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COMMON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACE USING GENERAL CONTRACTIVE CONDITION OF INTEGRAL TYPE

Abstract. The aim of this paper is to prove some common fixed point theorems for six discontinuous mappings in non complete intuitionistic fuzzy metric spaces using contractive condition of integral type.

Key words: intuitionistic fuzzy metric space, weakly compatible mapping, common fixed point.

AMS Mathematics Subject Classification: 47H10, 54H25.

1. Introduction

Motivated by the potential applicability of fuzzy topology to quantum particle physics particularly in connection with both string and $e^{(\infty)}$ theory developed by El Naschie [6], [7], Park introduced and discussed in [21] a notion of intuitionistic fuzzy metric spaces which is based on the idea of intuitionistic fuzzy sets due to Atanassov [2] and the concept of fuzzy metric space given by George and Veeramani [11]. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study the relationship between two probability functions. It has direct physics motivation in the context of the two-slit experiment as the foundation of E-infinity of high energy physics, recently studied by El Naschie [8], [9].

Alaca et al. [1] using the idea of intuitionistic fuzzy sets, they defined the notion of intuitionistic fuzzy metric space as Park [21] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach [3] and Edelstein [5] extended to intuitionistic fuzzy metric spaces with the help of Grabiec [10]. Turkoglu et al. [25] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces

and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β). Sharma and Tilwankar [24] and Kutukcu [18] proved fixed point theorems for multivalued mappings in intuitionistic fuzzy metric spaces.
Several authors [12], [13], [15], [23] proved some fixed point theorems for various generalizations of contraction mappings in probabilistic and fuzzy metric space. Branciari [4] obtained a fixed point theorem for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. Sedghi et al. [22] established a common fixed point theorem for weakly compatible mappings in intuitionistic fuzzy metric space satisfying a contractive condition of integral type. Muralisankar et al. [20] proved a common fixed point theorem in an intuitionistic fuzzy metric space for pointwise R-weakly commuting mappings using contractive condition of integral type and established a situation in which a collection of maps has a fixed point which is a point of discontinuity.
In this paper, we prove some common fixed point theorems for six mappings by using contractive condition of integral type for class of weakly compatible maps in noncomplete intuitionistic fuzzy metric spaces, without taking any continuous mapping. We improve and extend the results of Muralisankar and Kalpana [20].

2. Preliminaries

Definition 1 ([23]). *A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is continuous t-norm if* $*$ *is satisfying the following conditions:*

(*i*) $*$ *is commutative and associative,*
(*ii*) $*$ *is continuous,*
(*iii*) $a * 1 = a$ *for all* $a \in [0,1]$,
(*iv*) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$, $a, b, c, d \in [0,1]$.

Definition 2 ([23]). *A binary operation* $\diamond : [0,1] \times [0,1] \to [0,1]$ *is continuous t-conorm if* $\diamond$ *is satisfying the following conditions:*

(*i*) $\diamond$ *is commutative and associative,*
(*ii*) $\diamond$ *is continuous,*
(*iii*) $a \diamond 0 = a$ *for all* $a \in [0,1]$,
(*iv*) $a \diamond b \leq c \diamond d$ *whenever* $a \leq c$ *and* $b \leq d$, $a, b, c, d \in [0,1]$.

Remark 1. The concept of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [19] in his study of statistical metric spaces. Several examples for these concepts were proposed by many authors [16], [26].

Definition 3 ([1]). *A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric spaces if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,*

(*i*) $M(x, y, t) + N(x, y, t) \leq 1$,
(*ii*) $M(x, y, 0) = 0$,
(*iii*) $M(x, y, t) = 1$ *for all* $t > 0$ *if and only if* $x = y$,
(*iv*) $M(x, y, t) = M(y, x, t)$,
(*v*) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(*vi*) $M(x, y, .) : [0, \infty) \to [0, 1]$ *is left continuous,*
(*vii*) $\lim_{t\to\infty} M(x, y, t) = 1$ *for all* x, y *in* X,
(*viii*) $N(x, y, 0) = 1$,
(*ix*) $N(x, y, t) = 0$ *for all* $t > 0$ *if and only if* $x = y$,
(*x*) $N(x, y, t) = N(y, x, t)$,
(*xi*) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
(*xii*) $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ *is right continuous,*
(*xiii*) $\lim_{t\to\infty} N(x, y, t) = 0$ *for all* $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated, i.e., $x \diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Example 1. Let (X, d) be a metric space. Define t-norm $a*b = \min\{a, b\}$ and t-conorm $a \diamond b = \max\{a, b\}$ and for all $x, y \in X$ and t >0,

$$M_d(x, y, t) = \frac{t}{t\ +\ d(x,\ y)}, \qquad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)} \tag{2a}$$

Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric. On the other hand, note that there exists no metric on X satisfying $(2a)$.

Remark 3. In intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all x, $y \in X$.

Definition 4 ([1]). *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then*

(*i*) *A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim\limits_{n\to\infty} x_n = x$) if, for all t >0,*

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \qquad \lim_{n\to\infty} N(x_n, x, t) = 0.$$

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1, \qquad \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

Remark 4. Since $*$ and $\diamond$ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

Definition 5 ([1])**.** *An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.*

Lemma 1 ([1])**.** *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that*

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t), \quad N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all t >0 and $n = 1, 2, \ldots\ldots,$ then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2 ([1])**.** *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0, 1)$,*

$$M(x, y, kt) \geq M(x, y, t) \quad \text{and} \quad N(x, y, kt) \leq N(x, y, t),$$

then $x = y$.

