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ON UPPER AND LOWER FAINTLY δ-β-CONTINUOUS MULTIFUNCTIONS

Abstract. In this paper, we introduce and study a new class of multifunctions called faintly δ-β-continuous multifunctions in topological spaces.

Key words: topological space, δ-β-open set, δ-β-closed set, faintly δ-β-continuous multifunction.

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1. Introduction

It is well known that various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good number of them have been extended to the setting of multifunctions. This implies that both functions and multifunctions are important tools for studying other properties of spaces and for constructing new spaces from previously existing ones. In this paper, we introduce and study upper and lower faintly δ-β-continuous multifunctions in topological spaces and obtain some characterizations and properties of these new generalizations of continuous multifunctions.

2. Preliminaries

Let A be a subset of a topological space (X, τ). We denote the closure of A and the interior of A by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be regular open [9] if $A = \mathrm{Int}(\mathrm{Cl}(A))$. A set $A \subset X$ is said to be δ-open [10] if it is the union of regular open sets of X. The complement of a regular open (resp. δ-open) set is said to be regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure [10] of A and is denoted by $\mathrm{Cl}_\delta(A)$. A subset S of a topological space (X, τ) is said to be δ-β-open [2] if $S \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}_\delta(S)))$. The complement of a δ-β-open set is said to be δ-β-closed [2]. The intersection of all δ-β-closed sets containing S is called the δ-β-closure of S and is denoted by ${}_{\beta}\operatorname{Cl}_{\delta}(S)$. The δ-β-interior of S is defined by the union of all δ-β-open sets contained in S and is denoted by ${}_{\beta}\operatorname{Int}_{\delta}(S)$. The family of all δ-β-open sets of (X, τ) is denoted by $\delta\beta O(X)$. The family of all δ-β-open sets of (X, τ) containing a point $x \in X$ is denoted by $\delta\beta O(X, x)$. By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, also we always assume that $F(x) \neq \varnothing$ for all $x \in X$. For a multifunction $F : X \to Y$, the upper and lower inverse of any subset A of Y by $F^{+}(A)$ and $F^{-}(A)$, respectively, that is $F^{+}(A) = \{x \in X : F(x) \subseteq A\}$ and $F^{-}(A) = \{x \in X : F(x) \cap A \neq \varnothing\}$. In particular, $F^{-}(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$. A multifunction $F : X \to Y$ is said to be surjective if $F(X) = Y$. A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be lower δ-β-continuous [6] (resp. upper δ-β-continuous) if $F^{-}(V) \in \delta\beta O(X)$ (resp. $F^{+}(V) \in \delta\beta O(X)$) for every $V \in \sigma$.

3. Faintly δ-β-continuous multifunctions

Definition 1. *A multifunction $F : X \to Y$ is said to be :*

(*i*) *upper faintly δ-β-continuous at $x \in X$ if for each θ-open set V of Y containing $F(x)$, there exists $U \in \delta\beta O(X)$ containing x such that $F(U) \subset V$;*

(*ii*) *lower faintly δ-β-continuous at $x \in X$ if for each θ-open set V of Y such that $F(x) \cap V \neq \varnothing$, there exists $U \in \delta\beta O(X)$ containing x such that $F(u) \cap V \neq \varnothing$ for every $u \in U$;*

(*iii*) *upper* (*lower*) *faintly δ-β-continuous if it has this property at each point of X.*

Remark 1. It is clear that every upper δ-β-continuous multifunction is upper faintly δ-β-continuous. But the converse is not true in general, as the following example shows.

Example 1. Let $X = \{a, b, c\}$, $\tau = \{\varnothing$,{a},{b},{a, b},$X\}$ and $\sigma = \{\varnothing, \{a\}, X\}$. Then the multifunction $F : (X, \tau) \to (X, \sigma)$ defined by $F(a) = \{b\}$, $F(b) = \{c\}$ and $F(c) = \{a\}$ is upper faintly δ-β-continuous but it is not upper δ-β-continuous.

