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# $(\hat{g}, s)$-CONTINUOUS FUNCTIONS BETWEEN TOPOLOGICAL SPACES

Abstract. In this paper, we introduce ($\hat{g}$, s)-continuous functions between topological spaces, study some of its basic properties and discuss its relationships with other topological functions.

Key words: $\hat{g}$-closed set, regular open set, $\hat{g}$-$T_{1/2}$ space, $(\hat{g}, s)$-continuous function.

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## 1. Introduction

It is well known that the concept of closedness is fundamental with respect to the investigation of general topological spaces. Levine [26] initiated the study of generalized closed sets. The concept of $\hat{g}$-closed sets was introduced by Veerakumar [46]. Recently, this notion is further studied by Ravi et al [40]. Initiation of contra-continuity was due to Dontchev [10]. Many different forms of contra-continuous functions have been introduced over the years by various authors [5, 12, 14, 15, 17, 18, 22, 38].

In this paper, new generalizations of contra-continuity by using $\hat{g}$-closed sets called $(\hat{g}, s)$-continuity are presented. Characterizations and properties of $(\hat{g}, s)$-continuous functions are discussed in detail. Finally, we obtain many important results in topological spaces.

## 2. Preliminaries

In this paper, spaces $X$ and $Y$ mean topological spaces on which no separation axioms are assumed unless explicitely stated. For a subset $A$ of a space $X$, $cl(A)$ and $int(A)$ represent the closure of $A$ and the interior of $A$ respectively.

A subset $A$ of a space $X$ is said to be regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$) [43]. The $\delta$-interior [45] of a subset $A$ of $X$ is the union of all regular open sets of $X$ contained in $A$ and it is denoted by $\delta - int(A)$. A subset $A$ is called $\delta$-open [45] if $A = \delta - int(A)$.

The complement of $\delta$-open set is called $\delta$-closed. The $\delta$-closure of a set A in a space (X,$\tau$) is defined by $\delta - cl(A) = \{x \in X : A \cap int(cl(U)) \neq \phi$, $U \in \tau$ and $x \in U\}$ and it is denoted by $\delta - cl(A)$.

The finite union of regular open sets is said to be $\pi$-open [48]. The complement of $\pi$-open set is said to be $\pi$-closed. A subset $A$ is said to be semi-open [25] (resp. $\alpha$-open [31], preopen [30], $\beta$-open [1] or semi-preopen [2]) if $A \subset cl(int(A))$ (resp. $A \subset int(cl(int(A)))$, $A \subset int(cl(A))$, $A \subset cl(int(cl(A)))$). The complement of semi-open (resp. $\alpha$-open, preopen, $\beta$-open) is said to be semi-closed (resp. $\alpha$-closed, preclosed, $\beta$-closed). The union (resp. intersection) of all $\alpha$-open (resp. $\alpha$-closed) sets, each contained in (resp. containing) a set $S$ in a topological space $X$ is called $\alpha$-interior (resp. $\alpha$-closure) of $S$ and it is denoted by $\alpha int(S)$ (resp. $\alpha cl(S)$). The union (resp. intersection) of all semi-open (resp. semi-closed) sets, each contained in (resp. containing) a set $S$ in a topological space $X$ is called semi-interior (resp. semi-closure) of $S$ and it is denoted by $sint(S)$ (resp. $scl(S)$). The union (resp. intersection) of all preopen (resp. preclosed) sets, each contained in (resp. containing) a set $S$ in a topological space $X$ is called preinterior (resp. preclosure) of $S$ and it is denoted by $pint(S)$ (resp. $pcl(S)$).

A subset $A$ of a space $X$ is said to be generalized closed (briefly, $g$-closed) [26] (resp. $\pi g$-closed [13], $\hat{g}$-closed [46] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open (resp. $\pi$-open, semi-open) in $X$. The complement of $g$-closed (resp. $\pi g$-closed, $\hat{g}$-closed) is said to be $g$-open (resp. $\pi g$-open, $\hat{g}$-open). The union (resp. intersection) of all $\hat{g}$-open (resp. $\hat{g}$-closed) sets, each contained in (resp. containing) a set $S$ in a topological space $X$ is called $\hat{g}$-interior (resp. $\hat{g}$-closure) of $S$ and it is denoted by $\hat{g} - int(S)$ (resp. $\hat{g} - cl(S)$).

A point $x \in X$ is said to be a $\theta$-semi-cluster point [23] of a subset $A$ of $X$ if $cl(U) \cap A \neq \phi$ for every semi-open set $U$ containing $x$. The set of all $\theta$-semi-cluster points of $A$ is called the $\theta$-semi-closure of $A$ and is denoted by $\theta - s - cl(A)$. A subset $A$ is called $\theta$-semi-closed [23] if $A = \theta - s - cl(A)$. The complement of a $\theta$-semi-closed set is called $\theta$-semi-open.

The family of all $\delta$-open (resp. $\hat{g}$-open, $\hat{g}$-closed, $\pi$g-open, $\pi g$-closed, regular open, regular closed, semi-open, closed) sets of $X$ containing a point $x \in X$ is denoted by $\delta O(X, x)$ (resp. $\hat{G}O(X, x)$, $\hat{G}C(X, x)$, $\pi GO(X, x)$, $\pi GC(X, x)$, $RO(X, x)$, $RC(X, x)$, $SO(X, x)$, $C(X, x)$). The family of all $\delta$-open (resp. $\hat{g}$-open, $\hat{g}$-closed, $\pi g$-open, $\pi g$-closed, semi-open, $\beta$-open, preopen, regular open, regular closed) sets of $X$ is denoted by $\delta O(X)$ (resp $\hat{G}O(X)$, $\hat{G}C(X)$, $\pi GO(X)$, $\pi GC(X)$, $SO(X)$, $\beta O(X)$, $PO(X)$, $RO(X)$, $RC(X)$).

