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ON STRONGLY GENERALIZED PRECONTINUOUS MAPPINGS

Abstract. The aim of this paper is devoted to introduce and studya new class of generalized closed sets, namely strongly generalized preclosed sets which is weaker than both of preclosed sets and g^*-closed sets (M.K. Kumar, 1999). Moreover, we investigate some properties such as strongly generalized precontinuous and strongly generalized preclosed maps via strongly generalized preclosedness.

Key words: strongly generalized preclosed sets, strongly generalized preopen sets, strongly generalized precontinuous and strongly generalized preclosed mappings.

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1. Introduction and prelimenaries

In 1970, Levine [15] introduced and studied the concepts of generalized closed sets in topological spaces as a generalization of closed sets. Since the advent of this notion , many new notions by involving g-open and g-closed sets came to existence.

In the present paper, we define and study the properties of strongly generalized preclosed sets which is property placed in between the class of g^*-closed sets due to M. K. Mumer [14] and the class of $gp-$closed sets by Noiri, Maki and Umehara [18]. Also, many concepts such as, strongly generalized preopen sets, strongly generalized precontinuous and strongly generalized preclosed mappings are defined and discussed.

During this paper (X, τ) and (Y, σ) represent nonempty topological spaces on which no separation axioms are assumed unless after mentioned. For a subset A of (X, τ) the closure (resp. interior, derived) of A are denoted by $cl(A)$ (resp. $int(A)$, $d(A)$). (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 1. *For a space (X, τ) a subset A of X is said to be:*
(i) *preopen (resp. preclosed) [17] iff $A \subseteq int(cl(A))$*

(*resp.* $cl(int(A) \subseteq A)$,

(*ii*) γ-*open [8] (or b-open [3])* (*resp.* γ-*closed (or b-closed)) iff* $A \subseteq int(cl(A)) \cup cl(int(A))$ (*resp.* $cl(int(A)) \cap int(cl(A)) \subseteq A$,

(*iii*) *semi-preopen [2]* (*resp. semi-preclosed) iff* $A \subseteq cl(int(cl(A)))$ (*resp.* $int(cl(int(A))) \subseteq A$.

The intersection of all preclosed [1] (resp. γ-closed [8], semi-preclosed [2]) sets containing a subset A of (X,τ) is called preclosure (resp. γ-closure, semi-preclosure) of A and is denoted by $p{-}cl(A)$ (resp. $\gamma{-}cl(A)$, $sp{-}cl(A)$). The preinterior [1] (resp. γ-interior [8], semi-preinterior [2]) of $A \subseteq X$ is the largest preopen (resp. γ-open, semi-preopen) set contained in A and is denoted by p-$int(A)$ (resp. $\gamma - int(A)$, $sp - int(A)$). The intersection of an open set G and a preopen set H is preopen and also the union of a closed set A and a preclosed set B is preclosed [1]. For a space (X,τ) a point $p \in X$ is called pre-limit [1] point of a set A of X if every preopen set containing p contains some points of A other than p. Furthermore, the set of all pre-limit points of A is called the pre-derived set of A and is denoted by p-$d(A)$. The family of all preopen (resp. preclosed) subsets of (X,τ) will be as always denoted by $PO(X,\tau)$ (resp. $PC(X,\tau)$).

Definition 2. *A subset A of a space (X,τ) is called:*

(*i*) *generalized closed* (*briefly, g-closed*) *[15] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,*

(*ii*) *g^*-closed [14] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open,*

(*iii*) *generalized preclosed* (*briefly, gp-closed*) *[5] set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,*

(*iv*) *generalized preregular* (*briefly, gpr-closed*) *[12] set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open,*

(*v*) *γ-generalized closed* (*briefly, γg-closed*) *[11] set if $\gamma - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,*

(*vi*) *generalized semi-preclosed* (*briefly, gsp$-$closed*) *[7] set if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,*

(*vii*) *strongly γ-generalized closed* (*briefly, strongly γg-closed*) *[10] set if $\gamma - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.*

The complement of g-closed (resp. g^*-closed, gp-closed, gpr-closed, γg-closed, gsp-closed) set is called g-open (resp. g^*-open, gp-open, gpr-open, γg-open, gsp-open).

