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WELL-POSEDNESS OF FIXED POINT PROBLEM FOR A HYBRID PAIR OF MAPPINGS

Abstract. The purpose of this paper is to extend the notion of well-posedness of fixed point problem for a mapping to a hybrid pair of mappings. Also, we prove a general common fixed point theorem for a pair of D-mappings for which the fixed point problem is well posed.

Key words: well-posedness of fixed point problem, common fixed point, D-mappings, occasionally weakly compatible mappings, implicit relations.

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1. Introduction

Let (X, d) be a metric space and $B(X)$ the set of all nonempty bounded sets of X. As in [7] and [8], we define the functions $\delta(A, B)$ and $D(A, B)$ by

$$\delta(A, B) := \sup\{d(a, b) : a \in A,\ b \in B\},$$

$$D(A, B) := \inf\{d(a, b) : a \in A,\ b \in B\}.$$

If A consists of single point "a", we write $\delta(A, B) = \delta(a, B)$.
If B consists of single point "b", we write $\delta(A, B) = \delta(A, b)$.

It follows immediately from the definition of $\delta(A, B)$ that

$$\delta(A, B) = \delta(B, A), \quad \forall A, B \in B(X),$$

and

$$\delta(A, B) \leq \delta(A, C) + \delta(C, B), \quad \forall A, B, C \in B(X).$$

Throughout this paper, $\mathbb{N}$ will be the set of non negative integers.

Definition 1. *A sequence $\{A_n\}$ of nonempty subsets of X is said to converge to a subset A of X if:*

(i) each point $a \in A$ is the limit of a convergent sequence $\{a_n\}$, where $a_n \in A_n$, for all $n \in \mathbb{N}$.

(ii) For arbitrary $\epsilon > 0$ there exists an integer $m > 0$ such that $A_n \subset A(\epsilon)$ for all integer $n \geq m$, where

$$A(\epsilon) := \{x \in X : \exists a \in A : d(x, a) < \epsilon\}.$$

The set A is said to be the limit of the sequence $\{A_n\}$.

Lemma 1 (Fisher [7])**.** *If $\{A_n\}$ and $\{B_n\}$ are two sequences in $B(X)$ converging to the sets A and B respectively in $B(X)$, then the sequence $\{\delta(A_n, B_n)\}$ converges to $\delta(A, B)$.*

Lemma 2 (Fisher and Sessa [8])**.** *Let $\{A_n\}$ be a sequence in $B(X)$ and $y \in X$ such that $\lim_{n\to\infty} \delta(A_n, y) = 0$. Then the sequence $\{A_n\}$ converges to $\{y\}$ in $B(X)$.*

Let A and S be self-mappings of a metric space of a metric space (X, d). Jungck ([9]) defined A and S to be compatible if $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t,$$

for some $t \in X$.

A point $x \in X$ is a coincidence point of A and S if $Ax = Sx$. We denote by $C(A, S)$ the set of all coincidence points of A and S.

In [15], Pant defined A and S to be pointwise R-weakly commuting mappings, if for each $x \in X$, there exists $R > 0$ such that $d(ASx, SAx) \leq R$, $d(Ax, Sx)$.

It is proved in [16] that pointwise R-weakly commuting is equivalent to commutativity at coincidence points.

Definition 2 ([10])**.** *The pair $\{A, S\}$ is said to be weakly compatible if $ASu = SAu$ for all $u \in C(A, S)$.*

Definition 3 ([3])**.** *A and S are said to be occasionally weakly compatible mappings (owc) if $ASu = SAu$ for some $u \in C(A, S)$.*

Remark 1. If A and S are weakly compatible and $C(A, S) \neq \emptyset$ then A and S are occasionally weakly compatible (owc), but the the converse is not true (see Example in [3]).

Some fixed point theorems for owc mappings are proved in [13] and other papers.

