1, & ... \\ 0, & 0, & ...0, & 0, & ... \end{pmatrix}$$

with 1 in the $(m,n)^{th}$ position, -1 in the $(m+1, n+1)^{th}$ and zero other wise. An FK-space(or a metric space)X is said to have AK property if (δ_{mn}) is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$. An FDK-space is a double sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings $x = (x_k) \to (x_{mn})(m, n \in \aleph)$ are also continuous. If X is a sequence space, we give the following definitions:
(i) $X' =$ the continuous dual of X;
(ii) $X^\alpha = \left\{a = (a_{mn}) : \sum_{m,n=1}^{\infty} |a_{mn}x_{mn}| < \infty, \text{ for each } x \in X\right\}$
(iii) $X^\beta = \left\{a = (a_{mn}) : \sum_{m,n=1}^{\infty} a_{mn}x_{mn} \text{ is convegent,for each } x \in X\right\}$
(iv) $X^\gamma = \left\{a = (a_{mn}) : \sup_{m,n\geq 1} \left|\sum_{m,n=1}^{M,N} a_{mn}x_{mn}\right| < \infty, \text{ for each } x \in X\right\}$;
(v) let X be an FK-space $\supset \phi$; then $X^f = \{f(\delta_{mn}) : f \in X'\}$;
(vi) $X^\Lambda = \left\{a = (a_{mn}) : \sup_{mn} |a_{mn}x_{mn}|^{1/m+n} < \infty, \text{ for each } x \in X\right\}$;
X^α, X^β, X^γ are called $\alpha-$(*or Köthe-Toeplitz*) dual of X, $\beta-$(*or generalized-Köthe-Toeplitz*) dual of X, γ- dual of X, $\Lambda-$dual of X respectively.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [7] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c$, c_0 and ℓ_∞, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \aleph$. Here w, c, c_0 and ℓ_∞ denote the classes of all, convergent,null and bounded sclar valued single sequences respectively. The above spaces are Banach spaces normed by

$$\|x\| = |x_1| + \sup_{k\geq 1} |\Delta x_k|$$

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by

$$Z(\Delta) = \left\{x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z\right\}$$

where $Z = \Lambda^2$, χ^2 and $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m, n \in \aleph$.

Let $p = (p_{mn})$ be a sequence of real numbers such that $p_{mn} > 0$ for all m, n and $\sup_{mn} p_{mn} = H < \infty$, $v = (v_{mn})$ be any fixed sequence of non-zero complex numbers and $m \in \aleph$ be fixed. This assumption is made through out the rest of this paper.

2. Lemma

As in single sequences (*see* [11, *Theorem* 7.2.7])
(*i*) $X^{\gamma} \subset X^{f}$;
(*ii*) If X has AD, $X^{\beta} = X^{f}$;
(*iii*) If X has AD, $X^{\beta} = X^{f}$.

3. Definitions and preliminaries

Let w^2 denote the set of all complex double sequences. A sequence $x = (x_{mn})$ is said to be double analytic if $\sup_{mn} |x_{mn}|^{1/m+n} < \infty$. The vector space of all prime sense double analytic sequences will be denoted by Λ^2. A sequence $x = (x_{mn})$ is called prime sense double gai sequence if $((m+n)!\,|x_{mn}|)^{1/m+n} \to 0$ as $m, n \to \infty$. The double gai sequences will be denoted by χ^2. The space Λ^2 is a metric space with the metric

$$d(x,y) = \sup_{mn} \left\{ |x_{mn} - y_{mn}|^{1/m+n} : m, n : 1, 2, 3, ... \right\} \tag{2}$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in Λ^2.

The space χ^2 is a metric space with the metric

$$d(x,y) = \sup_{mn} \left\{ ((m+n)!\,|x_{mn} - y_{mn}|)^{1/m+n} : m, n : 1, 2, 3, ... \right\} \tag{3}$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in χ^2.

Throughout the article w^2, $\chi^2(\Delta)$, $\Lambda^2(\Delta)$ denote the spaces of all, prime sense double gai difference sequence spaces and prime sense double analytic difference sequence spaces respectively.

Let w^2 denote the set of all complex double sequences $x = (x_{mn})_{m,n=1}^{\infty}$.