Definition 2. *A net (x_{α}) is said to δ-β-converge to a point x if for every δ-β-open set V containing x, there exists an index α_0 such that for all $\alpha \geq \alpha_0$, $x_{\alpha} \in V$. This is denoted by $x_{\alpha} \underset{\to}{\delta\beta} x$.*

Theorem 1. *For a multifunction $F : X \to Y$, the following statements are equivalent :*

(*i*) *F is upper faintly δ-β-continuous;*

(*ii*) *For each $x \in X$ and for each θ-open set V such that $x \in F^{+}(V)$, there exists a δ-β-open set U containing x such that $U \subset F^{+}(V)$;*

(iii) For each $x \in X$ and for each θ-closed set V such that $x \in F^+(Y\backslash V)$, there exists a δ-β-closed set H such that $x \in X\backslash H$ and $F^-(V) \subset H$;

(iv) $F^+(V)$ is a δ-β-open set for any θ-open set V of Y;

(v) $F^-(V)$ is a δ-β-closed set for any θ-closed set V of Y;

(vi) $F^-(Y\backslash V)$ is a δ-β-closed set for any θ-open set V of Y;

(vii) $F^+(Y\backslash V)$ is a δ-β-open set for any θ-closed set V of Y;

(viii) For each $x \in X$ and for each net (x_α) which δ-β-converges to $x \in X$ and for each θ-open set V of Y such that $x \in F^+(V)$, the net (x_α) is eventually in $F^+(V)$.

Proof. $(i) \Leftrightarrow (ii)$: Clear.

$(ii) \Leftrightarrow (iii)$: Let $x \in X$ and V be a θ-closed set of Y such that $x \in F^+(Y\backslash V)$. By (ii), there exists a δ-β-open set U containing x such that $U \subset F^+(Y\backslash V)$. Then $F^-(V) \subset X\backslash U$. Take $H = X\backslash U$. We have $x \in X\backslash H$ and H is δ-β-closed. The converse is similar.

$(i) \Leftrightarrow (iv)$: Let $x \in F^+(V)$ and V be a θ-open set of Y. By (i), there exists a δ-β-open set U_x containing x such that $U_x \subset F^+(V)$. It follows that $F^+(V) = \bigcup_{x\in F^+(V)} U_x$. Since any union of δ-β-open sets is δ-β-open, $F^+(V)$ is δ-β-open.

The converse can be shown similarly.

$(iv) \Leftrightarrow (v) \Leftrightarrow (vi) \Leftrightarrow (vii) \Leftrightarrow (i)$: Clear.

$(i) \Rightarrow (viii)$: Let (x_α) be a net which δ-β-converges to x in X and let V be any θ-open set of Y such that $x \in F^+(V)$. Since F is an upper faintly δ-β-continuous multifunction, it follows that there exists a δ-β-open set U of X containing x such that $U \subset F^+(V)$. Since (x_α) δ-β-converges to x, it follows that there exists an index α_0 such that $x_\alpha \in U$ for all $\alpha \geq \alpha_0$. From here, we obtain that $x_\alpha \in U \subset F^+(V)$ for all $\alpha \geq \alpha_0$. Thus, the net (x_α) is eventually in $F^+(V)$.

$(viii) \Rightarrow (i)$: Suppose that (i) is not true. There exists a point x and a θ-open set V with $x \in F^+(V)$ such that $U \nsubseteq F^+(V)$ for each δ-β-open set U of X containing x. Let $x_U \in U$ and $x_U \notin F^+(V)$ for each δ-β-open set U of X containing x. Then for each δ-β-neighborhood net (x_U), $x_U \underset{\to}{\delta\beta} x$, but (x_U) is not eventually in $F^+(V)$. This is a contradiction. Thus, F is an upper faintly δ-β-continuous multifunction. ∎

Theorem 2. *For a multifunction $F : X \to Y$, the following statements are equivalent:*

(i) F is lower faintly δ-β-continuous;

(ii) For each $x \in X$ and for each θ-open set V such that $x \in F^-(V)$, there exists a δ-β-open set U containing x such that $U \subset F^-(V)$;

(iii) For each $x \in X$ and for each θ-open set V such that $x \in F^-(Y\backslash V)$, there exists a δ-β-closed set H such that $x \in X\backslash H$ and $F^+(Y\backslash V) \subset H$;