Definition 3. *A mapping $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called precontinuous [17]* (*resp. g^*-continuous [14], g-continuous [6], gp-continuous [4], gpr-continuous [12], γg-continuous [11], gsp-continuous [7]*)*, if the inverse image of each member of closed set of $(Y,\sigma)$is preclosed* (*resp. g^*- closed, g-closed, gp-closed, gpr-closed, γg-closed, gsp-closed*) *in (X,τ).*

Definition 4. *A mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called pre-irresolute [17] (resp. g^*-irresolute [14]) if the inverse image of each preclosed (resp. g^*-closed) set of (Y, σ) is preslosed (resp. g^*-closed) in (X, σ).*

Definition 5. *A mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be preclosed [17] (resp. g-closed [16], gp-closed [18], gpr-closed [12], γg-closed [11], gsp-closed [7]), if $f(V)$ is preclosed (resp. g-closed, gp-closed, gpr-closed, γg-closed, gsp-closed) in (Y, σ), for each closed set V in (X, τ).*

Proposition 1. *For any subset A of a space (X, τ), the following properties hold:*
(i) A *is preopen (resp. preclosed) iff* $A = p - int(A)$ (*resp.* $A = p - cl(A)$),
(ii) $int(A) \subseteq p{-}int(A) \subseteq \gamma{-}int(A) \subseteq sp{-}int(A) \subseteq A \subseteq sp{-}cl(A){-} \subseteq \gamma - cl(A) \subseteq p - cl(A) \subseteq cl(A)$,
(iii) *if* $A \subseteq B$, *then* $p - int(A) \subseteq p - int(B)$ (*resp.* $cl(A) \subseteq p - cl(B)$),
(iv) $p - cl(X - A) = X - (p - int(A))$,
(v) $p - int(X - A) = X - (p - cl(A))$,
(vi) $p - cl(A) = A \cup (p - d(A))$.

2. Strongly generalized preclosed sets

Definition 6. *A subset A of a space (X, τ) is called strongly generalized preclosed (briefly, strongly gp-closed) set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).*

Lemma 1. *For a space (X, τ), the following hold :*
(i) every preclosed (resp. g^*-closed) set is strongly gp-closed,
(ii) every strongly gp-closed set is gp-closed.

Remark 1. The notions of strongly *gp*-closed and *g*-closed sets are independent. Suppose that $X = \{a, b, c, d\}$ with a topology $\tau = \{X, \phi, \{c\}, \{c, d\}\}$. Then a subset $A = \{a, b\}$ is *g*-closed but not strongly *gp*-closed. Also, a subset $B = \{d\}$ is strongly *gp*-closed but not g-closed.

Remark 2. By Lemma 1 and Remark 1, we obtain the following diagram.

$$
\begin{array}{ccccccc}
\text{closed} & \longrightarrow & \text{g}^*\text{– closed} & \longrightarrow & \text{g-closed} & & \\
\downarrow & & \downarrow & & \downarrow & & \\
\text{preclosed} & \longrightarrow & \boxed{\text{strongly gp-closed}} & \longrightarrow & \text{gp-closed} & \longrightarrow & \text{gpr-closed} \\
\downarrow & & \downarrow & & \downarrow & & \updownarrow - \\
\gamma\text{–closed} & \longrightarrow & \text{strongly } \gamma\text{g-closed} & \longrightarrow & \gamma\text{g-losed} & \longrightarrow & \text{gsp-closed}
\end{array}
$$

The converses of these implications need not be true in general as it is shown by [8, 10, 11, 12, 14, 18] and the following examples.

Example 1. Let $X = \{a, b, c, d\}$ with topologies:

(i) $\tau_1 = \{X, \phi, \{a\}, \{a, b\}, \{a, b, d\}\}$. Then a subset $A = \{a, c\}$ is strongly *gp*-closed but not preclosed.