Given a double sequence $x \in w^2$, define the sets

$$\chi^2(\Delta) = \left\{ x \in w^2 : ((m+n)!\,|\Delta x_{mn}|)^{1/m+n} \to 0 \text{ as } m, n \to \infty \right\}$$

$$\Lambda^2(\Delta) = \left\{ x \in w^2 : \sup_{mn} |\Delta x_{mn}|^{1/m+n} < \infty \right\}.$$

The space $\Lambda^2(\Delta)$ is a metric space with the metric

$$d(x,y) = \sup_{mn} \left\{ |\Delta x_{mn} - \Delta y_{mn}|^{1/m+n} : m, n = 1, 2, \cdots \right\}$$

for all $x = (x_{mn})$ and $y = (y_{mn})$ in $\Lambda^2(\Delta)$.

The space $\chi^2(\Delta)$ is a metric space with the metric

$$d(x,y) = \sup_{mn}\left\{((m+n)!\,|\Delta x_{mn} - \Delta y_{mn}|)^{1/m+n} : m,n = 1,2,\cdots\right\}$$

for all $x = (x_{mn})$ and $y = (y_{mn})$ in $\chi^2(\Delta)$.

Now we define the following sequence spaces:

$$\chi^2(\Delta_v^m, s, p) = \left\{x = (x_{mn}) \in w^2 : (mn)^{-s}\left(((m+n)!\,|\Delta_v^m x_{mn}|)^{1/m+n}\right)^{p_{mn}} \to 0 \; (m,n \to \infty),\ s \geq 0\right\}$$

$$\Lambda^2(\Delta_v^m, s, p) = \left\{x = (x_{mn}) \in w^2 : \sup_{mn}(mn)^{-s}\left(|\Delta_v^m x_{mn}|^{1/m+n}\right)^{p_{mn}} < \infty,\ s \geq 0\right\}$$

where

$$\begin{aligned}
\Delta_v^0 x_{mn} &= (v_{mn}x_{mn}), \\
\Delta_v x_{mn} &= (v_{mn}x_{mn} - v_{mn+1}x_{mn+1} - v_{m+1n}x_{m+1n} + v_{m+1n+1}x_{m+1n+1}) \\
\Delta_v^m x_{mn} &= \Delta\Delta_v^{m-1}x_{mn} \\
&= \left(\Delta_v^{m-1}x_{mn} - \Delta_v^{m-1}x_{mn+1} - \Delta_v^{m-1}x_{m+1n} + \Delta_v^{m-1}x_{m+1n+1}\right)
\end{aligned}$$

we get the following sequence spaces from the above sequence spaces by choosing some special p, m, s and v.

If $s = 0$, $m = 1$ and

$$v = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix}$$

with 1 upto $(m,n)^{th}$ position and zero other wise and $p_{mn} = 1$ for all m, n. We have

$$\begin{aligned}
\chi^2(\Delta) &= \left\{x = (x_{mn}) : \Delta x \in \chi^2\right\}, \\
\Lambda^2(\Delta) &= \left\{x = (x_{mn}) : \Delta x \in \Lambda^2\right\}.
\end{aligned}$$

If $s = 0$ and $p_{mn} = 1$ for all m, n, we have the following sequence spaces

$$\begin{aligned}
\chi^2(\Delta_v^m) &= \left\{x = (x_{mn}) \in w^2 : \Delta_v^m x \in \chi^2\right\}, \\
\Lambda^2(\Delta_v^m) &= \left\{x = (x_{mn}) \in w^2 : \Delta_v^m x \in \Lambda^2\right\}.
\end{aligned}$$

If $s = 0$, $m = 0$ and

$$v = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix}$$

with 1 upto $(m, n)^{th}$ position and zero other wise. We have the following sequence spaces

$$\chi^2(p) = \left\{ x = (x_{mn}) \in w^2 : ((m+n)! \, |x_{mn}|)^{p_{mn}/m+n} \to 0, (m, n \to \infty) \right\}$$
$$\Lambda^2(p) = \left\{ x = (x_{mn}) \in w^2 : sup_{mn} |x_{mn}|^{p_{mn}/m+n} < \infty \right\}$$