Definition 4. *If $f : X \to X$ and $F : X \to B(X)$, then*

1) a point $x \in X$ is said to be a coincidence point of f and f if $fx \in Fx$. We denote by $C(f, F)$ the set of all coincidence points of f and F,

2) *a point $x \in X$ is said to be a strict coincidence point of f and F if $\{fx\} = Fx$,*
3) *a point $x \in X$ is a fixed point of F if $x \in Fx$,*
4) *a point $x \in X$ is a strict fixed point of F if $\{x\} = Fx$.*

Definition 5 ([11])**.** *The mappings $f : X \to X$ and $F : X \to B(X)$ are said to be δ-compatible if $\lim_{n\to\infty} \delta(Ffx_n, fFx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $fFx_n \in B(X)$, $fx_n \longrightarrow t$, and $Fx_n \longrightarrow \{t\}$ for some $t \in X$.*

Definition 6 ([12])**.** *Let $f : X \to X$ and $F : X \to B(X)$ be mappings. The hybrid pair $\{f, F\}$ is said to be weakly compatible if for all $x \in C(f, F)$, we have $fF(x) = Ff(x)$.*

If the pair $\{f, F\}$ is δ-compatible, then it is weakly compatible, but the converse is not true in general (see[12]).

Definition 7 ([1])**.** *Let S and T be two single valued self mappings of a metric sapce (X, d). We say that S and T satisfy property (E.A) if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t,$$

for some $t \in X$.

Remark 2. It is clear that two self mappings S and T of a metric space (X, d) will be noncompatible if there exists at least a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$, but $\lim_{n\to\infty} d(STx_n, TSx_n)$ is either non zero or does not exist. Therefore two noncompatible mappings of a metric space (X, d) satisfy property (E.A).

Recently, Djoudi and Khemis ([6]) introduced a generalization of pair of mappings satisfying property (E.A), named D-mappings.

Definition 8 ([6])**.** *Two mappings $f : X \to X$ and $F : X \to B(X)$ are said to be D-mappings if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = t$ and $\lim_{n\to\infty} Fx_n = \{t\}$ for some $t \in X$.*

Obviously, two mappings which are not δ-compatible are D-mappings.

Definition 9 ([2])**.** *The mappings $f : X \to X$ and $F : X \to B(X)$ are said to be occasionally weakly compatible (owc) if there exists $x \in C(f, F)$ such that $fFx = Ffx$.*

For a pair of D-mappings the following result is obtained in [4].

Lemma 3. *Let $f : X \to X$ be a self mapping of a metric space (X, d) and $F : X \to B(X)$ be a set valued map. Assume that f and F satisfy the conditions*

(i) $FX \subset fX$,

(ii) *the inequality*

$$\delta(Fx, Fy) < \alpha \max\{d(fx, fy), \delta(fx, Fx), \delta(fy, Fy)\} \\ + (1-\alpha)[aD(fx, Fy) + bD(fy, Fx)],$$

for all $x, y \in X$, where $0 \leq \alpha < 1$, $a \geq 0$, $b \geq 0$, $a + b < 1$, whenever the right hand side of inequality (ii) is positive.

If f and F are weakly compatible D-mappings and fX or FX is closed, then f and F have a unique common fixed point in X.

We obtain a similar lemma by Theorem 3.4 of [6] for mappings satisfying the inequality

$$(iii) \qquad \delta(Fx, Fy) < \max\{cd(fx, fy), c\delta(fx, Fx), c\delta(fy, Fy), \\ aD(fx, Fy) + bD(fy, Fx)\},$$

for all $x, y \in X$, where $0 \leq c < 1$, $a \geq 0$, $b \geq 0$ and $0 < a + b < 1$, whenever the right hand side of inequality (iii) is positive.

Definition 10 ([5]). *Let (X, d) be a metric space and $f : (X, d) \to (X, d)$ be a mapping. The fixed point problem of f is said to be well posed if:*

(i) *f has a unique fixed point x in X,*

(ii) *for any sequence $\{x_n\}$ of points in X such that $\lim_{n\to\infty} d(fx_n, x_n) = 0$, we have $\lim_{n\to\infty} d(x_n, x) = 0$.*

Recently, the well-posednes of the fixed point problem for certain types of mappings have been investigated in [5], [14], [19], [20], [21] and other papers.

We extend Definition 10 for a pair of hybrid mappings.