**Definition 1.** *A space $X$ is said to be*

1. *s-Urysohn [3] if for each pair of distinct points $x$ and $y$ in $X$, there exist $U \in SO(X, x)$ and $V \in SO(X, y)$ such that $cl(U) \cap cl(V) = \phi$;*

2. *weakly Hausdorff [41] if each element of $X$ is an intersection of regular closed sets.*

**Definition 2** ([19]). *Let $B$ be a subset of a space $X$. The set $\cap\{A \in RO(X) : B \subset A\}$ is called the $r$-kernel of $B$ and is denoted by $r - ker(B)$.*

**Proposition 1** ([19]). *The following properties hold for subsets $A$, $B$ of a space $X$:*

1. *$x \in r - ker(A)$ if and only if $A \cap K \neq \phi$ for any regular closed set $K$ containing $x$.*
2. *$A \subset r - ker(A)$ and $A = r - ker(A)$ if $A$ is regular open in $X$.*
3. *If $A \subset B$, then $r - ker(A) \subset r - ker(B)$.*

**Lemma 1** ([28]). *If $V$ is an open set, then $scl(V) = int(cl(V))$.*

**Definition 3.** *A space $X$ is said to be*

1. *S-closed [44] if every regular closed cover of $X$ has a finite subcover,*
2. *Countably S-closed [1] if every countable cover of $X$ by regular closed sets has a finite subcover,*
3. *S-Lindelof [27] if every cover of $X$ by regular closed sets has a countable subcover.*

**Theorem 1** ([46]). *Union (intersection) of any two $\hat{g}$-closed sets is again $\hat{g}$-closed.*

**Remark 1** ([13], [46]). We have the following relations:

$$\text{closed} \Rightarrow \hat{g}\text{-closed} \Rightarrow \text{g-closed} \Rightarrow \pi g\text{-closed}.$$

None of these implications are reversible. The subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of a function $f : X \to Y$ and is denoted by $G(f)$.

## 3. Characterizations of $\hat{g}$-open sets

**Lemma 2.** *For any subset $K$ of a topological space $X$, $X \backslash \hat{g} - cl(K) = \hat{g} - int(X \backslash K)$.*

**Lemma 3.** *If a subset $A$ is $\hat{g}$-closed in a space $X$, then $A = \hat{g} - cl(A)$.*

**Theorem 2.** *If $A$ is $\hat{g}$-closed and semi-open set, then $A$ is closed.*

**Theorem 3.** *A set $A$ is $\hat{g}$-open in $(X, \tau)$ if and only if $F \subseteq int(A)$ whenever $F$ is semi-closed in $X$ and $F \subseteq A$.*

**Proof.** Assume that $A$ is $\hat{g}$-open, $F \subseteq A$ and $F$ is semi-closed. Then $X \backslash F$ is semi-open and $X \backslash A \subseteq X \backslash F$. Since $X \backslash A$ is $\hat{g}$-closed, $cl(X \backslash A) \subseteq X \backslash F$. It implies that $X \backslash int(A) \subseteq X \backslash F$ and hence $F \subseteq int(A)$.

Conversely, put $X\backslash A = B$. Suppose $B \subseteq U$ where $U$ is semi-open. Now if $X\backslash A \subseteq U$, then $F = X\backslash U \subseteq A$ and $F$ is semi-closed. It implies that $F \subseteq int(A)$ and hence $X\backslash int(A) \subseteq X\backslash F = U$. Therefore $X\backslash int(X\backslash B) \subseteq U$ and consequently $cl(B) \subseteq U$. Hence $B$ is $\hat{g}$-closed and therefore $A$ is $\hat{g}$-open. ∎

**Theorem 4.** *Suppose that $A$ is $\hat{g}$-open in $X$ and that $B$ is $\hat{g}$-open in $Y$. Then $A \times B$ is $\hat{g}$-open in $X \times Y$.*

**Proof.** Suppose that $F$ is closed and hence semi-closed in $X \times Y$ and that $F \subseteq A \times B$. By the previous theorem, it suffices to show that $F \subseteq int(A \times B)$.

Let $(x, y) \in F$. Then, for each $(x, y) \in F$, $cl(\{x\}) \times cl(\{y\}) = cl(\{x\} \times \{y\}) = cl(\{x, y\}) \subset cl(F) = F \subset A \times B$. Two closed sets $cl(\{x\})$ and $cl(\{y\})$ are contained in $A$ and $B$ respectively. It follows from the assumption that $cl(\{x\}) \subseteq int(A)$ and that $cl(\{y\}) \subseteq int(B)$. Thus $(x, y) \in cl(\{x\}) \times cl(\{y\}) \subseteq int(A) \times int(B) \subseteq int(A \times B)$. It means that, for each $(x, y) \in F$, $(x, y) \in int(A \times B)$ and hence $F \subseteq int(A \times B)$. Therefore $A \times B$ is $\hat{g}$-open in $X \times Y$. ∎

**Definition 4.** *A function $f : X \to Y$ is called pre $\hat{g}$-closed if $f(V)$ is $\hat{g}$-closed set in $Y$ for each $\hat{g}$-closed set $V$ in $X$.*

**Theorem 5.** *If a function $f : X \to Y$ is pre $\hat{g}$-closed, then for each subset $B$ of $Y$ and each $\hat{g}$-open set $U$ of $X$ containing $f^{-1}(B)$, there exists a $\hat{g}$-open set $V$ in $Y$ containing $B$ such that $f^{-1}(V) \subset U$.*

**Proof.** Suppose that $f$ is pre $\hat{g}$-closed. Let $B$ be a subset of $Y$ and $U \in \hat{G}O(X)$ containing $f^{-1}(B)$. Put $V = Y\backslash f(X\backslash U)$, then $V$ is an $\hat{g}$-open set of $Y$ such that $B \subset V$ and $f^{-1}(V) \subset U$. ∎

## 4. Properties of $(\hat{g}, s)$-continuous functions

**Definition 5.** *A function $f : X \to Y$ is called $(\hat{g}, s)$-continuous if the inverse image of each regular open set of $Y$ is $\hat{g}$-closed in $X$.*