(ii) $\tau_2 = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{b, d\}, \{b, c, d\}\}$. Hence a subset $B = \{d\}$ is strongly *gp*-closed but not g^*-closed,

(iii) $\tau_3 = \{X, \phi, \{a\}\}$. Further a subset $C = \{a, d\}$is *gp*-closed but not strongly *gp*-closed.

Example 2. Let $X = \{a, b, c, d\}$ with a topology $\tau = \{X, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$. Then a subset $A = \{a, b\}$ is strongly γg-closed but not strongly *gp*-closed.

Remark 3. The union of two strongly *gp*-closed sets need not be strongly *gp*-closed. Let $X = \{a, b, c, d, e\}$ with a topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, e\}\}$. Then two *subsets*$A = \{c\}$ and $B = \{d\}$ of X are strongly *gp*-closed sets but the union $A \cup B = \{c, d\}$ is not strongly *gp*-closed.

Theorem 1. *A subset A of a space (X, τ) is strongly gp-closed if and only if $A \subseteq G$ and G is g-open, there exists a preclosed set F such that $A \subseteq F \subseteq G$.*

Proof. The Necessity. Suppose that A is a strongly *gp*-closed set, $A \subseteq G$ and G is a g-open set. Then $p - cl(A) \subseteq G$. We put $F = p - cl(A)$. Hence by Proposition [1,(ii)], $A \subseteq F \subseteq G$.

The Sufficiency. Assume that $A \subseteq G$ and G is a g-open set. Then by hypothesis, there exists a preclosed set F such that $A \subseteq F \subseteq G$. Therefore by Proposition [1,(i)], $A \subseteq p - cl(A) \subseteq F$ and hence $p - cl(A) \subseteq G$. Thus A is strongly *gp*-closed. ■

Proposition 2. *If A is closed and B is strongly gp-closed sets, then $A \cup B$ is strongly gp-closed.*

Proof. Suppose that $A \cup B \subseteq U$ and U is a g-open set. Then $A \subseteq U$ and $B \subseteq U$. But B is strongly *gp*-closed. Then $p - cl(B) \subseteq U$ and hence $A \cup B \subseteq A \cup p - cl(B) \subseteq U$. Then $F = A \cup p - cl(B)$ is a preclosed set and therefore there exists a preclosed set F such that $A \cup B \subseteq F \subseteq U$. Thus by Theorem 1, $A \cup B$ is strongly *gp*-closed. ■

Theorem 2. *If A and B are strongly gp-closed sets, $d(A) \subseteq p - d(A)$ and $d(B) \subseteq p - d(B)$, then $A \cup B$ is strongly gp-closed.*

Proof. Let $A \cup B \subseteq U$ and U is a g-open set. Then $A \subseteq U$ and $B \subseteq U$. Observe that A and B are strongly gp-closed sets and hence $p - cl(A) \subseteq U$ and $p - cl(B) \subseteq U$, but $d(A) \subseteq p - d(A)$ and $d(B) \subseteq p - d(B)$. Therefore by Proposition 1, $d(A) = p - d(A)$ and $d(B) = p - d(B)$. This implies that $p - cl(A \cup B) \subseteq cl(A \cup B) \subseteq U$ and thus $A \cup B$ is strongly gp-closed. ■

Theorem 3. *If A is a strongly gp-closed set, then $p - cl(A) - A$ contains no non-empty g-closed set.*

Proof. Assume that F is a g-closed set such that $F \subseteq p - cl(A) - A$ that is,

$$F \subseteq p - cl(A). \tag{1}$$

Then $F \subseteq X - A$. This implies that $A \subseteq X - F$ which is g-open. But A is a strongly gp-closed set, then $p - cl(A) \subseteq X - F$. Thus,

$$F \subseteq X - (p - cl(A)). \tag{2}$$

Hence from (1) and (2), we have $F = \phi$. Therefore $p - cl(A) - A$ contains no non-empty g-closed set. ■