Definition 11. *Let (X, d) be a metric space. Let $f : X \to X$ be a self mapping of X and let $F : X \to B(X)$ be a multifunction. The common fixed point problem of f and F is said to be well-posed if:*

(i) *f and F have a unique common fixed point x in X which is a strict fixed point of F,*

(ii) *for any sequence $\{x_n\}$ of points in X such that*

$$\lim_{n\to\infty} d(fx_n, x_n) = 0 \quad \textit{and} \quad \lim_{n\to\infty} \delta(x_n, Fx_n) = 0,$$

we have $\lim_{n\to\infty} d(x_n, x) = 0$.

In [17] and [18] the study of fixed points for mappings satisfying implicit relations was introduced.

The purpose of this paper is to prove a general common fixed point theorem for a hybrid pair of D-mappings (satisfying an implicit relation) for which the fixed point problem is well-posed.

2. Implicit relations

Definition 12. *Let $F(t_1, \ldots, t_6) : \mathbb{R}^6 \to \mathbb{R}$ be a mapping. We define the following properties:*

(F_m) : *F is non-increasing in the variable t_5.*

(F_1) : *$F(t, 0, 0, t, t, 0) > 0$, for every $t > 0$.*

(F_2) : *$F(t, t, 0, 0, t, t) \geq 0$, for every $t > 0$.*

Example 1. $F(t_1, \ldots, t_6) = t_1 - c \max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\}$, where $0 < c \leq 1$.

(F_m) : is obvious.

(F_1) : $F(t, 0, 0, t, t, 0) = t(1 - \frac{c}{2}) > 0$, for every $t > 0$.

(F_2) : $F(t, t, 0, 0, t, t) = t(1 - c) \geq 0$, for every $t > 0$.

Example 2. $F(t_1, \ldots, t_6) = t_1^3 - a t_1^2 t_2 - b t_1 t_3 t_4 - c t_5^2 t_6 - d t_5 t_6^2$, where $a, b, c, d \geq 0$ and $0 < a + c + d \leq 1$.

(F_m) : is obvious.

(F_1) : $F(t, 0, 0, t, t, 0) = t^3 > 0$, for every $t > 0$.

(F_2) : $F(t, t, 0, 0, t, t) = t^3(1 - (a + c + d)) \geq 0$, for every $t > 0$.

Example 3. $F(t_1, \ldots, t_6) = t_1^6 - c \dfrac{t_3^3 t_4^3 + t_1 t_2 t_5^2 t_6^2}{1 + [t_3 + t_4]^6}$, where $0 < c \leq 1$.

(F_m) : is obvious.

(F_1) : $F(t, 0, 0, t, t, 0) = t^6 > 0$, for every $t > 0$.

(F_2) : $F(t, t, 0, 0, t, t) = t^6(1 - c) \geq 0$, for every $t > 0$.

Example 4. $F(t_1, \ldots, t_6) = t_1 - \alpha \max\{t_2, t_3, t_4\} - (1 - \alpha)(a t_5 + b t_6)$, where $0 \leq \alpha < 1$, $a, b \geq 0$ and $0 < a + b < 1$.

(F_m) : is obvious.

(F_1) : $F(t, 0, 0, t, t, 0) = (1 - \alpha)(1 - a)t > 0$, for every $t > 0$.

(F_2) : $F(t, t, 0, 0, t, t) = t(1 - \alpha)(1 - (a + b)) \geq 0$, for every $t > 0$.

Example 5. $F(t_1, \ldots, t_6) = t_1 - \max\{c t_2, c t_3, c t_4, a t_5 + b t_6\}$, where $0 \leq c < 1$, $a, b \geq 0$ and $0 < a + b < 1$.

(F_m) : is obvious.

(F_1) : $F(t, 0, 0, t, t, 0) = t(1 - \max\{a, c\}) > 0$, for every $t > 0$.

(F_2) : $F(t, t, 0, 0, t, t) = t(1 - \max\{c, a + b\}) \geq 0$, for every $t > 0$.