**Theorem 6.** *The following are equivalent for a function $f : X \to Y$:*

(1) *$f$ is $(\hat{g}, s)$-continuous,*

(2) *The inverse image of a regular closed set of $Y$ is $\hat{g}$-open in $X$,*

(3) *$f^{-1}(int(cl(V)))$ is $\hat{g}$-closed in $X$ for every open subset $V$ of $Y$,*

(4) *$f^{-1}(cl(int(F)))$ is $\hat{g}$-open in $X$ for every closed subset $F$ of $Y$,*

(5) *$f^{-1}(cl(U))$ is $\hat{g}$-open in $X$ for every $U \in \beta O(Y)$,*

(6) *$f^{-1}(cl(U))$ is $\hat{g}$-open in $X$ for every $U \in SO(Y)$,*

(7) *$f^{-1}(int(cl(U)))$ is $\hat{g}$-closed in $X$ for every $U \in PO(Y)$.*

**Proof.** (1) ⇔ (2): Obvious.

(1) ⇔ (3): Let $V$ be an open subset of $Y$. Since $int(cl(V))$ is regular open, $f^{-1}(int(cl(V)))$ is $\hat{g}$-closed. The converse is similar.

(2) $\Leftrightarrow$ (4): Similar to 1) $\Leftrightarrow$ 3).

(2) $\Rightarrow$ (5): Let $U$ be any $\beta$-open set of Y. By Theorem 2.4 of [2] that $cl(U)$ is regular closed. Then by (2) $f^{-1}(cl(U))$ is $\hat{g}$-open in $X$.

(5) $\Rightarrow$ (6): Obvious from the fact that $SO(Y) \subset \beta O(Y)$.

(6) $\Rightarrow$ (7): Let $U \in PO(Y)$. Then $Y \backslash int(cl(U))$ is regular closed and hence it is semi-open. Then, we have $X \backslash f^{-1}(int(cl(U))) = f^{-1}(Y \backslash int(cl(U)))$ $= f^{-1}(cl(Y \backslash int(cl(U))))$ is $\hat{g}$-open in $X$. Hence $f^{-1}(int(cl(U)))$ is $\hat{g}$-closed in $X$.

(7) $\Rightarrow$ (1): Let $U$ be any regular open set of Y. Then U $\in$ PO(Y) and hence $\mathrm{f}^{-1}(\mathrm{U}) = \mathrm{f}^{-1}(\mathrm{int}(\mathrm{cl}(\mathrm{U})))$ is $\hat{g}$-closed in X. ■

**Lemma 4** ([35]). *For a subset $A$ of a topological space $(Y, \sigma)$ the following properties hold:*

(1) *$\alpha cl(A) = cl(A)$ for every $A \in \beta O(Y)$,*

(2) *$pcl(A) = cl(A)$ for every $A \in SO(Y)$,*

(3) *$scl(A) = int(cl(A))$ for every $A \in PO(Y)$.*

**Corollary 1.** *The following are equivalent for a function $f : X \to Y$:*

(1) *$f$ is $(\hat{g}, s)$-continuous,*

(2) *$f^{-1}(\alpha cl(A))$ is $\hat{g}$-open in $X$ for every $A \in \beta O(Y)$,*

(3) *$f^{-1}(pcl(A))$ is $\hat{g}$-open in $X$ for every $A \in SO(Y)$,*

(4) *$f^{-1}(scl(A))$ is $\hat{g}$-closed in $X$ for every $A \in PO(Y)$.*

**Proof.** It follows from Lemma 4. ■

**Theorem 7.** *Suppose that $\hat{G}C(X)$ is closed under arbitrary intersections. The following are equivalent for a function $f : X \to Y$:*

(1) *$f$ is $(\hat{g}, s)$-continuous,*

(2) *the inverse image of a $\theta$-semi-open set of $Y$ is $\hat{g}$-open,*

(3) *the inverse image of a $\theta$-semi-closed set of $Y$ is $\hat{g}$-closed,*

(4) *$f(\hat{g} - cl(U)) \subset r - ker(f(U))$ for every subset $U$ of $X$,*

(5) *$\hat{g}$-cl($f^{-1}(V)$) $\subset f^{-1}$(r-ker($V$)) for every subset $V$ of $Y$,*

(6) *for each $x \in X$ and each $V \in SO(Y, f(x))$, there exists a $\hat{g}$-open set $U$ in $X$ containing $x$ such that $f(U) \subset cl(V)$,*

(7) *$f^{-1}(V) \subset \hat{g} - int(f^{-1}(cl(V)))$ for every $V \in SO(Y)$,*

(8) *$f(\hat{g} - cl(A)) \subset \theta - s - cl(f(A))$ for every subset $A$ of $X$,*

(9) *$\hat{g} - cl(f^{-1}(B)) \subset f^{-1}(\theta - s - cl(B))$ for every subset $B$ of $Y$,*

(10) *$\hat{g} - cl(f^{-1}(V)) \subseteq f^{-1}(\theta - s - cl(V))$ for every open subset $V$ of $Y$,*

(11) *$\hat{g} - cl(f^{-1}(V)) \subseteq f^{-1}(scl(V))$ for every open subset $V$ of $Y$,*

(12) *$\hat{g} - cl(f^{-1}(V)) \subseteq f^{-1}(int(cl(V))$ for every open subset $V$ of $Y$.*

**Proof.** (1) $\Rightarrow$ (2): Since any $\theta$-semi-open set is a union of regular closed sets, by using Theorem 6, (2) holds.

(2) $\Rightarrow$ (6): Let $x \in X$ and $V \in SO(Y)$ containing $f(x)$. Since $cl(V)$ is $\theta$-semi-open in $Y$, there exists a $\hat{g}$-open set $U$ in $X$ containing $x$ such that $x \in U \subset f^{-1}(cl(V))$. Hence $f(U) \subset cl(V)$.