Proposition 3. *For any space (X, τ). If A is a strongly gp-closed set and $A \subseteq B \subseteq p - cl(A)$, then B is strongly gp-closed.*

Proof. Let $B \subseteq G$ and G is a g-open set. Then $p - cl(A) \subseteq G$, where A is strongly gp-closed and $A \subseteq B$, hence $p - cl(B) \subseteq G$. Then B is a strongly gp-closed set. ■

3. Strongly generalized preopen sets

This article is devoted to give and study the concept of a strongly generalized peropen set. Also, many properties of this concept are discussed.

Definition 7. *A subset A of a space (X, τ) is called strongly generalized preopen (briefly, strongly gp-open) set if $X - A$ strongly gp-closed in (X, τ).*

Lemma 2. *For a topological space (X, τ), the following hold:*
(i) every preopen (resp. g^*- open) set is strongly gp-open,
(ii) every strongly gp-open set is gp-open.

Corollary 1. *The concepts of strongly gp-open and g-open sets are independent.*

The following is indicated the implications between the above notions in Lemma 2 and Corollary 1.

$$\begin{tikzcd}
\text{open} \arrow[r] \arrow[d] & \text{g}^*\text{-open} \arrow[r] \arrow[d] & \text{g-open} \arrow[d] & \\
\text{preopen} \arrow[r] \arrow[d] & \boxed{\text{strongly gp-open}} \arrow[r] \arrow[d] & \text{gp-open} \arrow[r] \arrow[d] & \text{gpr-open} \arrow[d, leftrightarrow, "-"] \\
\gamma-\text{open} \arrow[r] & \text{strongly } \gamma\text{g-open} \arrow[r] & \gamma \text{ g-open} \arrow[r] & \text{gsp-open}
\end{tikzcd}$$

The converses of the previous implications need not be true in general as it is shown by [8, 10, 11, 12, 14, 18] and the following examples.

Example 3. In Example 1, $A = \{b, d\}$ with respect to τ_1 is strongly gp-open but not preopen. Also, $B = \{a, b, c\}$ with respect to τ_2 is gp-open but not g-open. While $H = \{b, c\}$ with respect to τ_3 is gp-open but not strongly gp-open.

Example 4. In Remark 2, the subset $A = \{a, d\}$ is g-open but not strongly gp-open. Also, the subset $B = \{a, b, c\}$ is strongly gp-open but not g-open.

Example 5. In Example 2, the subset $A = \{c, d\}$ is strongly γg-open but not strongly gp-open.

The intersection of two strongly gp-open sets need not be strongly gp-open. In Remark 3, Two subsets $A = \{a, b, d, e\}$ and $B = \{a, b, c, e\}$ of X are strongly gp-open sets but $A \cap B = \{a, b, e\}$ is not strongly gp-open.

Theorem 4. *In a topological space (X, τ) and $A \subseteq X$, the following statements are equivalent :*

(i) A is a strongly gp-open set,

(ii) for each g-closed set $F \subseteq X$ contained in A, then $F \subseteq p - int(A)$,

(iii) for each g-closed set $F \subseteq X$ contained in A, there exists a preopen set G such that $F \subseteq G \subseteq A$.

Proof. $(i) \Rightarrow (ii)$. Let $F \subseteq A$ and F be a g-closed set. Then $X - A \subseteq X - F$ which is g-open in X. Hence, $p - cl(X - A) \subseteq X - F$. Thus $F \subseteq p - int(A)$.

$(ii) \Rightarrow (iii)$. Assume that $F \subseteq A$ and F is a g-closed set. Then by hypotesis, $F \subseteq p - int(A)$. Set $G = p - int(A)$ and hence $F \subseteq G \subseteq A$.