Definition 6 ([14])**.** *Two self mappings S and T are said to be weakly compatible if they commute at their coincidence points; i.e., if $Tu = Su$ for some $u \in X$, then $TSu = STu$.*

In this paper, we prove some common fixed point theorems for six mappings by using contractive condition of integral type for class of weakly compatible maps in non complete intuitionistic fuzzy metric spaces, without taking any continuous mapping. We improve and extend the results of Muralisankar and Kalpana [20].

3. Main result

Theorem 1. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamond$ defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0, 1]$. Let A, B, S, T, P and Q be mappings from X into itself such that*

(a) $P(X) \subset AB(X)$ and $Q(X) \subset ST(X)$

(b) there exists a constant $k \in (0, 1)$ such that

$$\int_0^{M(Px,Qy,kt)} \varphi(t)dt \geq \int_0^{m(x,y,t)} \varphi(t)dt$$

and

$$\int_0^{N(Px,Qy,kt)} \varphi(t)dt \le \int_0^{n(x,y,t)} \varphi(t)dt$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ *is a Lebesque-integrable mapping which is summable, nonnegative, and such that*

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \quad \text{for each } \varepsilon > 0,$$

where

$$\begin{aligned} m(x,y,t) \; = \; & \min\{M(ABy,Qy,t), M(STx,Px,t), M(STx,Qy,\alpha t), \\ & M(ABy,Px,(2-\alpha)t), M(ABy,STx,t)\} \end{aligned}$$

and

$$\begin{aligned} n(x,y,t) \; = \; & \max\{N(ABy,Qy,t), N(STx,Px,t), N(STx,Qy,\alpha t), \\ & N(ABy,Px,(2-\alpha)t), N(ABy,STx,t)\} \end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ *and* $t > 0$, *and*

(c) *if one of* $P(X)$, $AB(X)$, $ST(X)$ *or* $Q(X)$ *is a complete subspace of* X, *then*

(i) P *and* ST *have a coincidence point, and*

(ii) Q *and* AB *have a coincidence point.*

Further, if

(d) $AB = BA$, $QB = BQ$, $QA = AQ$, $PT = TP$, $ST = TS$, *and*

(e) *the pair* $\{P, ST\}$ *is weakly compatible,*

then

(iii) A, B, S, T, P *and* Q *have a unique common fixed point in* X.

Proof. By (a), since $P(X) \subset AB(X)$, for any point $x_0 \in X$, there exists a point $x_1 \in X$ such that $Px_0 = ABx_1$. Since $Q(X) \subset ST(X)$, for this point x_1 we can choose a point $x_2 \in X$ such that $Qx_1 = STx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that for $n = 0, 1, \ldots,$

$$y_{2n} = Px_{2n} = ABx_{2n+1}$$

and

$$y_{2n+1} = Qx_{2n+1} = STx_{2n+2}.$$

By (b), for all $t > 0$ and $\alpha = 1 - q$, with $q \in (0,1)$, we have

$$\begin{aligned} \int_0^{M(y_{2n+1},y_{2n+2},kt)} \varphi(t)dt \; &= \int_0^{M(Qx_{2n+1},Px_{2n+2},kt)} \varphi(t)dt \\ &= \int_0^{M(Px_{2n+2},Qx_{2n+1},kt)} \varphi(t)dt \\ &\ge \int_0^{m(x_{2n+2},x_{2n+1},t)} \varphi(t)dt, \end{aligned}$$

and

$$\begin{aligned}\int_0^{N(y_{2n+1},y_{2n+2},kt)} \varphi(t)dt &= \int_0^{N(Qx_{2n+1},Px_{2n+2},kt)} \varphi(t)dt \\ &= \int_0^{N(Px_{2n+2},Qx_{2n+1},kt)} \varphi(t)dt \\ &\leq \int_0^{n(x_{2n+2},x_{2n+1},t)} \varphi(t)dt,\end{aligned}$$

$$\begin{aligned}m(x_{2n+2}, x_{2n+1}, t) = \min\{&M(ABx_{2n+1}, Qx_{2n+1}, t), \\ &M(STx_{2n+2}, Px_{2n+2}, t), M(STx_{2n+2}, Qx_{2n+1}, \alpha t), \\ &M(ABx_{2n+1}, Px_{2n+2}, (2-\alpha)t), \\ &M(ABx_{2n+1}, STx_{2n+2}, t)\} \\ = \min\{&M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), \\ &M(y_{2n+1}, y_{2n+1}, \alpha t), M(y_{2n}, y_{2n+2}, (1+q)t), \\ &M(y_{2n}, y_{2n+1}, t)\} \\ \geq \min\{&M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), 1, \\ &M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, qt), \\ &M(y_{2n}, y_{2n+1}, t)\} \\ \geq \min\{&M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), \\ &M(y_{2n+1}, y_{2n+2}, qt)\}\end{aligned} \tag{1a}$$