If $m = 0$ and

$$v = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix}$$

with 1 upto $(m, n)^{th}$ position and zero other wise. We have the following sequence spaces

$$\chi^2(p, s) = \left\{ x = (x_{mn}) \in w^2 : (mn)^{-s} ((m+n)! \, |x_{mn}|)^{p_{mn}/m+n} \to 0, (m, n \to \infty), \; s \geq 0 \right\},$$
$$\Lambda^2(p, s) = \left\{ x = (x_{mn}) \in w^2 : sup_{mn} (mn)^{-s} |x_{mn}|^{p_{mn}/m+n} < \infty, \; s \geq 0 \right\},$$

If $s = 0$, $m = 0$ and $p_{mn} = 1$

$$v = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix}$$

for all m, n with 1 upto $(m, n)^{th}$ position and zero other wise. We have χ^2 and Λ^2. If $s = 0$ we have $\chi^2(\Delta_v^m, p)$ and $\Lambda^2(\Delta_v^m, p)$

For a subspace ψ of a linear space is said to be sequence algebra if $x, y \in \psi$ implies that $x \cdot y = (x_{mn} y_{mn}) \in \psi$, see Kamptan and Gupta [13].

A sequence E is said to be solid (or normal) if $(\lambda_{mn} x_{mn}) \in E$, whenever $(x_{mn}) \in E$ for all sequences of scalars $(\lambda_{mn} = k)$ with $|\lambda_{mn}| \leq 1$.

If X is a linear space over the field C, then a paranorm on X is a function $g : g(\theta) = 0$ where $\theta = (0, 0, 0, \cdots)$, $g(-x) = g(x)$, $g(x + y) \leq g(x) + g(y)$ and $|\lambda - \lambda_0| \to 0$, $g(x - x_0)$ imply $g(\lambda x - \lambda_0 x_0) \to 0$, where $\lambda, \lambda_0 \in C$ and $x, x_0 \in X$. A paranormed space is a linear space X with a paranorm g and is written (X, g).

4. Main results

Theorem 1. *The following statements are hold*

(i) $\chi^2(\Delta_v^m, s) \subset \Lambda^2(\Delta_v^m, s)$ *and the inclusion is strict.*

(ii) $X(\Delta_v^m, s, p) \subset X\left(\Delta_v^{m+1}, s, p\right)$ *does not hold in general for any* $X = \chi^2$ *and* Λ^2.

Proof. (i) If we choose $s = 0$,

$$x = \begin{pmatrix} 1, & 0, & ...1, & 0, & 0, ... \\ 1, & 0, & ...1, & 0, & 0, ... \\ . \\ . \\ . \\ 1, & 0, & ...1, & 0, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . \\ . \\ . \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix}$$

Hence $x \in \Lambda^2(\Delta_v^m, s)$, but $x \notin \chi^2(\Delta_v^m, s)$

$$(ii) \text{ Let } v = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . \\ . \\ . \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix}, \ p = (p_{mn}) \text{ and } x = (x_{mn}) \text{ given}$$

by

$$p_{mn} = 1, \ \left((m+n)! \, |x_{mn}|\right)^{1/m+n} = m^2 n^2 \text{ if } m, \, n \text{ is odd}$$
$$p_{mn} = 2, \ \left((m+n)! \, |x_{mn}|\right)^{1/m+n} = mn \text{ if } m, \, n \text{ if even}$$
$$0 \text{ otherwise.}$$

Since for $m, n \geq 1$,

$$\left((m+n)! \left|\Delta_v^0 x_{mn}\right|\right)^{p_{mn}/m+n} = \left((m+n)! \, |x_{mn}|\right)^{p_{mn}/m+n} = m^2 n^2$$

$$m^{-3}n^{-3}\left((m+n)!\left|\Delta_v^0 x_{mn}\right|\right)^{p_{mn}/m+n} = m^{-3}n^{-3}m^2n^2 = m^{-1}n^{-1} \to 0$$

$(m,n \to \infty)$ and for $j \geq 1$

$$((4j)!\left|\Delta_v x_{2j,2j}\right|)^{p_{2j,2j}/4j} = \left(4j^3+4j^2+1\right)^2,$$
$$(4j)^{-3}\left((4j)!\left|\Delta_v x_{2j,2j}\right|\right)^{p_{2j,2j}/4j} \geq 4j \to \infty \quad (j \to \infty).$$