$(iii) \Rightarrow (i)$. Suppose that $X - A \subseteq U$ and U is a g-open set. Then $X - U \subseteq A$ and by hypothesis, there is a preopen set G such that $X - U \subseteq G \subseteq A$, that is, $X - A \subseteq X - G \subseteq U$. Therefore, by Theorem 1, $X - A$ is strongly gp-closed and by Definition 7, A is strongly gp-open. ■

Corollary 2. *If A is open and B is strongly gp-open sets, then $A \cap B$ is strongly gp-open.*

Proof. Obvious from Proposition 2. ■

Theorem 5. *For a topological space (X, τ), if A and B are strongly gp-open sets, $d(X - A) \subseteq p - d((X - A))$ and $d(X - B) \subseteq p - d(X - B)$, then $A \cap B$ is strongly gp-open.*

Proof. Obvious from Theorem 2. ■

Corollary 3. *If $p - int(A) \subseteq B \subseteq A$ and A is a strongly gp-open set, then B is strongly gp-open.*

Proof. By the daulity of Proposition 3. ■

4. Strongly generalized precontinuous mappings

This section is devoted to give and study the notion of a strongly generalized precontinuous mapping. Also, some characterizations and properties of this notion are investigated.

Definition 8. *A mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called strongly generalized precontinuous (briefly, st.gp-cont.) if the inverse image of every closed set of (Y, σ) is strongly gp-closed in (X, τ).*

From the implication between strongly *gp*-closed sets with each of g^*-closed, *gp*-closed, *gpr*-closed, γg-closed and *gsp*-closed which studied previously, one can show the relationships of strongly *gp*-continuous with other corresponding types as given throughout the next diagram.

$$
\begin{tikzcd}
\text{continuity} \arrow[r] \arrow[d] & \text{g}^*\text{-continuity} \arrow[r] \arrow[d] & \text{g-continuity} \arrow[d] & \\
\text{precontinuity} \arrow[r] & \boxed{\text{strongly gp-continuity}} \arrow[r] & \text{gp-continuity} \arrow[r] \arrow[d] & \gamma\text{g-continuity} \arrow[d] \\
& & \text{gpr-continuity} \arrow[r, phantom, "\nleftrightarrow"] & \text{gsp-continuity}
\end{tikzcd}
$$

Now, by [4, 6, 11, 12, 13, 17] and the following examples which given the converses of these implications are not be true in general.

Example 6. Let $X = Y = \{a, b, c, d\}$ with two topologies $\tau_X = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$, $\tau_Y = \{Y, \phi, \{b, c\}\}$ and a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ which defined by $f(a) = a$, $f(b) = c$, $f(c) = b$ and $f(d) = d$. Then f is strongly *gp*-continuous but not precontinuous.

Example 7. Let $X = \{a, b, c\}$ and $Y = \{a, b, c, d\}$ with two topologies $\tau_X = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_Y = \{Y, \phi, \{b, c, d\}\}$and a mapping $g : (X, \tau_X) \longrightarrow$

(Y, τ_Y) which defined by $g(a) = d$, $g(b) = a$ and $g(c) = c$. Then g is strongly gp-continuous but not g^*-continuous.

Example 8. If $X = \{a, b, c, d\}$ and $Y = \{a, b, c\}$ with two topologies $\tau_X = \{X, \phi, \{a\}\}$, $\tau_Y = \{Y, \phi, \{a, c\}\}$ and a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ is defined by $f(a) = f(d) = b$, $f(b) = a$ and $f(c) = c$, then f is gp-continuous but not strongly gp-continuous.

Remark 4. g-continuity and strongly gp-continuity are independent of each other as it is shown in the following two examples.

Example 9. Let $X = \{a, b, c, d\}$ and $Y = \{a, b, c\}$ with two topologies $\tau = \{X, \phi, \{c\}, \{c, d\}\}$, $\sigma = \{Y, \phi, \{b, c\}\}$ and a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ which defined by $f(a) = c$, $f(b) = f(c) = a$ and $f(d) = b$. Then f is g-continuous but not strongly gp-continuous.