and

$$\begin{aligned}n(x_{2n+2}, x_{2n+1}, t) = \max\{&N(ABx_{2n+1}, Qx_{2n+1}, t), \\ &N(STx_{2n+2}, Px_{2n+2}, t), N(STx_{2n+2}, Qx_{2n+1}, \alpha t), \\ &N(ABx_{2n+1}, Px_{2n+2}, (2-\alpha)t), \\ &N(ABx_{2n+1}, STx_{2n+2}, t)\} \\ = \max\{&N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), \\ &N(y_{2n+1}, y_{2n+1}, \alpha t), N(y_{2n}, y_{2n+2}, (1+q)t), \\ &N(y_{2n}, y_{2n+1}, t)\} \\ \leq \max\{&N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), 0, \\ &N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, qt), \\ &N(y_{2n}, y_{2n+1}, t)\} \\ \leq \max\{&N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), \\ &N(y_{2n+1}, y_{2n+2}, qt)\}.\end{aligned} \tag{1b}$$

Since the t-norm $*$ and t-conorm $\diamond$ are continuous, $M(x,y,\cdot)$ is left continuous and $N(x,y,.)$ is right continuous, letting $q \to 1$ in $(1a)$ and $(1b)$, we have

$$m(x_{2n+2}, x_{2n+1}, t) \geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)\}$$

and

$$n(x_{2n+2}, x_{2n+1}, t) \leq \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t)\}.$$

Therefore,

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \varphi(t)dt \geq \int_0^{\min\{M(y_{2n},y_{2n+1},t),M(y_{2n+1},y_{2n+2},t)\}} \varphi(t)dt,$$

and

$$\int_0^{N(y_{2n+1},y_{2n+2},kt)} \varphi(t)dt \leq \int_0^{\max\{N(y_{2n},y_{2n+1},\ t),N(y_{2n+1},y_{2n+2},t)\}} \varphi(t)dt.$$

Similarly, we also have

$$\int_0^{M(y_{2n+2},y_{2n+3},kt)} \varphi(t)dt \geq \int_0^{\min\{M(y_{2n+1},y_{2n+2},t),M(y_{2n+2},y_{2n+3},t)\}} \varphi(t)dt,$$

and

$$\int_0^{N(y_{2n+2},y_{2n+3},kt)} \varphi(t)dt \leq \int_0^{\max\{N(y_{2n+1},y_{2n+2},t),N(y_{2n+2},y_{2n+3},t)\}} \varphi(t)dt.$$

In general, we have for $n = 1, 2, \ldots$

$$\int_0^{M(y_{n+1},y_{n+2},kt)} \varphi(t)dt \geq \int_0^{\min\{M(y_n,y_{n+1},t),M(y_{n+1},y_{n+2},t)\}} \varphi(t)\ dt,$$

and

$$\int_0^{N(y_{n+1},y_{n+2},kt)} \varphi(t)dt \leq \int_0^{\max\{N(y_n,y_{n+1},t),N(y_{n+1},y_{n+2},t)\}} \varphi(t)dt.$$

Consequently, it follows that for $n = 1, 2, \ldots, p = 1, 2, \ldots.$

$$\int_0^{M(y_{n+1},y_{n+2},kt)} \varphi(t)dt \geq \int_0^{\min\{M(y_n,y_{n+1},t),M(y_{n+1},y_{n+2},t/k^p)\}} \varphi(t)dt,$$

and

$$\int_0^{N(y_{n+1},y_{n+2},kt)} \varphi(t)dt \leq \int_0^{\max\{N(y_n,y_{n+1},t),N(y_{n+1},y_{n+2},t/k^p)\}} \varphi(t)dt.$$

By noting that $M(y_{n+1}, y_{n+2}, t/k^p) \to 1$ and $N(y_{n+1}, y_{n+2}, t/k^p) \to 0$ as $p \to \infty$, we have for $n = 1, 2, \ldots$

$$\int_0^{M(y_{n+1},y_{n+2},kt)} \varphi(t)dt \geq \int_0^{M(y_n,y_{n+1},t)} \varphi(t)dt,$$

and

$$\int_0^{N(y_{n+1},y_{n+2},kt)} \varphi(t)dt \leq \int_0^{N(y_n,y_{n+1},t)} \varphi(t)dt.$$

Hence by Lemma 1, $\{y_n\}$ is a Cauchy sequence. Now suppose $ST(X)$ is complete. Note that the subsequence $\{y_{2n+1}\}$ is contained in $ST(X)$ and has a limit in $ST(X)$. Call it z. Let $u \in ST^{-1}z$. Then $STu = z$. We shall use the fact that the subsequence $\{y_{2n}\}$ also converges to z. By (b), we have

$$\begin{aligned}\int_0^{M(Pu,y_{2n+1},kt)} \varphi(t)dt &= \int_0^{M(Pu,Qx_{2n+1},kt)} \varphi(t)dt \\ &\geq \int_0^{m(u,\ x_{2n+1},\ t)} \varphi(t)dt,\end{aligned}$$

and

$$\begin{aligned}\int_0^{N(Pu,y_{2n+1},kt)} \varphi(t)dt &= \int_0^{N(Pu,Qx_{2n+1},kt)} \varphi(t)dt \\ &\leq \int_0^{n(u,x_{2n+1},t)} \varphi(t)dt.\end{aligned}$$