(6) $\Rightarrow$ (7): Let $V \in SO(Y)$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. By (6), there exists a $\hat{g}$-open set $U$ in $X$ containing $x$ such that $f(U) \subset cl(V)$. It follows that $x \in U \subset f^{-1}(cl(V))$. Hence $x \in \hat{g} - int(f^{-1}(cl(V)))$. Thus, $f^{-1}(V) \subset \hat{g} - int(f^{-1}(cl(V)))$.

(7) $\Rightarrow$ (1): Let $F$ be any regular closed set of $Y$. Since $F \in SO(Y)$, then by (7), $f^{-1}(F) \subset \hat{g} - int(f^{-1}(F))$. This shows that $f^{-1}(F)$ is $\hat{g}$-open in $X$. Hence, by Theorem 6, (1) holds.

(2) $\Leftrightarrow$ (3): Obvious.

(1) $\Rightarrow$ (4): Let $U$ be any subset of $X$. Let $y \notin r - ker(f(U))$. Then there exists a regular closed set $F$ containing $y$ such that $f(U) \cap F = \phi$. Hence, we have $U \cap f^{-1}(F) = \phi$ and $\hat{g} - cl(U) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f(\hat{g} - cl(U)) \cap F = \phi$ and $y \notin f(\hat{g} - cl(U))$. Thus, $f(\hat{g} - cl(U)) \subset r - ker(f(U))$.

(4) $\Rightarrow$ (5): Let $V$ be any subset of $Y$. By (4), $f(\hat{g} - cl(f^{-1}(V))) \subset r - ker(V)$ and $\hat{g} - cl(f^{-1}(V)) \subset f^{-1}(r - ker(V))$.

(5) $\Rightarrow$ (1): Let $V$ be any regular open set of $Y$. By (5), $\hat{g} - cl(f^{-1}(V)) \subset f^{-1}(r - ker(V)) = f^{-1}(V)$ and $\hat{g} - cl((f^{-1}(V)) = f^{-1}(V)$. We obtain that $f^{-1}(V)$ is $\hat{g}$-closed in $X$.

(6) $\Rightarrow$ (8): Let $A$ be any subset of $X$. Suppose that $x \in \hat{g} - cl(A)$ and $G$ is any semi-open set of $Y$ containing $f(x)$. By (6), there exists $U \in \hat{G}O(X, x)$ such that $f(U) \subset cl(G)$. Since $x \in \hat{g} - cl(A)$, $U \cap A \neq \phi$ and hence $\phi \neq f(U) \cap f(A) \subset cl(G) \cap f(A)$. Therefore, we obtain $f(x) \in \theta - s - cl(f(A))$ and hence $f(\hat{g} - cl(A)) \subset \theta - s - cl(f(A))$.

(8) $\Rightarrow$ (9): Let $B$ be any subset of $Y$. Then $f(\hat{g} - cl(f^{-1}(B))) \subset \theta - s - cl(f(f^{-1}(B)) \subset \theta - s - cl(B)$ and $\hat{g} - cl(f^{-1}(B)) \subset f^{-1}(\theta - s - cl(B))$.

(9) $\Rightarrow$ (6): Let $V$ be any semi-open set of $Y$ containing $f(x)$. Since $cl(V) \cap (Y \backslash cl(V)) = \phi$, we have $f(x) \notin \theta - s - cl(Y \backslash cl(V))$ and $x \notin f^{-1}(\theta - s - cl(Y \backslash cl(V)))$. By (9), $x \notin \hat{g} - cl(f^{-1}(Y \backslash cl(V)))$. Hence, there exists $U \in \hat{G}O(X, x)$ such that $U \cap f^{-1}(Y \backslash cl(V)) = \phi$ and $f(U) \cap (Y \backslash cl(V)) = \phi$. It follows that $f(U) \subset cl(V)$. Thus,(6) holds.

(9) $\Rightarrow$ (10): Obvious.

(10) $\Rightarrow$ (11): Obvious from the fact that $\theta - s - cl(V) = scl(V)$ for an open set $V$.

(11) $\Rightarrow$ (12): Obvious from Lemma 1.

(12) $\Rightarrow$ (1): Let $V \in RO(Y)$. Then by (12) $\hat{g} - cl(f^{-1}(V)) \subset f^{-1}(int(cl(V))) = f^{-1}(V)$. Hence, $f^{-1}(V)$ is $\hat{g}$-closed which proves that $f$ is $(\hat{g}, s)$-continuous. ■

**Corollary 2.** *Assume that $\hat{G}C(X)$ is closed under arbitrary intersections. The following are equivalent for a function $f : X \to Y$:*

(1) $f$ *is* $(\hat{g}, s)$*-continuous.*
(2) $\hat{g} - cl(f^{-1}(B)) \subset f^{-1}(\theta - s - cl(B))$ *for every* $B \in SO(Y)$,
(3) $\hat{g} - cl(f^{-1}(B)) \subset f^{-1}(\theta - s - cl(B))$ *for every* $B \in PO(Y)$,
(4) $\hat{g} - cl(f^{-1}(B)) \subset f^{-1}(\theta - s - cl(B))$ *for every* $B \in \beta O(Y)$.