Now, we can see that $x \in \chi^2\left(\Delta_v^0, 3, p\right)$ and $x \notin \Lambda^2\left(\Delta_v^0, 3, p\right)$, which imply that $X\left(\Delta_v^m, s, p\right)$ is not a subset of $X\left(\Delta_v^{m+1}, s, p\right)$. This completes the proof. ■

Theorem 2. *$\chi^2\left(\Delta_v^m, s, p\right)$ and $\Lambda^2\left(\Delta_v^m, s, p\right)$ are linear spaces over the complex field C.*

Proof. Suppose that $M = max\left(1, sup_{m,n \geq \aleph} p_{mn}\right)$ Since $p_{mn}/M \leq 1$, we have for all m, n

$$(4) \qquad \left|\Delta_v^m\left(x_{mn}+y_{mn}\right)\right|^{p_{mn}/M} \leq \left(\left|\Delta_v^m x_{mn}\right|^{p_{mn}/M} + \left|\Delta_v^m y_{mn}\right|^{p_{mn}/M}\right)$$

and for all $\lambda \in C$

$$(5) \qquad |\lambda|^{p_{mn}/M} \leq Max\left(1, |\lambda|\right)$$

Now the linearity follows from (4) and (5). This completes the proof. ■

Theorem 3. *Let*
$N_1 = \min\left\{n_0 : \sup_{m,n \geq n_0} (mn)^{-s}\left(\left((m+n)!\left|\Delta_v^m x_{mn}\right|\right)^{1/m+n}\right)^{p_{mn}} < \infty\right\}$,
$N_2 = \min\left\{n_0 : \sup_{m,n \geq n_0} p_{mn} < \infty\right\}$ *and* $N = \max\left\{N_1, N_2\right\}$, $\chi^2\left(\Delta_v^m, s, p\right)$ *is a paranormed space with*

$$(6) \qquad g(x) = \sum_{m=1}^{i}\sum_{n=1}^{j}(m+n)!\left|x_{mn}\right| + \lim_{N\to\infty}\sup_{m,n\geq N}(mn)^{-S/M}\left((m+n)!\left|\Delta_v^m x_{mn}\right|\right)^{p_{mn}/M}$$

if and only if $\mu > 0$, *where* $\mu = \lim_{N\to\infty}\inf_{m,n\geq N} p_{mn}$ *and* $M = \max\left(1, \sup_{m,n\geq N} p_{mn}\right)$.

Proof.

(i) **Necessity:** Let $\chi^2\left(\Delta_v^m, s, p\right)$ be a paranormed space with (6) and suppose that $\mu = 0$. Then $\alpha = \inf_{m,n\geq N} p_{mn} = 0$ for all $N \in \aleph$ and hence we obtain

$$g(\lambda x) = \sum_{m=1}^{i}\sum_{n=1}^{j}(m+n)!\left|x_{mn}\right| + \lim_{N\to\infty}\sup_{m,n\geq N}(mn)^{-s}|\lambda|^{p_{mn}/M} = 1$$

for all $\lambda \in (0,1]$, where $x = \alpha \in \chi^2(\Delta_v^m, s, p)$. Whence $\lambda \to 0$ does not imply $\lambda x \to \theta$, when x is fixed. But this contradicts to (6) to be a paranorm.

Sufficiency: Let $\mu > 0$. It is trivial that $g(\theta) = 0$, $g(-x) = g(x)$ and $g(x+y) \le g(x) + g(y)$. Since $\mu > 0$ there exists a positive number β such that $p_{mn} > \beta$ for sufficiently large positive integer m, n. Hence for any $\lambda \in C$, we may write $|\lambda|^{p_{mn}} \le \max(|\lambda|^M, |\lambda|^\beta)$ for sufficiently large positive integers $m, n \ge \aleph$. Therefore, we obtain that $g(\lambda x) \le \max(|\lambda|, |\lambda|^{\beta/M}) g(x)$ using this, one can prove that $\lambda x \to \theta$, whenever x is fixed and $\lambda \to 0$ (or) $\lambda \to 0$ and $x \to \theta$ or λ is fixed and $x \to \theta$. This completes the proof. ■

Theorem 4. *Let* $0 < p_{mn} \le q_{mn} \le 1$ *then*
(i) $\Lambda^2(\Delta_v^m, s, p) \subseteq \Lambda^2(\Delta_v^m, s, q)$
(ii) $\chi^2(\Delta_v^m, s, p) \subseteq \chi^2(\Delta_v^m, s, q)$.