Definition 13. *Let $F(t_1, \ldots, t_6) : \mathbb{R}^6 \to \mathbb{R}$ be a mapping. We define the following property:*

(F_p) : *There exists* $p \in (0,1)$ *such that for every* $u, v, w \geq 0$,

$$F(u, v, 0, w, u, v) \leq 0 \quad \Longrightarrow \quad u \leq p \max\{v, w\}.$$

Example 6. $F(t_1, \ldots, t_6) = t_1 - c \max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\}$, where $0 < c < 1$.
(F_p) : For all $u, v, w \geq 0$, we have

$$F(u, v, 0, w, u, v) = u - c \max\{v, \frac{w}{2}, \frac{u+v}{2}\}.$$

Suppose that $F(u, v, 0, w, u, v) \leq 0$, with $u > 0$ and $u \geq \max\{v, w\}$. Then we have

$$\max\{v, \frac{w}{2}, \frac{u+v}{2}\} \leq \max\{u, \frac{u}{2}, \frac{u+v}{2}\} = u.$$

Therefore we get $u(1-c) \leq 0$, a contradiction. Hence, $0 < u \leq \max\{v, w\}$, which implies that

$$u \leq c \max\{v, \frac{w}{2}, \frac{u+v}{2}\} \leq c \max\{v, w\}.$$

Hence $u \leq c \max\{v, w\}$. This inequality is true if $u = 0$. We conclude that (F_p) is satisfied with $p := c \in (0,1)$.

Example 7. $F(t_1, \ldots, t_6) = t_1^3 - a t_1^2 t_2 - b t_1 t_3 t_4 - c t_5^2 t_6 - d t_5 t_6^2$, where $a, b, c, d \geq 0$ and $0 < a + c + d < 1$.
(F_p) : For all $u, v, w \geq 0$, we have

$$F(u, v, 0, w, u, v) = u^3 - a u^2 v - c u^2 v - d u v^2.$$

Suppose that $F(u, v, 0, w, u, v) \leq 0$. If $u > 0$ then $u^2 \leq auv + cuv + dv^2$. If $u \geq v$ then $u^2 \leq (a + c + d)u^2 < u^2$, a contradiction. Hence $u < v$ and then we have $u^2 \leq (a + c + d)v^2$, which implies $u \leq p\, v \leq p \max\{v, w\}$, where $0 < p := \sqrt{a + c + d} < 1$. If $u = 0$ then evidently, $u \leq p \max\{v, w\}$. Thus (F_p) is satisfied.

Example 8. $F(t_1, \ldots, t_6) = t_1^6 - c \dfrac{t_3^3 t_4^3 + t_1 t_2 t_5^2 t_6^2}{1 + [t_3 + t_4]^6}$, where $0 < c < 1$.
(F_p) : For all $u, v, w \geq 0$, we have

$$F(u, v, 0, w, u, v) = u^6 - c \frac{u^3 v^3}{1 + w^6}.$$

Suppose that $F(u, v, 0, w, u, v) \leq 0$, then $u^6 \leq c \frac{u^3 v^3}{1+w^6}$, which implies $u^6 \leq c u^3 v^3$. If $u > 0$, then $u \leq c^{\frac{1}{3}} v \leq c^{\frac{1}{3}} \max\{v, w\}$. Hence we have $u \leq p \max\{v, w\}$, where $0 < p := c^{\frac{1}{3}} < 1$. If $u = 0$ then $u \leq p \max\{v, w\}$. Thus the property (F_p) is satisfied.

Example 9. $F(t_1, \ldots, t_6) = t_1 - \alpha \max\{t_2, t_3, t_4\} - (1-\alpha)(at_5 + bt_6)$, where $0 \le \alpha < 1$, $a, b \ge 0$ and $0 < a + b < 1$.

(F_p) : For all $u, v, w \ge 0$, $F(u, v, 0, w, u, v) \le 0$ implies $u \le p \max\{v, w\}$, where $0 < p := \alpha + (1-\alpha)(a+b) < 1$.

Example 10. $F(t_1, \ldots, t_6) = t_1 - \max\{ct_2, ct_3, ct_4, at_5 + bt_6\}$, where $0 < c < 1$, $a, b \ge 0$ and $0 < a + b < 1$.

(F_p) : For all $u, v, w \ge 0$, $F(u, v, 0, w, u, v) \le 0$ implies $u \le p \max\{v, w\}$, where $0 < p := \max\{c, (a+b)\} < 1$.