**Proof.** In Theorem 7, we have proved that the following are equivalent.
(1) $f$ is $(\hat{g}, s)$-continuous.
(2) $\hat{g} - cl(f^{-1}(B)) \subset f^{-1}(\theta - s - cl(B))$ for every subset $B$ of $Y$.
Hence the corollary is proved. ■

## 5. The related functions with $(\hat{g}, s)$-continuous functions

**Definition 6.** *A function* $f : X \to Y$ *is said to be*

1. *perfectly continuous [33] if* $f^{-1}(V)$ *is clopen in* $X$ *for every open set* $V$ *of* $Y$,

2. *regular set-connected [11, 16] if* $f^{-1}(V)$ *is clopen in* $X$ *for every* $V \in RO(Y)$,

3. *almost s-continuous [6, 37] if for each* $x \in X$ *and each* $V \in SO(Y, f(x))$, *there exists an open set* $U$ *in* $X$ *containing* $x$ *such that* $f(U) \subset scl(V)$,

4. *strongly continuous [24] if the inverse image of every set in* $Y$ *is clopen in* $X$,

5. *RC-continuous [11] if* $f^{-1}(V)$ *is regular closed in* $X$ *for each open set* $V$ *of* $Y$,

6. *contra R-map [17] if* $f^{-1}(V)$ *is regular closed in* $X$ *for each regular open set* $V$ *of* $Y$,

7. *contra-super-continuous [22] if for each* $x \in X$ *and for each* $F \in C(Y, f(x))$, *there exists a regular open set* $U$ *in* $X$ *containing* $x$ *such that* $f(U) \subset F$,

8. *almost contra-super-continuous [15] if* $f^{-1}(V)$ *is* $\delta$*-closed in* $X$ *for every regular open set* $V$ *of* $Y$,

9. *contra continuous [10] if* $f^{-1}(V)$ *is closed in* $X$ *for every open set* $V$ *of* $Y$,

10. *contra g-continuous [5] if* $f^{-1}(V)$ *is g-closed in* $X$ *for every open set* $V$ *of* $Y$,

11. $(\theta, s)$*-continuous [23, 38] if for each* $x \in X$ *and each* $V \in SO(Y, f(x))$, *there exists an open set* $U$ *in* $X$ *containing* $x$ *such that* $f(U) \subset cl(V)$,

12. *contra* $\pi g$*-continuous [18] if* $f^{-1}(V)$ *is* $\pi g$*-closed in* $X$ *for each open set* $V$ *of* $Y$,

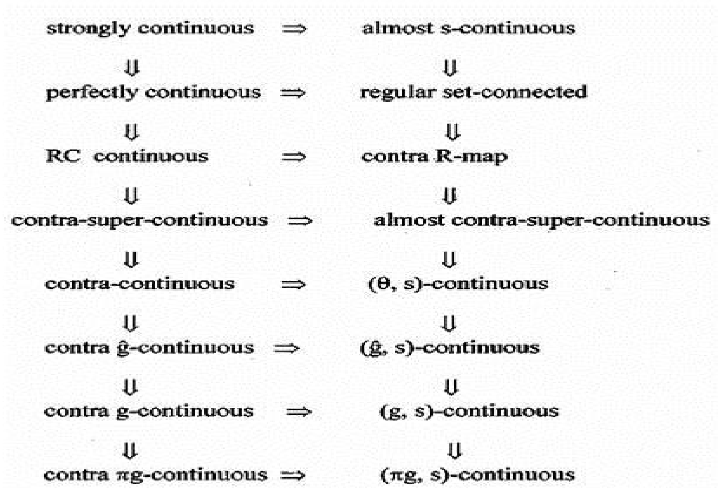
13. $\hat{g}$*-continuous [46] if* $f^{-1}(V)$ *is* $\hat{g}$*-closed in* $X$ *for each closed set* $V$ *of* $Y$,

14. $(g, s)$*-continuous [14] if* $f^{-1}(V)$ *is g-closed in* $X$ *for each regular open set* $V$ *of* $Y$,

15. *$(\pi g, s)$-continuous [14] if $f^{-1}(V)$ is $\pi g$-closed in $X$ for each regular open set $V$ of $Y$.*

**Definition 7.** *A function $f : X \to Y$ is said to be contra $\hat{g}$-continuous if $f^{-1}(V)$ is $\hat{g}$-closed in $X$ for each open set $V$ of $Y$.*

**Remark 2.** The following diagram holds for a function $f : X \to Y$:




None of these implications is reversible as shown in the following examples and in the related paper [14].

**Example 1.** Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $\sigma = \{\phi, Y, \{a, b\}\}$. Then the identity function $f : X \to Y$ is both contra $g$-continuous and $(\hat{g}, s)$-continuous but it is not contra $\hat{g}$-continuous.

**Example 2.** Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{a, b\}\}$. Then the identity function $f : X \to Y$ is contra $\hat{g}$-continuous but it is not contra continuous.

**Example 3.** Let $X = Y = \{a, b, c, d\}$, $\tau = \sigma = \{\phi, X = Y, \{c\}, \{a, d\}, \{a, c, d\}\}$. Then the function $f : X \to Y$ which is defined as $f(a) = c$, $f(b) = c$, $f(c) = b$, $f(d) = b$ is $(\hat{g}, s)$-continuous but it is not $(\theta, s)$-continuous.

**Example 4.** Let X = {a, b, c}, Y = {p, q, r}, $\tau$ = {$\phi$, X, {a}, {a, c}} and $\sigma$ = {$\phi$, Y, {p}, {q}, {p,q}}. Then the function f: X $\to$ Y which is defined as f(a) = p; f(b) = p; f(c) = r is (g, s)-continuous but it is not ($\hat{g}$, s)-continuous.

A topological space $(X, \tau)$ is said to be extremely disconnected [4] if the closure of every open set of $X$ is open in $X$.

**Definition 8.** *A function $f : X \to Y$ is said to be almost $\hat{g}$-continuous if $f^{-1}(V)$ is $\hat{g}$-open in $X$ for every regular open set $V$ of $Y$.*

**Theorem 8.** *Let $(Y, \sigma)$ be extremely disconnected. Then, the following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*
(1) *$f$ is $(\hat{g}, s)$-continuous,*
(2) *$f$ is almost $\hat{g}$-continuous.*

**Proof.** (1) $\Rightarrow$ (2): Let $x \in X$ and $U$ be any regular open set of $Y$ containing $f(x)$. Since $Y$ is extremely disconnected, by Lemma 5.6 of [39] $U$ is clopen and hence $U$ is regular closed. Then $f^{-1}(U)$ is $\hat{g}$-open in $X$. Thus, $f$ is almost $\hat{g}$-continuous.