Proof. (i) Let $x \in \Lambda^2(\Delta_v^m, s, p)$. Then there exists a constant $M > 1$ such that

$$(mn)^{-s} |\Delta_v^m x_{mn}|^{q_{mn}/m+n} \le M \quad \text{for all } m, n$$

and so

$$(mn)^{-s} |\Delta_v^m x_{mn}|^{q_{mn}/m+n} \le M \quad \text{for all } m, n$$

suppose that $x^i \in \Lambda^2(\Delta_v^m, s, q)$ and $x^i \to x \in \Lambda^2(\Delta_v^m, s, p)$. Then for every $0 < \epsilon < 1$, there exist $\aleph$ such that for all m, n

$$(mn)^{-s} \left|\Delta_v^m \left(x_{mn}^{(i)} - x_{mn}\right)\right|^{p_{mn}/m+n} < \epsilon \quad \text{for all } m, n$$

Now,

$$\begin{aligned}(mn)^{-s} \left|\Delta_v^m \left(x_{mn}^{(i)} - x_{mn}\right)\right|^{q_{mn}/m+n} &< (mn)^{-s} \left|\Delta_v^m \left(x_{mn}^{(i)} - x_{mn}\right)\right|^{p_{mn}/m+n} \\ &< \epsilon \quad \text{for all } i > N.\end{aligned}$$

Therefore $x \in \Lambda^2(\Delta_v^m, s, q)$. This completes the proof.

(ii) It is easy. Therefore omit the proof. ■

Proposition 1. *For* $X = \chi^2$ *and* Λ^2*, then we obtain*
(i) $X(\Delta_v^m, s, p)$ *is not sequence algebra, in general*
(ii) $X(\Delta_v^m, s, p)$ *is not solid, in general.*

Proof. (i) This result is clear from the following example :

Example 1. Let $p_{mn} = 1, (m+n)! v_{mn} = \frac{1}{(mn)^{2(m+n)}}, (m+n)! x_{mn} = (mn)^{2(m+n)}$ and $(m+n)! y_{mn} = (mn)^{2(m+n)}$ for all m, n. Then we have $x, y \in \chi^2(\Delta, 0, p)$ but $x, y \notin \chi^2(\Delta, 0, p)$ with $m = 1$ and $s = 0$.

(ii) This result is clear from the following example

Example 2. Consider $x_{mn} = \begin{pmatrix} 1, & 1, & ...1, & 1, & 0, ... \\ 1, & 1, & ...1, & 1, & 0, ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1, & 1, & ...1, & 1, & 0, ... \\ 0, & 0, & ...0, & 0, & 0, ... \end{pmatrix} \in \chi^2 (\Delta_v^m, sp)$

Let $p_{mn} = 1$, $\alpha_{mn} = (-1)^{m+n}$, then $\alpha_{mn} x_{mn} \notin \chi^2 (\Delta_v^m, s, p)$ with $m = 1$ and $s = 0$.

■

The following proposition's proof is a routine verification.

Proposition 2. *For $X = \chi^2$ and Λ^2, then we obtain*

(i) *$s_1 < s_2$ implies $X (\Delta_v^m, s_1, p) \subset X (\Delta_v^m, s_2, p)$,*

(ii) *Let $0 < inf p_{mn} < p_{mn} \leq 1$ then $X (\Delta_v^m, s, p) \subset X (\Delta_v^m, s)$,*

(iii) *Let $1 \leq p_{mn} \leq sup_{mn} p_{mn} < \infty$, then $X (\Delta_v^m, s) \subset X (\Delta_v^m, s, p)$,*

(iv) *Let $0 < p_{mn} \leq q_{mn}$ and $\left(\frac{q_{mn}}{p_{mn}}\right)$ be bounded, then $X (\Delta_v^m, s, q) \subset X (\Delta_v^m, s, p)$.*

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