Take $\alpha = 1$,

$$\begin{aligned}m(u, x_{2n+1}, t) &= \min\{M(ABx_{2n+1}, Qx_{2n+1}, t), M(STu, Pu, t), \\ &\qquad M(STu, Qx_{2n+1}, t), M(ABx_{2n+1}, Pu, t), \\ &\qquad M(ABx_{2n+1}, STu, t)\} \\ &= \min\{M(y_{2n}, y_{2n+1}, t), M(z, Pu, t), M(z, y_{2n+1}, t), \\ &\quad M(y_{2n}, Pu, t), M(y_{2n}, z, t)\}\end{aligned}$$

and

$$\begin{aligned}n(u, x_{2n+1}, t) &= \max\{N(ABx_{2n+1}, Qx_{2n+1}, t), N(STu, Pu, t), \\ &\qquad N(STu, Qx_{2n+1}, t), N(ABx_{2n+1}, Pu, t), \\ &\qquad N(ABx_{2n+1}, STu, t)\} \\ &= \max\{N(y_{2n}, y_{2n+1}, t), N(z, Pu, t), N(z, y_{2n+1}, t), \\ &\qquad N(y_{2n}, Pu, t), N(y_{2n}, z, t)\}\end{aligned}$$

which implies that, as $n \to \infty$

$$\begin{aligned} m(u, x_{2n+1}, t) &= \min\{1, M(z, Pu, t), 1, M(z, Pu, t), 1\} \\ &= M(z, Pu, t) \end{aligned}$$

and

$$\begin{aligned} n(u, x_{2n+1}, t &= \max\{0, N(z, Pu, t), 0, N(z, Pu, t), 0\} \\ &= N(z, Pu, t) \end{aligned}$$

Therefore,

$$\int_0^{M(Pu,z,kt)} \varphi(t)dt \geq \int_0^{M(Pu,z,t)} \varphi(t)dt$$

and

$$\int_0^{N(Pu,z,kt)} \varphi(t)dt \leq \int_0^{N(Pu,z,t)} \varphi(t)dt.$$

Therefore, by Lemma 2, we have $Pu = z$. Since $STu = z$ thus $Pu = z = STu$, i.e. u is a coincidence point of P and ST. This proves (i).
Since $P(X) \subset AB(X)$, $Pu = z$ implies that $z \in AB(X)$. Let $v \in AB^{-1}z$. Then $ABv = z$. By (b), we have

$$\begin{aligned} \int_0^{M(y_{2n},Qv,kt)} \varphi(t)dt &= \int_0^{M(Px_{2n},Qv,kt)} \varphi(t)dt, \\ &\geq \int_0^{m(x_{2n},v,t)} \varphi(t)dt, \end{aligned}$$

and

$$\begin{aligned} \int_0^{N(y_{2n},Qv,kt)} \varphi(t)dt &= \int_0^{N(Px_{2n},Qv,kt)} \varphi(t)dt, \\ &\leq \int_0^{n(x_{2n},v,t)} \varphi(t)dt. \end{aligned}$$

Take $\alpha = 1$,

$$\begin{aligned} m(x_{2n}, v, t) &= \min\{M(ABv, Qv, t), M(STx_{2n}, Px_{2n}, t), \\ &\quad M(STx_{2n}, Qv, t), M(ABv, Px_{2n}, t), \\ &\quad M(ABv, STx_{2n}, t)\} \\ &= \min\{M(z, Qv, t), M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, Qv, t), \\ &\quad M(z, y_{2n}, t), M(z, y_{2n-1}, t)\} \end{aligned}$$

and

$$\begin{aligned} n(x_{2n}, v, t) &= \max\{N(ABv, Qv, t), N(STx_{2n}, Px_{2n}, t), \\ &\qquad N(STx_{2n}, Qv, t), N(ABv, Px_{2n}, t), \\ &\qquad N(ABv, STx_{2n}, t)\} \\ &= \max\{N(z, Qv, t), N(y_{2n-1}, y_{2n}, t), N(y_{2n-1}, Qv, t), \\ &\qquad N(z, y_{2n}, t), N(z, y_{2n-1}, t)\} \end{aligned}$$

which implies that, as $n \to \infty$ and

$$\begin{aligned} m(x_{2n}, v, t) &= \min\{M(z, Qv, t), 1, M(z, Qv, t), 1, 1\} \\ &= M(z, Qv, t) \end{aligned}$$

and

$$\begin{aligned} n(x_{2n}, v, t) &= \max\{N(z, Qv, t), 0, N(z, Qv, t), 0, 0\} \\ &= N(z, Qv, t). \end{aligned}$$

Therefore,

$$\int_0^{M(z,Qv,kt)} \varphi(t)dt \geq \int_0^{M(z,Qv,t)} \varphi(t)dt$$

and

$$\int_0^{N(z,Qv,kt)} \varphi(t)dt \leq \int_0^{N(z,Qv,t)} \varphi(t)dt.$$

Therefore, by Lemma 2, we have $Qv = z$. Since $ABv = z$ thus $Qv = z = ABv$, i.e., v is a coincidence point of Q and AB. This proves (ii).

The remaining two cases pertain essentially to the previous cases. Indeed if $P(X)$ or $Q(X)$ is complete, then by (a) $z \in P(X) \subset AB(X)$ or $z \in Q(X) \subset ST(X)$. Thus $(i)$and (ii) are completely established.