(2) $\Rightarrow$ (1): Let $K$ be any regular closed set of $Y$. Since $Y$ is extremely disconnected, $K$ is regular open and $f^{-1}(K)$ is $\hat{g}$-open in $X$. Thus, $f$ is $(\hat{g}, s)$-continuous. ■

**Definition 9.** *A space is said to be $P_\Sigma$ [47] or strongly s-regular [21] if for any open set $V$ of $X$ and each $x \in V$, there exists $K \in RC(X, x)$ such that $x \in K \subset V$.*

**Definition 10.** *A space $(X, \tau)$ is called $\hat{g} - T_{1/2}$ if every $\hat{g}$-closed set is closed.*

**Theorem 9.** *Let $f : X \to Y$ be a function from a $\hat{g} - T_{1/2}$- space $X$ to a topological space $Y$. The following are equivalent.*
(1) *$f$ is $(\theta, s)$-continuous.*
(2) *$f$ is $(\hat{g}, s)$-continuous.*

**Theorem 10.** *Let $f : X \to Y$ be a function. Then, if $f$ is $(\hat{g}, s)$-continuous, $X$ is $\hat{g} - T_{1/2}$ and $Y$ is $P_\Sigma$, then $f$ is continuous.*

**Proof.** Let $G$ be any open set of $Y$. Since $Y$ is $P_\Sigma$, there exists a subfamily $\Phi$ of $RC(Y)$ such that $G = \cup\{A : A \in \Phi\}$. Since $X$ is $\hat{g} - T_{l/2}$ and $f$ is $(\hat{g}, s)$-continuous, $f^{-1}(A)$ is open in $X$ for each $A \in \Phi$ and $f^{-1}(G)$ is open in $X$. Thus, $f$ is continuous. ■

**Theorem 11.** *Let $f : X \to Y$ be a function from a $\hat{g} - T_{1/2}$ -space $(X, \tau)$ to an extremely disconnected space $(Y, \sigma)$. Then the following are equivalent.*
(1) *$f$ is $(\hat{g}, s)$-continuous.*
(2) *$f$ is $(\theta, s)$-continuous.*
(3) *$f$ $f$ is almost contra-super-continuous.*
(4) *$f$ is contra R-map.*
(5) *$f$ is regular set-connected.*
(6) *$f$ is almost s-continuous.*

**Proof.** (6) $\Rightarrow$ (5) $\Rightarrow$ (4) $\Rightarrow$ (3) $\Rightarrow$ (2) $\Rightarrow$ (1): Obvious.

(1) $\Rightarrow$ (6): Let $V$ be any semi-open and semi-closed set of $Y$. Since $V$ is semi-open, $cl(V) = cl(int(V))$ and hence $cl(V)$ is open in $Y$. Since

$V$ is semi-closed, $int(cl(V)) \subset V \subset cl(V)$ and hence $int(cl(V)) = V = cl(V)$. Therefore, $V$ is clopen in $Y$ and $V \in RO(Y) \cap RC(Y)$. Since $f$ is $(\hat{g}, s)$-continuous, $f^{-1}(V)$ is $\hat{g}$-open and $\hat{g}$-closed in $X$. Since $X$ is $\hat{g} - T_{1/2}$-space, $\tau = \hat{G}O(X)$. Thus, $f^{-1}(V)$ is clopen in $X$ and hence $f$ is almost $s$-continuous [36, Theorem 3.1]. ∎

**Definition 11.** *A space is said to be weakly $P_\Sigma$ [34] if for any $V \in RO(X)$ and each $x \in V$, there exists $F \in RC(X, x)$ such that $x \in F \subset V$.*

**Theorem 12.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a $(\hat{g}, s)$-continuous function and $\hat{G}C(X)$ be closed under arbitrary intersections. If $Y$ is weakly $P_\Sigma$ and $X$ is $\hat{g} - T_{1/2}$, then $f$ is regular set-connected.*

**Proof.** Let $V$ be any regular open set of $Y$. Since $Y$ is weakly $P_\Sigma$, there exists a subfamily $\Phi$ of $RC(Y)$ such that $V = \cup\{A : A \in \Phi\}$. Since $f$ is $(\hat{g}, s)$-continuous, $f^{-1}(A)$ is $\hat{g}$-open in $X$ for each $A \in \Phi$ and $f^{-1}(V)$ is $\hat{g}$-open in $X$. Also $f^{-1}(V)$ is $\hat{g}$-closed in $X$ since $f$ is $(\hat{g}, s)$-continuous. Since $X$ is $\hat{g} - T_{1/2}$ space, then $\tau = \hat{G}O(X)$. Hence $f^{-1}(V)$ is clopen in $X$ and then $f$ is regular set-connected. ∎

**Definition 12.** *A function $f : X \to Y$ is said to be $\hat{g}$-irresolute [46] if $f^{-1}(V)$ is $\hat{g}$-open in $X$ for every $V \in \hat{G}O(X)$.*

**Theorem 13.** *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*

1. *If $f$ is $\hat{g}$-irresolute and $g$ is $(\hat{g}, s)$-continuous, then $gof : X \to Z$ is $(\hat{g}, s)$-continuous.*
2. *If $f$ is $(\hat{g}, s)$-continuous and $g$ is contra $R$-map, then $gof : X \to Z$ is almost $\hat{g}$-continuous.*
3. *If $f$ is $\hat{g}$-continuous and $g$ is $(\theta, s)$-continuous, then $gof : X \to Z$ is $(\hat{g}, s)$-continuous.*
4. *If $f$ is $(\hat{g}, s)$-continuous and $g$ is $RC$-continuous, then $gof : X \to Z$ is $\hat{g}$-continuous.*
5. *If $f$ is almost $\hat{g}$-continuous and $g$ is contra $R$-map, then $gof : X \to Z$ is $(\hat{g}, s)$-continuous.*

**Theorem 14.** *Let $Y$ be a regular space and $f : X \to Y$ be a function. Suppose that the collection of $\hat{g}$-closed sets of $X$ is closed under arbitrary intersections. Then if $f$ is $(\hat{g}, s)$-continuous, $f$ is $\hat{g}$-continuous.*