(iv) $F^-(V)$ *is a* δ-β-*open set for any* θ-*open set* V *of* Y*;*

(v) $F^+(V)$ *is a* δ-β-*closed set for any* θ-*closed set* V *of* Y*;*

(vi) $F^+(Y\backslash V)$ *is a* δ-β-*closed set for any* θ-*open set* V *of* Y*;*

(vii) $F^-(Y\backslash V)$ *is a* δ-β-*open set for any* θ-*closed set* V *of* Y*;*

(viii) For each $x \in X$ *and for each net* (x_α) *which* δ-β-*converges to* $x \in X$ *and for each* θ-*open set* V *of* Y *such that* $x \in F^-(V)$ *the net* (x_α) *is eventually in* $F^-(V)$.

Proof. The proof is similar to that of Theorem 1. ■

Lemma 1 ([3])**.** *If* A *is* δ-*open in* X *and* $B \in \delta\beta O(X)$*, then* $A \cap B \in \delta\beta O(A)$.

Theorem 3. *Let* $F : X \to Y$ *be a multifunction and* U *a* δ-*open set of* X*. If* F *is a lower (upper) faintly* δ-β-*continuous multifunction, then the restriction* $F_{|_U} : U \to Y$ *is a lower (upper) faintly* δ-β-*continuous multifunction.*

Proof. Let $x \in U$ and V be any θ-open set of Y such that $x \in F^{|_U}(V)$. Since F is a lower faintly δ-β-continuous multifunction, it follows that there exists a δ-β-open set G containing x such that $G \subset F^-(V)$. From here by Lemma 1, we obtain that $x \in G \cap U \in \delta\beta O(U)$ and $G \cap U \subset F^{|_U}(V)$. This shows that the restriction $F_{|_U}$ is lower faintly δ-β-continuous.

The proof of the upper faintly δ-β-continuity of $F_{|_U}$ can be done by the same token. ■

Recall that the graph multifunction $G_F : X \to X \times Y$ of a multifunction $F : X \to Y$ is defined by $G_F(x) = \{x\} \times F(x)$ for every $x \in X$.

Lemma 2 ([5])**.** *The following hold for a multifunction* $F : X \to Y$*:*

(i) $G_F^+(A \times B) = A \cap F^+(B)$

(ii) $G_F^-(A \times B) = A \cap F^-(B)$

for each subsets $A \subset X$ *and* $B \subset Y$.

Theorem 4. *Let* $F : X \to Y$ *be a multifunction. If the graph multifunction* G_F *of* F *is upper faintly* δ-β-*continuous, then* F *is upper faintly* δ-β-*continuous.*

Proof. Let $x \in X$ and V be any θ-open subset of Y such that $x \in F^+(V)$. We obtain that $x \in G_F^+(X \times V)$ and that $X \times V$ is a θ-open set. Since the graph multifunction G_F is upper faintly δ-β-continuous, it follows that there exists a δ-β-open set U of X containing x such that $U \subset G_F^+(X \times V)$. Since $U \subset G_F^+(X \times V) = X \cap F^+(V) = F^+(V)$. We obtain that $U \subset F^+(V)$. Thus, F is upper faintly δ-β-continuous. ■

Theorem 5. *A multifunction* $F : X \to Y$ *is lower faintly* δ-β-*continuous if* G_F *is lower faintly* δ-β-*continuous.*

Proof. Suppose that G_F is lower faintly δ-β-continuous. Let $x \in X$ and V be any θ-open set of Y such that $x \in F^-(V)$. Then $X \times V$ is θ-open in $X \times Y$ and $G_F(x) \cap (X \times V) = (\{x\} \times F(x)) \cap (X \times V) = \{x\} \times (F(x) \cap V) \neq \varnothing$. Since G_F is lower faintly δ-β-continuous, there exists a δ-β-open U containing x such that $U \subset G_F^-(X \times V)$; hence $U \subset F^-(V)$. This shows that F is lower faintly δ-β-continuous. ∎