Since the pair $\{P, ST\}$ is weakly compatible therefore P and ST commute at their coincidence point, i.e., $P(STu) = (ST)Pu$ or $Pz = STz$. By (d), we have

$$Q(ABv) = (AB)Qv \quad \text{or} \quad Qz = ABz.$$

Now, we prove that $Pz = z$, by (b), we have

$$\begin{aligned} \int_0^{M(Pz,y_{2n+1},kt)} \varphi(t)dt &= \int_0^{M(Pz,Qx_{2n+1},kt)} \varphi(t)dt, \\ &\geq \int_0^{m(z,x_{2n+1},t)} \varphi(t)dt, \end{aligned}$$

and

$$\int_0^{N(Pz,y_{2n+1},kt)} \varphi(t)dt = \int_0^{N(Pz,Qx_{2n+1},kt)} \varphi(t)dt \leq \int_0^{n(z,x_{2n+1},t)} \varphi(t)dt.$$

Take $\alpha = 1$,

$$\begin{aligned} m(z, x_{2n+1}, t) &= \min\{M(ABx_{2n+1}, Qx_{2n+1}, t), M(STz, Pz, t), \\ &\qquad M(STz, Qx_{2n+1}, t), M(ABx_{2n+1}, Pz, t), \\ &\qquad M(ABx_{2n+1}, STz, t)\} \\ &= \min\{M(y_{2n}, y_{2n+1}, t), M(Pz, Pz, t), M(Pz, y_{2n+1}, t), \\ &\qquad M(y_{2n}, Pz, t), M(y_{2n}, Pz, t)\} \end{aligned}$$

and

$$\begin{aligned} n(z, x_{2n+1}, t) &= \max\{N(ABx_{2n+1}, Qx_{2n+1}, t), N(STz, Pz, t), \\ &\qquad N(STz, Qx_{2n+1}, t), N(ABx_{2n+1}, Pz, t), \\ &\qquad N(ABx_{2n+1}, STz, t)\} \\ &= \max\{N(y_{2n}, y_{2n+1}, t), N(Pz, Pz, t), N(Pz, y_{2n+1}, t), \\ &\qquad N(y_{2n}, Pz, t), N(y_{2n}, Pz, t)\}. \end{aligned}$$

Proceeding limit as $n \to \infty$, we have

$$\begin{aligned} m(z, x_{2n+1}, t) &= \min\{1, 1, M(Pz, z, t), M(z, Pz, t), M(z, Pz, t)\} \\ &= M(Pz, z, t) \end{aligned}$$

and

$$\begin{aligned} n(z, x_{2n+1}, t) &= \max\{0, 0, N(Pz, z, t), N(z, Pz, t), N(z, Pz, t)\} \\ &= N(Pz, z, t). \end{aligned}$$

Therefore,

$$\int_0^{M(Pz,z,kt)} \varphi(t)dt \geq \int_0^{M(Pz,z,t)} \varphi(t)dt,$$

and

$$\int_0^{N(Pz,z,kt)} \varphi(t)dt \leq \int_0^{N(Pz,z,t)} \varphi(t)dt.$$

Therefore, by Lemma 2, we have $Pz = z$ so $Pz = STz = z$. By (b), we have

$$\begin{aligned}\int_0^{M(y_{2n+2},Qz,kt)} \varphi(t)dt &= \int_0^{M(Px_{2n+2},Qz,kt)} \varphi(t)dt,\\ &\geq \int_0^{m(x_{2n+2},z,t)} \varphi(t)dt,\end{aligned}$$

and

$$\begin{aligned}\int_0^{N(y_{2n+2},Qz,kt)} \varphi(t)dt &= \int_0^{N(Px_{2n+2},Qz,kt)} \varphi(t)dt,\\ &\leq \int_0^{n(x_{2n+2},z,t)} \varphi(t)dt.\end{aligned}$$

Take $\alpha = 1$,

$$\begin{aligned}m(x_{2n+2},z,t) &= \min\{M(ABz,Qz,t), M(STx_{2n+2},Px_{2n+2},t),\\ &\quad M(STx_{2n+2},Qz,t), M(ABz,Px_{2n+2},t),\\ &\quad M(ABz,STx_{2n+2},t)\}\\ &= \min\{M(Qz,Qz,t), M(y_{2n+1},y_{2n+2},t), M(y_{2n+1},Qz,t),\\ &\quad M(Qz,y_{2n+2},t), M(Qz,y_{2n+1},t)\}\end{aligned}$$

and

$$\begin{aligned}n(x_{2n+2},z,t) &= \max\{N(ABz,Qz,t), N(STx_{2n+2},Px_{2n+2},t),\\ &\quad N(STx_{2n+2},Qz,t), N(ABz,Px_{2n+2},t),\\ &\quad N(ABz,STx_{2n+2},t)\}\\ &= \max\{N(Qz,Qz,t), N(y_{2n+1},y_{2n+2},t), N(y_{2n+1},Qz,t),\\ &\quad N(Qz,y_{2n+2},t), N(Qz,y_{2n+1},t)\}.\end{aligned}$$