3. Common fixed points

Theorem 1. *Let (X, d) be a metric space. Let $f : (X, d) \to (X, d)$ and $F : (X, d) \to B(X)$ be occasionally weakly compatible mappings. If f and F have a unique point of strict coincidence z (i.e., $\{z\} = \{ft\} = Ft$, for some $t \in X$). Then the point z is the unique common fixed point of f and F which is a strict fixed point of F.*

Proof. Since f and F are occasionally weakly compatible, there exists a point $x \in X$ such that $\{fx\} = Fx$ implies $fFx = Ffx$. Then $z := fx$ and $\{fz\} = Fz$, hence fz is another point of strict coincidence for f and F. By hypothesis, we have $fz = z$. Hence $\{z\} = \{fz\} = Fz$. Thus z is a common fixed point for f and F which is a strict fixed point of F. Suppose $v \ne z$ another common fixed point for f and F which is a strict fixed point of F. Hence $\{v\} = \{fv\} = Fv$. Therefore v is a point of strict coincidence for f and F. By hypothesis, we have $v = z$. ∎

Theorem 2. *Let $f : (X, d) \to (X, d)$ and $F : (X, d) \to B(X)$ be mappings such that*

$$\phi\left(\delta(Fx, Fy), d(fx, fy), \delta(fx, Fx), \delta(fy, Fy), D(fx, Fy), D(fy, Fx)\right) < 0, \tag{1}$$

for all $x, y \in X$ such that $x \ne y$, where ϕ satisfies property (F_2). If u is a point of strict coincidence of f and F, then u is the unique point of strict coincidence of f and F.

Proof. Let $\{u\} = \{fx\} = Fx$ a point of strict coincidence of f and F. Suppose that $\{v\} = \{fy\} = Fy$ is another point of strict coincidence of f and F. Then we have $u = fx$ and $v = fy$. Suppose that $u \ne v$. Therefore $x \ne y$. By using inequality (1), we obtain

$$\phi\left(d(fx, fy), d(fx, fy), 0, 0, d(fx, fy), d(fx, fy)\right) < 0,$$

with $d(fx, fy) = d(u, v) > 0$, a contradiction of property (F_2). Hence $u = v$ and u is an unique point of strict coincidence of f and F. ■

Theorem 3. *Let $f : (X, d) \to (X, d)$ and $F : (X, d) \to B(X)$ be mappings such that*
1) *f and F are D-mappings,*
2) *the inequality*

$$\phi\left(\delta(Fx, Fy), d(fx, fy), \delta(fx, Fx), \delta(fy, Fy), D(fx, Fy), D(fy, Fx)\right) < 0, \tag{2}$$

holds for all $x, y \in X$ such that $x \neq y$, where ϕ is a continuous function which satisfies properties (F_m), (F_1) and (F_2),
3) *$F(X) \subset f(X)$.*

If $f(X)$ or $F(X)$ is closed then f and F have a strict coincidence point.

Moreover, if f and F are occasionally weakly compatible, then f and F have a unique common fixed point which is a strict fixed point of F.

Proof. Suppose that the pair f and F are D-mappings, then there is a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = t$ and $\lim_{n\to\infty} Fx_n = \{t\}$ for some $t \in X$.

Suppose that $F(X)$ or $f(X)$ is closed. Then there exists $u \in X$ such that $t = fu$.

If the sequence $\{x_n\}$ is stationary then we have $x_n = x$ for $n \geq n_0$ for some integer n_0. In this case we have $fx = t$ and $Fx = \{t\}$. Hence $\{t\} = \{fx\} = Fx$ is a point of strict coincidence.

If the sequence $\{x_n\}$ is not stationary, then by considering subsequences, we may suppose that $x_n \neq u$ for all integers $n \in \mathbb{N}$. By inequality (3.1), for all integer $n \in \mathbb{N}$, we have

$$\phi\left(\delta(Fx_n, Fu), d(fx_n, fu), \delta(fx_n, Fx_n), \delta(fu, Fu), D(fx_n, Fu), D(fu, Fx_n)\right) < 0.$$

Letting n tend to infinity and using the continuity of ϕ, we obtain

$$\phi\left(\delta(fu, Fu), 0, 0, \delta(fu, Fu), D(fu, Fu), 0\right) \leq 0.$$

Since $D(fu, Fu) \leq \delta(fu, Fu)$ and ϕ is non-increasing in the fifth variable, then we obtain

$$\phi\left(\delta(fu, Fu), 0, 0, \delta(fu, Fu), \delta(fu, Fu), 0\right) \leq 0,$$

a contradiction of property (F_1) if $\delta(fu, Fu) > 0$. Hence $\delta(fu, Fu) = 0$ which implies that $\{fu\} = Fu$. So u is a strict coincidence point of f and F.