Theorem 6. *Suppose that (X, τ) and (X_α, τ_α) are topological spaces where $\alpha \in J$. Let $F : X \to \prod_{\alpha \in J} X_\alpha$ be a multifunction from X to the product space $\prod_{\alpha \in J} X_\alpha$ and let $P_\alpha : \prod_{\alpha \in J} X_\alpha \to X_\alpha$ be the projection for each $\alpha \in J$ which is defined by $P_\alpha((x_\alpha)) = x_\alpha$. If F is an upper (lower) faintly δ-β-continuous multifunction, then $P_\alpha \circ F$ is an upper (lower) faintly δ-β-continuous multifunction for each $\alpha \in J$.*

Proof. Take any $\alpha_0 \in J$. Let V_{α_0} be a θ-open set in $(X_{\alpha 0}, \tau_{\alpha 0})$. Then $(P_{\alpha 0} \circ F)^+(V_{\alpha 0}) = F^+(P_{\alpha 0}^+(V_{\alpha 0})) = F^+(V_{\alpha 0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$ (resp. $(P_{\alpha 0} \circ F)^-(V_{\alpha 0}) = F^-(P_{\alpha 0}^-(V_{\alpha 0})) = F^-(V_{\alpha 0} \times \prod_{\alpha \neq \alpha 0} X_\alpha)$). Since F is an upper (lower) faintly δ-β-continuous multifunction and since $V_{\alpha 0} \times \prod_{\alpha \neq \alpha_0} X_\alpha$ is a θ-open set, it follows that $F^+(V_{\alpha 0} \times \prod_{\alpha \neq 0} X_\alpha)$ (resp. $F^-(V_{\alpha 0} \times \prod_{\alpha \neq \alpha 0} X_\alpha)$) is a δ-β-open set in (X, τ). This shows that $P_{\alpha_0} \circ F$ is an upper (lower) faintly δ-β-continuous multifunction. Hence, we obtain that $P_\alpha \circ F$ is an upper (lower) faintly δ-β-continuous multifunction for each $\alpha \in J$. ∎

Theorem 7. *Suppose that for each $\alpha \in J$, (X_α, τ_α) and $(Y_\alpha, \sigma_\alpha)$ are topological spaces. Let $F_\alpha : X_\alpha \to Y_\alpha$ be a multifunction for each $\alpha \in J$ and let $F : \prod_{\alpha \in J} X_\alpha \to \prod_{\alpha \in J} Y_\alpha$ be a multifunction defined by $F((x_\alpha)) = \prod_{\alpha \in J} F_\alpha(x_\alpha)$. If F is an upper (lower) faintly δ-β-continuous multifunction, then each F_α is an upper (lower) faintly δ-β-continuous multifunction for each $\alpha \in J$.*

Proof. Let V_α be a θ-open set of Y_α. Then $V_\alpha \times \prod_{\alpha \neq \beta} Y_\beta$ is a θ-open set. Since F is an upper (lower) faintly δ-β-continuous multifunction, it follows that $F^+(V_\alpha \times \prod_{\alpha \neq \beta} Y_\beta) = F_\alpha^+(V_\alpha) \times \prod_{\alpha \neq \beta} X_\beta$ (resp. $F^-(V_\alpha \times \prod_{\alpha \neq \beta} Y_\beta) = F_\alpha^-(V_\alpha) \times \prod_{\alpha \neq \beta} X_\beta$) is a δ-β-open set. Consequently, we obtain that $F_\alpha^+(V_\alpha)$ (resp. $F_\alpha^-(V_\alpha)$) is a δ-β-open set. Thus, F_α is an upper (lower) faintly δ-β-continuous multifunction. ∎

We say that a space X is a θ-normal space if for any disjoint θ-closed sets E and F, there exist disjoint θ-open sets U and V of X such that $E \subset U$ and $F \subset V$.

Recall that a multifunction $F : X \to Y$ is said to be punctually closed if for each $x \in X$, $F(x)$ is closed.