Example 10. If $X = Y = \{a, b, c\}$ with two topologies $\tau = \{X, \phi, \{a, b\}\}$, $\tau = \{Y, \phi, \{b\}, \{a, b\}\}$ and a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is defined by $f(a) = c$, $f(b) = b$ and $f(c) = a$, then f is strongly gp-continuous but not g-continuous.

Theorem 6. Let $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping. Then the following statements are equivalent:

(i) f is strongly gp-continuous,

(ii) The inverse image of each member of τ_Y is strongly gp-open in (X, τ_X).

Proof. $(i) \Rightarrow (ii)$. Suppose that $A \subseteq Y$ is an open set. Then $X - A \subseteq Y$ is a closed set and hence $f^{-1}(X - A) = Y - f^{-1}(A)$ is strongly gp-closed. Therefore, $f^{-1}(A)$ is strongly gp-open.

$(ii) \Rightarrow (i)$. Assume that $A \subseteq Y$ is a closed set. Then $X - A \subseteq Y$ is an open set and therefore $f^{-1}(X - A) = Y - f^{-1}(A)$ is strongly gp-open and hence $f^{-1}(A)$ is strongly gp-closed. ■

Theorem 7. *If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is strongly gp-continuous, then for each $x \in X$ and $V \in \sigma$ containing $f(x)$, there exists a strongly gp-open set W containing x such that $f(W) \subseteq V$.*

Proof. Let $V \subseteq Y$ be an opn set containing $f(x)$. Then $x \in f^{-1}(V)$ which is a strongly gp-open set in X. Set $W = f^{-1}(V)$ and hence $f(W) \subseteq V$. ■

Remark 5. The composition of two strongly gp-continuous mappings is not always strongly gp-continuous. Let $X = Y = Z = \{a, b, c, d\}$ with $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$ and $\xi = \{Z, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = f(c) = c$, $f(d) = d$ and the mapping $g : (Y, \sigma) \to (Z, \xi)$ be defined by $g(a) = g(b) = a$, $g(c) = d$ and $g(d) = b$. Then f and g are strongly gp-continuous, but $g \circ f$ is not strongly *gp*-continuous.

Theorem 8. *If a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is pre-irresolut (resp. g^*-irresolute) and $g : (Y, \sigma) \to (Z, \xi)$ is precontinuous (resp. g^* -continuous), then $g \circ f : (X, \tau) \longrightarrow (Z, \xi)$ is strongly gp-continuous.*

Proof. Assume that $G \subseteq Z$ is a closed set. Then $g^{-1}(G) \subseteq Y$ is preclosed (resp. g^*-closed) and hence by Lemma 2, $(g \circ f\ )^{-1}(G)$ is a strongly *gp*-closed set in (X, τ). Therefore, $g \circ f$ is strongly *gp*-continuous. ■

5. Strongly generalized preclosed mappings

In this article, we give and study the concepts of strongly generalized preclosed (briefly, strongly gp-closed) and strongly generalized preopen (briefly, strongly gp-open) mappings. Also, some properties of these concepts are introduced.

Definition 9. *A mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be:*

(*i*) *strongly generalized preclosed (briefly, strongly gp-preclosed) if the image of each closed set of (X, τ) is strongly gp-closed in (Y, σ),*

(*ii*) *strongly generalized preopen (briefly, strongly gp-preopen) if the image of each open set of (X, τ) is strongly gp-open in (Y, σ).*

Remark 6. It is clear that strongly *gp*-closed (resp. strongly *gp*-open) mappings is stronger than *gp*-closed (resp. *gp*-open). Hence the implications between these mappings and other correspinding ones are given by the following diagram.

$$
\begin{tikzcd}
\text{preclosed} \arrow[d] & \text{closed} \arrow[l] \arrow[r] & \text{g-coclosed} \arrow[d] & \\
\boxed{\text{strongly gp-closed}} \arrow[rr] & & \text{gp-closed} \arrow[r] \arrow[d] & \gamma\text{g-closed} \arrow[d] \\
 & & \text{gpr-closed} \arrow[r, phantom, "\nleftrightarrow"] & \text{gsp-closed}
\end{tikzcd}
$$

The converses of these implications are not true in general as it is shown by [9, 11, 12, 15, 18] and the following examples.