Proceeding limit as $n \to \infty$, we have

$$\begin{aligned}m(x_{2n+2},z,t) &= \min\{1,1,M(z,Qz,t), M(Qz,z,t), M(Qz,z,t)\}\\ &= M(z,Qz,t)\end{aligned}$$

and

$$\begin{aligned}n(x_{2n+2},z,t) &= \max\{0,0,N(z,Qz,t), N(Qz,z,t), N(Qz,z,t)\}\\ &= N(z,Qz,t).\end{aligned}$$

Therefore,

$$\int_0^{M(z,Qz,kt)} \varphi(t)dt \geq \int_0^{M(z,Qz,t)} \varphi(t)dt,$$

and

$$\int_0^{N(z,Qz,kt)} \varphi(t)dt \le \int_0^{N(z,Qz,t)} \varphi(t)dt.$$

Therefore, by Lemma 2, we have $Qz = z$ so $Qz = ABz = z$. By (b) and using (d), we have

$$\begin{aligned}\int_0^{M(z,Bz,kt)} \varphi(t)dt &= \int_0^{M(Pz,\ Q(Bz),kt)} \varphi(t)dt,\\ &\ge \int_0^{m(z,Bz,t)} \varphi(t)dt,\end{aligned}$$

and

$$\begin{aligned}\int_0^{N(z,Bz,kt)} \varphi(t)dt &= \int_0^{N(Pz,Q(Bz),kt)} \varphi(t)dt,\\ &\le \int_0^{n(z,Bz,t)} \varphi(t)dt.\end{aligned}$$

Take $\alpha = 1$,

$$\begin{aligned}m(z,Bz,t) &= \min\{M(AB(Bz),Q(Bz),t), M(STz,Pz,t),\\ &\qquad M(STz,Q(Bz),t), M(ABz,Pz,t),\\ &\qquad M(AB(Bz),STz,t)\}\\ &= \min\{M(Bz,Bz,t), M(z,z,t), M(z,Bz,t), M(z,z,t),\\ &\qquad M(Bz,z,t)\}\\ &= \min\{1,1,M(z,Bz,t),1,M(Bz,z,t)\}\\ &= M(z,Bz,t)\end{aligned}$$

and

$$\begin{aligned}n(z,Bz,t) &= \max\{N(AB(Bz),Q(Bz),t), N(STz,Pz,t),\\ &\qquad N(STz,Q(Bz),t), N(ABz,Pz,t),\\ &\qquad N(AB(Bz),STz,t)\}\\ &= \max\{N(Bz,Bz,t), N(z,z,t), N(z,Bz,t), N(z,z,t),\\ &\qquad N(Bz,z,t)\}\\ &= \max\{0,0,N(z,Bz,t),0,N(Bz,z,t)\}\\ &= N(z,Bz,t).\end{aligned}$$

Therefore,

$$\int_0^{M(z,Bz,kt)} \varphi(t)dt \ge \int_0^{M(z,Bz,t)} \varphi(t)dt,$$

and

$$\int_0^{N(z,Bz,kt)} \varphi(t)dt \leq \int_0^{N(z,Bz,t)} \varphi(t)dt.$$

Therefore, by Lemma 2, we have $Bz = z$. Since $ABz = z$, therefore $Az = z$. Again by (b) and using (d), we have

$$\begin{aligned}\int_0^{M(Tz,z,kt)} \varphi(t)dt &= \int_0^{M(P(Tz),Qz,kt)} \varphi(t)dt,\\ &\geq \int_0^{m(Tz,z,t)} \varphi(t)dt,\end{aligned}$$

and

$$\begin{aligned}\int_0^{N(Tz,z,kt)} \varphi(t)dt &= \int_0^{N(P(Tz),Qz,kt)} \varphi(t)dt,\\ &\leq \int_0^{n(Tz,z,t)} \varphi(t)dt.\end{aligned}$$

Take $\alpha = 1$,

$$\begin{aligned}m(Tz,z,t) &= \min\{M(ABz,Qz,t), M(ST(Tz),P(Tz),t),\\ &\qquad M(ST(Tz),Qz,t), M(ABz,P(Tz),t),\\ &\qquad M(ABz,ST(Tz),t)\}\\ &= \min\{M(Qz,Qz,t), M(Tz,Tz,t), M(Tz,z,t), M(z,Tz,t),\\ &\qquad M(z,Tz,t)\}\\ &= \min\{1,1,M(Tz,z,t), M(z,Tz,t), M(z,Tz,t)\}\\ &= M(Tz,z,t)\end{aligned}$$

and

$$\begin{aligned}n(Tz,z,t) &= \max\{N(ABz,Qz,t), N(ST(Tz),P(Tz),t),\\ &\qquad N(ST(Tz),Qz,t), N(ABz,P(Tz),t),\\ &\qquad N(ABz,ST(Tz),t)\}\\ &= \max\{N(Qz,Qz,t), N(Tz,Tz,t), N(Tz,z,t), N(z,Tz,t),\\ &\qquad N(z,Tz,t)\}\\ &= \max\{0,0,N(Tz,z,t), N(z,Tz,t), N(z,Tz,t)\}\\ &= N(Tz,z,t).\end{aligned}$$

Therefore,

$$\int_0^{M(Tz,z,kt)} \varphi(t)dt \geq \int_0^{M(Tz,z,t)} \varphi(t)dt,$$

and

$$\int_0^{N(Tz,\ ,kt)} \varphi(t)dt \leq \int_0^{N(Tz,z,t)} \varphi(t)dt.$$

Therefore, by Lemma 2, we have $Tz = z$. Since $STz = z$, therefore $Sz = z$.