Lemma 3 ([3]). *If $A \in \delta\beta O(X)$ and $B \in \delta\beta O(Y)$, then $A \times B \in \delta\beta O(X \times Y)$.*

Theorem 8. *If Y is a θ-normal space and $F_i : X_i \to Y$ is an upper faintly δ-β-continuous multifunction such that F_i is punctually closed for i = 1, 2, then a set $\{(x_1, x_2) \in X_1 \times X_2$: $F_1(x_1) \cap F_2(x_2) \neq \varnothing\}$ is a δ-β-closed set in $X_1 \times X_2$.*

Proof. Let $A = \{(x_1, x_2) \in X_1 \times X_2$: $F_1(x_1) \cap F_2(x_2) \neq \varnothing\}$ and $(x_1, x_2) \in (X_1 \times X_2)\backslash A$. Then $F_1(x_1) \cap F_2(x_2) = \varnothing$. Since Y is θ-normal and F_i is punctually closed for $i = 1, 2$, there exist disjoint θ-open sets V_1, V_2 such that $F_i(x_i) \subset V_i$ for $i = 1, 2$. Since F_i is upper faintly δ-β-continuous, $F_i^+(V_i)$ is a δ-β-open set for $i = 1, 2$. Put $U = F_1^+(V_1) \times F_2^+(V_2)$, then by Lemma 3 U is a δ-β-open set and $(x_1, x_2) \in U \subset (X_1 \times X_2) \backslash A$. This shows that $(X_1, \times X_2) \backslash A$ is δ-β-open and hence A is δ-β-closed in $X_1 \times X_2$. ■

Definition 3. *A topological space X is said to be finitely δ-β-Alexandroff [4] if every finite intersection of δ-β-open sets is δ-β-open.*

Theorem 9. *Let F and G be upper faintly δ-β-continuous and punctually closed multifunctions from a faintly δ-β-Alexandroff topological space (X, τ) to a θ-normal space (Y, σ). Then the set $K = \{x \in X : F(x) \cap G(x) \neq \varnothing\}$ is δ-β-closed in X.*

Proof. Let $x \in X\backslash K$. Then $F(x) \cap G(x) = \varnothing$. Since F and G are punctually closed multifunctions and Y is a θ-normal space, it follows that there exist disjoint θ-open sets U and V containing $F(x)$ and $G(x)$, respectively. Since F and G are upper faintly δ-β-continuous multifunctions, then the sets $F^+(U)$ and $G^+(V)$ are δ-β-open sets containing x. Let $H = F^+(U) \cap G^+(V)$. Then H is a δ-β-open set containing x and $H \cap K = \varnothing$; hence K is δ-β-closed in X. ■

Definition 4. *A topological space (X, τ) is said to be δ-β-T_2 [3] (resp. θ-T_2 [7]) if for each pair of distinct points x and y in X, there exists disjoint δ-β-open (resp. θ-open) sets U and V in X such that $x \in U$ and $y \in V$.*

Theorem 10. *Let $F : X \to Y$ be an upper faintly δ-β-continuous and punctually closed multifunction from a topological space X to a θ-normal space Y and let $F(x) \cap F(y) = \varnothing$ for each pair of distinct points x and y of X. Then X is a δ-β-T_2 space.*

Proof. Let x and y be any two distinct points in X. Then we have $F(x) \cap F(y) = \varnothing$. Since Y is θ-normal, it follows that there exist disjoint

θ-open sets U and V containing $F(x)$ and $F(y)$, respectively. Thus $F^+(U)$ and $F^+(V)$ are disjoint δ-β-open sets containing x and y, respectively and hence (X, τ) is δ-β-T_2. ■

Definition 5. *A topological space (X, τ) is said to be θ-compact [7] (resp. δ-β-compact [3]) if every θ-open (resp. δ-β-open) cover of X has a finite subcover. A subset A of a topological space X is said to be θ-compact relative to X if every cover of A by θ-open sets of X has a finite subcover.*