**Proof.** Let $x$ be an arbitrary point of $X$ and $V$ an open set of $Y$ containing $f(x)$. Since $Y$ is regular, there exists an open set $G$ in $Y$ containing $f(x)$ such that $cl(G) \subset V$. Since $f$ is $(\hat{g}, s)$-continuous, there exists $U \in \hat{G}O(X, x)$ such that $f(U) \subset cl(G)$. Then $f(U) \subset cl(G) \subset V$. Hence, $f$ is $\hat{g}$-continuous. ∎

## 6. Fundamental properties

**Definition 13.** *A space $X$ is said to be*

1. $\hat{g}$-$T_2$ *if for each pair of distinct points $x$ and $y$ in $X$, there exist $U \in \hat{G}O(X, x)$ and $V \in \hat{G}O(X, y)$ such that $U \cap V = \phi$.*

2. $\hat{g} - T_1$ *if for each pair of distinct points $x$ and $y$ in $X$, there exist $\hat{g}$-open sets $U$ and $V$ containing $x$ and $y$, respectively, such that $y \notin U$ and $x \notin V$.*

**Remark 3.** The following implications are hold for a topological space $X$.

1. $T_2 \Rightarrow \hat{g} - T_2$.
2. $T_1 \Rightarrow \hat{g} - T_1$.

These implications are not reversible.

**Example 5.** Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. Then $X$ is both $\hat{g} - T_2$ and $\hat{g} - T_1$ but it is neither $T_1$ nor $T_2$.

**Theorem 15.** *The following properties hold for a function $f : X \to Y$:*

1. *If $f$ is a $(\hat{g}, s)$-continuous injection and $Y$ is s-Urysohn, then $X$ is $\hat{g} - T_2$.*
2. *If $f$ is a $(\hat{g}, s)$-continuous injection and $Y$ is weakly Hausdorff, then $X$ is $\hat{g} - T_1$.*

**Proof.** (1): Let $Y$ be $s$-Urysohn. By the injectivity of $f$, $f(x) \neq f(y)$ for any distinct points $x$ and $y$ in $X$. Since $Y$ is $s$-Urysohn, there exist $A \in SO(Y, f(x))$ and $B \in SO(Y, f(y))$ such that $cl(A) \cap cl(B) = \phi$. Since $f$ is a $(\hat{g}, s)$-continuous, by Theorem 7, there exist $\hat{g}$-open sets $C$ and $D$ in $X$ containing $x$ and $y$, respectively, such that $f(C) \subset cl(A)$ and $f(D) \subset cl(B)$ such that $C \cap D = \phi$. Thus, $X$ is $\hat{g} - T_2$.

(2): Let $Y$ be weakly Hausdorff. For $x \neq y$ in $X$, there exist $A, B \in RC(Y)$ such that $f(x) \in A$, $f(y) \notin A$, $f(x) \notin B$ and $f(y) \in$B. Since $f$ is $(\hat{g}, s)$-continuous, $f^{-1}(A)$ and f$^{-1}$(B) are $\hat{g}$-open subsets of $X$ such that $x \in f^{-1}(A)$, $y \notin f^{-1}(A)$, $x \notin f^{-1}(B)$ and $y \in f^{-1}(B)$. Hence, $X$ is $\hat{g} - T_1$. ■

**Theorem 16.** *A space $X$ is $\hat{g} - T_2$ if and only if any pair of distinct points $x$, $y$ of $X$ there exist $\hat{g}$-open sets $U$ and $V$ such that $x \in U$ and $y \in V$ and $\hat{g} - cl(U) \cap \hat{g} - cl(V) = \phi$.*

**Proof.** In $\hat{g} - T_2$, $\hat{g}$-open sets and $\hat{g}$-closed sets are coincide. ■

**Definition 14.** *A graph $G(f)$ of a function $f : X \to Y$ is said to be $(\hat{g}, s)$-graph if there exist a $\hat{g}$-open set $A$ in $X$ containing $x$ and a semi-open set $B$ in $Y$ containing $y$ such that $(A \times cl(B)) \cap G(f) = \phi$ for each $(x, y) \in (X \times Y) \backslash G(f)$.*

**Proposition 2.** *The following properties are equivalent for a function $f : X \to Y$:*

1. *$G(f)$ is $(\hat{g}, s)$-graph,*

2. *For each $(x, y) \in (X \times Y)\backslash G(f)$, there exist a $\hat{g}$-open set $A$ in $X$ containing $x$ and a semi-open set $B$ in $Y$ containing $y$ such that $f(A) \cap cl(B) = \phi$.*

3. *For each $(x, y) \in (X \times Y)\backslash G(f)$, there exist a $\hat{g}$-open set $A$ in $X$ containing $x$ and a regular closed set $K$ in $Y$ containing $y$ such that $f(A) \cap K = \phi$.*

**Definition 15.** *A subset $S$ of a space $X$ is said to be $S$-closed relative to $X$ [32] if for every cover $\{A_i : i \in I\}$ of $S$ by semi-open sets of $X$, there exists a finite subset $I_0$ of $I$ such that $S \subset \cup\{cl(A_i) : i \in I_0\}$.*

**Theorem 17.** *If a function $f : X \to Y$ has a $(\hat{g}, s)$-graph and the collection of $\hat{g}$-closed sets of $X$ is closed under arbitrary intersections, then $f^{-1}(A)$ is $\hat{g}$-closed in $X$ for every subset $A$ which is $S$-closed relative to $Y$.*