By combining the above results, we have

$$Az = Bz = Sz = Tz = Pz = Qz = z,$$

that is z is a common fixed point of A, B, S, T, P and Q. The uniqueness of the common fixed point of A, B, S, T, P and Q follows easily from (b). This completes the proof. ∎

If we put $P = Q$ in Theorem 1, we have the following result:

Corollary 1. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamond$defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0,1]$. Let A, B, S, T and P be mappings from X into itself such that*

(a) $P(X) \subset AB(X)$ and $P(X) \subset ST(X)$,

(b) there exists a constant $k \in (0,1)$ such that

$$\int_0^{M(Px,Py,kt)} \varphi(t)dt \geq \int_0^{m(x,y,t)} \varphi(t)dt$$

and

$$\int_0^{N(Px,Py,kt)} \varphi(t)dt \leq \int_0^{n(x,y,t)} \varphi(t)dt,$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque-integrable mapping which is summable, nonnegative, and such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \quad \text{for each} \quad \varepsilon > 0,$$

where

$$\begin{aligned} m(x,y,t) \;=\; & \min\{M(ABy,Py,t), M(STx,Px,t), M(STx,Py,\alpha t), \\ & M(ABy,Px,(2-\alpha)t), M(ABy,STx,t)\} \end{aligned}$$

and

$$\begin{aligned} n(x,y,t) \;=\; & \max\{N(ABy,Py,t), N(STx,Px,t), N(STx,Py,\alpha t), \\ & N(ABy,Px,(2-\alpha)t), N(ABy,STx,t)\} \end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ and $t > 0$, and *(c) if one of $P(X)$, $AB(X)$ or $ST(X)$ is a complete subspace of X, then*

(i) P and ST have a coincidence point, and

(ii) P and AB have a coincidence point.

Further, if

(d) $AB = BA$, $PB = BP$, $PA = AP$, $PT = TP$, $ST = TS$, and

(e) the pair $\{P, ST\}$ is weakly compatible,

then

(iii) A, B, S, T and P have a unique common fixed point in X.

If we put $B = T = Ix$ (the identity mapping on X) in Theorem 1, we have the following result:

Corollary 2. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamond$ defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0,1]$. Let A, S, P and Q be mappings from X into itself such that*

(a) $P(X) \subset A(X)$ and $Q(X) \subset S(X)$,

(b) there exists a constant $k \in (0,1)$ such that

$$\int_0^{M(Px,Qy,kt)} \varphi(t)dt \geq \int_0^{m(x,y,t)} \varphi(t)dt$$

and

$$\int_0^{N(Px,Qy,kt)} \varphi(t)dt \leq \int_0^{n(x,y,t)} \varphi(t)dt$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable mapping which is summable, nonnegative, and such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \quad \text{for each} \quad \varepsilon > 0,$$

where

$$\begin{aligned} m(x,y,t) \;=\; & \min\{M(Ay,Qy,t), M(Sx,Px,t), M(Sx,Qy,\alpha t), \\ & M(Ay,Px,(2-\alpha)t), M(Ay,Sx,t)\} \end{aligned}$$

and

$$\begin{aligned} n(x,y,t) \;=\; & \max\{N(Ay,Qy,t), N(Sx,Px,t), N(Sx,Qy,\alpha t), \\ & N(Ay,Px,(2-\alpha)t), N(Ay,Sx,t)\} \end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ and $t > 0$, and

(c) if one of $P(X)$, $A(X)$, $S(X)$ or $Q(X)$ is a complete subspace of X, then

(i) P and S have a coincidence point, and

(ii) Q and A have a coincidence point.

Further, if

(d) $QA = AQ$, and

(e) the pair $\{P, S\}$ is weakly compatible,

then

(iii) A, S, P and Q have a unique common fixed point in X.

If we put $A = S$ in Corollary 2, we have the following result:

Corollary 3. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamond$ defined by $t*t \geq t$ and $(1-t)\diamond(1-t) \leq (1-t)$ for all $t \in [0,1]$. Let A, P and Q be mappings from X into itself such that*

(a) $P(X) \subset A(X)$ and $Q(X) \subset A(X)$,

(b) there exists a constant $k \in (0,1)$ such that

$$\int_0^{M(Px,Qy,kt)} \varphi(t)dt \geq \int_0^{m(x,y,t)} \varphi(t)dt$$

and

$$\int_0^{N(Px,Qy,kt)} \varphi(t)dt \leq \int_0^{n(x,y,t)} \varphi(t)dt,$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque-integrable mapping which is summable, nonnegative, and such that

$$\int_0^{\varepsilon} \varphi(t)dt>0 \quad \text{for each} \quad \varepsilon > 0,$$

where

$$\begin{aligned} m(x,y,t) \;=\; \min\{&M(Ay,Qy,t), M(Ax,Px,t), M(Ax,Qy,\alpha t),\\ &M(Ay,Px,(2-\alpha)t), M(Ay,Ax,t)\} \end{aligned}$$

and

$$\begin{aligned} n(x,y,t) \;=\; \max\{&N(Ay,Qy,t), N(Ax,Px,t), N(Ax,Qy,\alpha t),\\ &N(Ay,Px,(2-\alpha)t), N(Ay,Ax,t)\} \end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ and $t > 0$, and

(c) if one of $P(X)$, $Q(X)$ or $A(X)$ is a complete subspace of X, then

(i) P and A have a coincidence point, and

(ii) Q and A have a coincidence point.