Theorem 11. *Let $F : X \to Y$ be an upper faintly δ-β-continuous surjective multifunction such that $F(x)$ is θ-compact relative to Y for each $x \in X$. If X is a δ-β-compact space, then Y is θ-compact.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a θ-open cover of Y. Since $F(x)$ is θ-compact relative to Y for each $x \in X$, there exists a finite subset $\Lambda(x)$ of Λ such that $F(x) \subset \bigcup\{V_\alpha \colon \alpha \in \Lambda(x)\}$. Put $V(x) = \bigcup\{V_\alpha \colon \alpha \in \Lambda(x)\}$. Since F is upper faintly δ-β-continuous, there exists a δ-β-open set $U(x)$ of X containing x such that $F(U(x)) \subset V(x)$. Then the family $\{U(x) : x \in X\}$ is a δ-β-open cover of X and since X is δ-β-compact, there exists a finite number of points, say, $x_1, x_2, x_3,..., x_n$ in X such that $X = \bigcup\{U(x_i) \colon i = 1, 2,..., n\}$. Hence we have $Y = F(X) = F(\bigcup\limits_{i=1}^{n} U(x_i)) = \bigcup\limits_{i=1}^{n} F(U(x_i)) \subset \bigcup\limits_{i=1}^{n} V(x_i) = \bigcup\limits_{i=1}^{n} \bigcup\limits_{\alpha\in\Lambda(x_i)} V_\alpha$. This shows that Y is θ-compact. ■

Definition 6. *Let $F : X \to Y$ be a multifunction. The graph $G(F)$ is said to be δ-β-θ-closed if for each $(x, y) \notin G(F)$, there exist a δ-β-open set U and a θ-open set V containing x and y, respectively, such that $(U \times V) \cap G(F) = \varnothing$.*

Theorem 12. *If $F : X \to Y$ is an upper faintly δ-β-continuous multifunction such that $F(x)$ is θ-compact relative to Y for each $x \in X$ and Y is a θ-T_2 space, then $G(F)$ is δ-β-θ-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(F)$. That is $y \notin F(x)$. Since Y is θ-T_2, for each $z \in F(x)$, there exist disjoint θ-open sets $V(z)$ and $U(z)$ of Y such that $z \in U(z)$ and $y \in V(z)$. Then $\{U(z) : z \in F(x)\}$ is a θ-open cover of $F(x)$ and since $F(x)$ is θ-compact relative to Y, there exists a finite number of points, say, $z_1, z_2, \ldots, z_n$ in $F(x)$ such that $F(x) \subset \bigcup\{U(z_i) : i = 1, 2, \ldots, n\}$. Put $U = \bigcup\{U(z_i) \colon i = 1, 2, \ldots, n\}$ and $V = \bigcap \{V(y_i) \colon i = 1, 2, \ldots, n\}$. Then U and V are θ-open sets in Y such that $F(x) \subset U$, $y \in V$ and $U \cap V = \varnothing$. Since F is an upper faintly δ-β-continuous multifunction, there exists a δ-β-open set W of X containing x such that $F(W) \subset U$. We have $(x, y) \in W \times V \subset (X \times Y) \setminus G(F)$. We obtain that $(W \times V) \cap G(F) = \varnothing$ and hence $G(F)$ is δ-β-θ-closed in $X \times Y$. ■

Theorem 13. *Let X be finitely δ-β-Alexandroff and $F : X \to Y$ be a multifunction having the δ-β-θ-closed graph $G(F)$. If B is θ-compact relative to Y, then $F^{-}(B)$ is δ-β-closed in X.*

Proof. Let $x \in X \backslash F^{-}(B)$. For each $y \in B$, $(x, y) \notin G(F)$ and there exist a δ-β-open set $U(y) \subset X$ and a θ-open set $V(y) \subset Y$ containing x and y, respectively, such that $F(U(y)) \cap V(y) = \varnothing$. That is, $U(y) \cap F^{-}(V(y)) = \varnothing$. Then $\{V(y)\colon y \in B\}$ is a θ-open cover of B and since B is θ-compact relative to Y, there exists a finite subset B_0 of B such that $B \subset \bigcup\{V(y) : y \in B_0\}$. Put $U = \bigcap\{U(y) : y \in B_0\}$. Then U is δ-β-open in X, $x \in U$ and $U \cap F^{-}(B) = \varnothing$; that is, $x \in U \subset X\backslash F^{-}(B)$. This shows that $F^{-}(B)$ is δ-β-closed in X. ■

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