**Proof.** Suppose that $A$ is $S$-closed relative to $Y$ and $x \notin f^{-1}(A)$. We have $(x, y) \in (X \times Y)\backslash G(f)$ for each $y \in A$ and there exist $\hat{g}$-open set $B_y$ containing $x$ and a semi-open set $C_y$ containing $y$ such that $f(B_y) \cap cl(C_y)) = \phi$. Since $\{C_y : y \in A\}$ is a cover by semi-open sets of $Y$, there exists a finite subset $\{y_1, y_2, \ldots, y_n\}$ of $A$ such that $A \subset \cup\{cl(C_{yi}) : i = 1, 2, ..., n\}$. Take $B = \cap\{B_{yi} : i = 1, 2, \ldots, n\}$. Then $B$ is a $\hat{g}$-open containing $x$ and $f(B) \cap A = \phi$. Thus, $B \cap f^{-1}(A) = \phi$ and hence $f^{-1}(A)$ is $\hat{g}$-closed in $X$. ■

**Theorem 18.** *Let $f : X \to Y$ be a $(\hat{g}, s)$-continuous function. Then the following properties hold:*

1. *$G(f)$ is a $(\hat{g}, s)$-graph if $Y$ is an $s$-Urysohn.*

2. *$f$ is almost $\hat{g}$-continuous if $Y$ is $s$-Urysohn $S$-closed space and $\hat{G}C(X)$ is closed under arbitrary intersections.*

**Proof.** (1): Let $Y$ be $s$-Urysohn and $(x, y) \in (X \times Y)\backslash G(f)$. Then $f(x) \neq y$. Since $Y$ is $s$-Urysohn, there exist $M \in SO(Y, f(x))$ and $N \in SO(Y, y)$ such that $cl(M) \cap cl(N) = \phi$. Since $f$ is $(\hat{g}, s)$-continuous, by theorem there exists $\hat{g}$-open set $A$ in $X$ containing $x$ such that $f(A) \subset cl(M)$. Hence, $f(A) \cap cl(N) = \phi$ and $G(f)$ is $(\hat{g}, s)$-graph in $X \times Y$.

(2): Let $F$ be a regular closed set in $Y$. By Theorem 3.3 and 3.4 [32], $F$ is $S$-closed relative to $Y$. Hence, by Theorem 17 and (1), $f^{-1}(F)$ is $\hat{g}$-closed in $X$ and hence $f$ is almost $\hat{g}$-continuous. ■

**Theorem 19.** *Let $f, g : X \to Y$ be functions and $\hat{g} - cl(S)$ be $\hat{g}$-closed for each $S \subset X$. If*

1. *$f$ and $g$ are $(\hat{g}, s)$-continuous,*

2. *$Y$ is $s$-Urysohn,*
*then $A = \{x \in X : f(x) = g(x)\}$ is $\hat{g}$-closed in $X$.*

**Proof.** Let $x \in X \backslash A$, then it follows that $f(x) \neq g(x)$. Since $Y$ is $s$-Urysohn, there exist $M \in SO(Y, f(x))$ and $N \in SO(Y, g(x))$ such that $cl(M) \cap cl(N) = \phi$. Since $f$ and $g$ are $(\hat{g}, s)$-continuous, there exist $\hat{g}$-open sets $U$ and $V$ containing $x$ such that $f(U) \subset cl(M)$ and $g(V) \subset cl(N)$. Hence, $U \cap V = P \in \hat{G}O(X)$, $f(P) \cap g(P) = \phi$ and then $x \notin \hat{g} - cl(A)$. Thus, $A$ is $\hat{g}$-closed in $X$. ■

**Definition 16.** *A subset $A$ of a topological space $X$ is said to be $\hat{g}$-dense in $X$ if $\hat{g} - cl(A) = X$.*

**Theorem 20.** *Let $f, g : X \to Y$ be functions and $\hat{g} - cl(S)$ be $\hat{g}$-closed for each $S \subset X$. If*
1. *$Y$ is $s$-Urysohn,*
2. *$f$ and $g$ are $(\hat{g}, s)$-continuous,*
3. *$f = g$ on $\hat{g}$-dense set $A \subset X$, then $f = g$ on $X$.*

**Proof.** Since $f$ and $g$ are $(\hat{g}, s)$-continuous and $Y$ is $s$-Urysohn, by Theorem 19 $B = \{x \in X : f(x) = g(x)\}$ is $\hat{g}$-closed in $X$. We have $f = g$ on $\hat{g}$-dense set $A \subset X$. Since $A \subset B$ and $A$ is $\hat{g}$-dense set in $X$, then $X = \hat{g} - cl(A) \subset \hat{g} - cl(B) = B$. Hence $f = g$ on $X$. ■

**Definition 17.** *A space $X$ is said to be*
1. *countably $\hat{g}$-compact if every countable cover of $X$ by $\hat{g}$-open sets has a finite subcover,*
2. *$\hat{g}$-Lindelof if every $\hat{g}$-open cover of $X$ has a countable subcover.*

**Theorem 21.** *Let $f : X \to Y$ be a $(\hat{g}, s)$-continuous surjection. Then the following statements hold:*
1. *if $X$ is $\hat{g}$-Lindelof, then $Y$ is $S$-Lindelof.*
2. *if $X$ is countably $\hat{g}$-compact, then $Y$ is countably $S$-closed.*

**Definition 18.** *A space $X$ is called $\hat{g}$-connected if $X$ is not the union of two disjoint nonempty $\hat{g}$-open sets.*

**Example 6.** Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. Then $X$ is not $\hat{g}$-connected.

**Example 7.** Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then $X$ is $\hat{g}$-connected.

**Theorem 22.** *Let $f : X \to Y$ be a $(\hat{g}, s)$-continuous surjection. If $X$ is $\hat{g}$-connected, then $Y$ is connected.*

**Proof.** Assume that $Y$ is not connected space. Then there exist nonempty disjoint open sets $A$ and $B$ such that $Y = A \cup B$. Also $A$ and $B$ are clopen in $Y$. Since $f$ is $(\hat{g}, s)$-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are $\hat{g}$-open in $X$. Moreover $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty disjoint and $X = f^{-1}(A) \cup f^{-1}(B)$. This shows that $X$ is not $\hat{g}$-connected. This contradicts the assumption that $Y$ is not connected. ■

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