Further, if

(d) $QA = AQ$, and

(e) the pair $\{P, A\}$ is weakly compatible,

then

(iii) A, P and Q have a unique common fixed point in X.

In Theorem 1, if we replace the condition $QA = AQ$ by weak compatibility of the pair $\{Q, AB\}$ then we have the following theorem:

Theorem 2. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamond$ defined by $t*t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0,1]$. Let A, B, S, T, P and Q be mappings from X into itself such that*

(a) $P(X) \subset AB(X)$ and $Q(X) \subset ST(X)$,

(b) there exists a constant $k \in (0,1)$ such that

$$\int_0^{M(Px,Qy,kt)} \varphi(t)dt \geq \int_0^{m(x,y,t)} \varphi(t)dt$$

and

$$\int_0^{N(Px,Qy,kt)} \varphi(t)dt \leq \int_0^{n(x,y,t)} \varphi(t)dt,$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque-integrable mapping which is summable, nonnegative, and such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \quad \textit{for each} \quad \varepsilon > 0,$$

where

$$\begin{aligned} m(x,y,t) \;=\; & \min\{M(ABy,Qy,t), M(STx,Px,t), M(STx,Qy,\alpha t), \\ & M(ABy,Px,(2-\alpha)t), M(ABy,STx,t)\} \end{aligned}$$

and

$$\begin{aligned} n(x,y,t) \;=\; & \max\{N(ABy,Qy,t), N(STx,Px,t), N(STx,Qy,\alpha t), \\ & N(ABy,Px,(2-\alpha)t), N(ABy,STx,t)\} \end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ and $t > 0$, and

(c) if one of $P(X)$, $AB(X)$, $ST(X)$ or $Q(X)$ is a complete subspace of X,

then

(i) P and ST have a coincidence point, and

(ii) Q and AB have a coincidence point.

Further, if

(d) $AB = BA$, $QB = BQ$, $PT = TP$, $ST = TS$, and

(*e*) *the pairs* $\{P, ST\}$ *and* $\{Q, AB\}$ *are weakly compatible,*
then

(*iii*) A, B, S, T, P *and* Q *have a unique common fixed point in* X.

By using Theorem 2, we have the following theorem:

Theorem 3. *Let* $(X, M, N, *, \diamond)$ *be an intuitionistic fuzzy metric space with continuous t-norm* $*$ *and continuous t-conorm* $\diamond$ *defined by* $t * t \geq t$ *and* $(1-t) \diamond (1-t) \leq (1-t)$ *for all* $t \in [0,1]$. *Let* A, B, S, T *and* P_i, *for* $i = 0, 1, 2\ldots$, *be mappings from* X *into itself such that*

(*a*) $P_0(X) \subset AB(X)$ *and* $P_i(X) \subset ST(X)$, *for* $i \in \mathbb{N}$,

(*b*) *there exists a constant* $k \in (0,1)$ *such that*

$$\int_0^{M(P_0x, P_iy, kt)} \varphi(t)dt \geq \int_0^{m(x,y,t)} \varphi(t)dt$$

and

$$\int_0^{N(P_0x, P_iy, kt)} \varphi(t)dt \leq \int_0^{n(x,y,t)} \varphi(t)dt,$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ *is a Lebesque-integrable mapping which is summable, nonnegative, and such that*

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \quad \text{for each} \quad \varepsilon > 0,$$

where

$$\begin{aligned} m(x,y,t) \; = \; \min\{&M(ABy, P_iy, t), M(STx, P_0x, t), M(STx, P_iy, \alpha t), \\ &M(ABy, P_0x, (2-\alpha)t), M(ABy, STx, t)\} \end{aligned}$$

and

$$\begin{aligned} n(x,y,t) \; = \; \max\{&N(ABy, P_iy, t), N(STx, P_0x, t), N(STx, P_iy, \alpha t), \\ &N(ABy, P_0x, (2-\alpha)t), N(ABy, STx, t)\} \end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ *and* $t > 0$, *and*

(*c*) *if one of* $P_0(X)$, $AB(X)$ *or* $ST(X)$*is a complete subspace of* X *or alternatively,* P_i, *for* $i \in \mathbb{N}$, *are complete subspace of* X,
then

(*i*) P_0 *and* ST *have a coincidence point, and*

(*ii*) *for* $i \in \mathbb{N}$, P_i *and* AB *have a coincidence point.*

Further, if

(*d*) $AB = BA$, $P_iB = BP_i (i \in \mathbb{N})$, $P_0T = TP_0$, $ST = TS$, *and*

(*e*) *the pairs* $\{P_0, ST\}$ *and* $\{P_i(i \in \mathbb{N}), AB\}$ *are weakly compatible,*
then

(*iii*) A, B, S, T *and* P_i, *for* $i = 0, 1, 2\ldots$, *have a unique common fixed point in* X.

Conclusion: This paper is to present some common fixed point theorems by using contractive condition of integral type for class of weakly compatible maps in noncomplete intuitionistic fuzzy metric spaces, without taking any continuous mapping.

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