Example 11. Let $X = \{a, b, c\}$ and $Y = \{a, b, c, d\}$ with two topologies $\tau_X = \{X, \phi, \{b\}\}$, $\tau_Y = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and a mapping $g : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ which defined by $g(a) = g(b) = a$ and $g(c) = d$. Then g is strongly *gp*-closed but not preclosed.

Example 12. Suppose that $X = Y = \{a, b, c, d\}$ with two topologies $\tau_X = \{X, \phi, \{b, d\}\}$, $\tau_Y = \{Y, \phi, \{a\}\}$ and a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ is defined by $f(a) = a$, $f(b) = c$, $f(c) = b$ and $f(d) = d$. Then f is *gp*-closed but not strongly gp-closed.

Example 13. g-closedness and strongly *gp*-closedness are independent. Let $X = Y = \{a, b, c, d\}$ with two topologies $\tau_X = \{X, \phi, \{c, d\}\}$, $\tau_Y = \{Y, \phi, \{c\}, \{c, d\}\}$ and a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ which defined by $f(a) = a$, $f(b) = c$ and $f(c) = f(d) = d$. Then f is g-closed but not strongly *gp*-closed. Also, a mapping $g : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ which defined by $g(a) = g(b) = d$, $g(c) = a$ and $g(d) = b$. Then g is strongly *gp*-closed but not g-closed.

Example 14. If $X = Y = \{a, b, c, d\}$ with two topologies $\tau_X = \{X, \phi, \{b, c, d\}\}$, $\tau_Y = \{Y, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$ and a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ is defined by $f(a) = c$, $f(b) = f(c) = a$ and $f(d) = d$. Then f is γg-closed but not *gpr*-closed.

Example 15. Suppose that $X = Y = \{a, b, c\}$ with two topologies $\tau_X = \{X, \phi, \{b\}\}$, $\tau_Y = \{Y, \phi, \{a\}, \{a, b\}\}$ and a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ is defined by $f(a) = f(c) = a$ and $f(b) = c$. Then f is *gpr*-closed but not γ g-closed.

Theorem 9. *Let $f : X \longrightarrow Y$ be a bijective mapping. Then the following statements are equivalent :*

(i) *f is strongly gp-closed,*
(ii) *f is strongly gp-open,*
(iii) *$f^{-1} : Y \longrightarrow X$ is strongly gp-continuous.*

Remark 7. The composition of two strongly *gp*-closed (resp. strongly *gp*-open) mappings is not always strongly *gp*-closed (resp. strongly *gp*-open). Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau_X = \{X, \phi, \{b\}, \{c\} \{b, c\}, \{b, d\}, \{b, c, d\}\}$, $\tau_Y = \{Y, \phi, \{a\}\}$ and $\tau_Z = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping defined by $f(a) = b$ and $f(b) = f(c) = f(d) = c$ and $g : (Y, \tau_Y) \rightarrow (Z, \tau_Z)$ be a mapping defined by $g(a) = b$, $g(b) = a$, $g(c) = c$ and $g(d) = d$. Then f and g are strongly *gp*-closed, but $g \circ f$ is not strongly *gp*-closed.

Theorem 10. *Let $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ and $g : (Y, \tau_Y) \rightarrow (Z, \tau_Z)$ be two mappings. Then $g \circ f : (X, \tau_X) \longrightarrow (Z, \tau_Z)$ is strongly gp-closed (resp. strongly gp-open), if f is closed (resp. open) and g is strongly gp-closed (resp. strongly gp-open).*

Proof. Suppose that $G \subseteq X$ is a closed set. Since f is a closed mapping, then $f(G)$ is a closed set in Y and $(g \circ f)(G)$ is strongly *gp*-closed in Z, where g is strongly *gp*-closed. Therefore $(g \circ f)$ is strongly *gp*-closed.

The other case is similar. ■

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