In all cases, f and F have at least a strict coincidence point $u \in X$.

Moreover, if f and F are occasionally weakly compatible, then by Theorem 2 $\{v\} := \{fu\} = Fu$ is the unique point of strict coincidence of f and F. By Theorem 1 v is the unique common fixed point of f and F which is a strict fixed point for the multifunction F. ■

Corollary 1. *a) By using Theorem 3 and Example 4, we obtain a generalization for Lemma 3.*

b) By Theorem 3 and Example 5, we obtain a similar generalization for a result of [6].

4. Well-posedness of common fixed point problem

Theorem 4. *Let $f : (X, d) \to (X, d)$ and $F : (X, d) \to B(X)$ be mappings such that*

1) *f and F are D-mappings,*

2) *the inequality*

$$\phi\left(\delta(Fx, Fy), d(fx, fy), \delta(fx, Fx), \delta(fy, Fy), D(fx, Fy), D(fy, Fx)\right) < 0, \tag{3}$$

holds for all $x, y \in X$ such that $x \neq y$, where ϕ is a continuous function which satisfies properties (F_m), (F_1), (F_2) and (F_p),

3) *$F(X) \subset f(X)$.*

If $f(X)$ or $F(X)$ is closed and f and F are occasionally weakly compatible, then the common fixed point problem is well-posed.

Proof. By Theorem 3 the mappings f and F have a unique common fixed point $x \in X$ which is a strict fixed point of F. Let $\{x_n\}$ be a sequence of points in X such that

$$\lim_{n\to\infty} d(fx_n, x_n) = 0 \quad \text{and} \quad \lim_{n\to\infty} \delta(x_n, Fx_n) = 0.$$

Without loss of generality, we may suppose that $x \neq x_n$ for every non-negative integer n. Then by inequality (3) we have

$$\begin{aligned} &F\left(\delta(Fx, Fx_n), d(fx, fx_n), \delta(fx, Fx), \delta(fx_n, Fx_n), D(fx, Fx_n), D(fx_n, Fx)\right) \\ &\quad = F\left(\delta(x, Fx_n), d(x, fx_n), 0, \delta(fx_n, Fx_n), D(x, Fx_n), d(x, fx_n)\right) < 0. \end{aligned}$$

By property (F_m) we deduce that

$$F\left(\delta(x, Fx_n), d(x, fx_n), 0, \delta(fx_n, Fx_n), \delta(x, Fx_n), d(x, fx_n)\right) < 0.$$

By property (F_p) we have

$$\begin{aligned}\delta(x, Fx_n) &\le p \max\{d(x, fx_n), \delta(fx_n, Fx_n)\} \\ &\le p\,[d(x, fx_n) + \delta(fx_n, Fx_n)].\end{aligned}$$

Therefore

$$\begin{aligned}d(x, x_n) &\le \delta(x, Fx_n) + \delta(Fx_n, x_n) \\ &\le p\,[d(x, fx_n) + \delta(fx_n, Fx_n)] + \delta(Fx_n, x_n) \\ &\le p\,[d(x, x_n) + d(x_n, fx_n) + \delta(Fx_n, x_n) + d(x_n, fx_n)] \\ &\quad + \delta(Fx_n, x_n),\end{aligned}$$

which implies

$$d(x, x_n) \le \frac{2p}{1-p} d(x_n, fx_n) + \frac{p+1}{1-p} \delta(Fx_n, x_n) \longrightarrow 0 \quad \text{as } n \longrightarrow \infty.$$

Hence $\lim_{n\to\infty} d(x, x_n) = 0$. Consequenltly the common fixed point problem of the mappings f and F is well-posed. ■

Corollary 2. *By using Theorem 4 and the examples 6–10, we obtain new results.*

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