F A S C I C U L I M A T H E M A T I C I

Nr 48 2012

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ON WEAKLY $\widetilde{g}$-CONTINUOUS FUNCTIONS

Abstract. In this paper, we introduce a new class of functions called weakly $\widetilde{g}$-continuous functions and investigate some of their fundamental properties.

Key words: topological spaces, $\widetilde{g}$-open sets, $\widetilde{g}$-closure, weakly $\widetilde{g}$-continuous.

AMS Mathematics Subject Classification: 54D10.

1. Introduction

Recent progress in the study of characterizations and generalizations of continuity, compactness, connectedness, separation axioms etc. has been done by means of several generalized closed sets. The notion of generalized closed sets has been studied extensively in recent years by many topologists [see [3], [10]- [15]) because generalized closed sets are only natural generalization of closed sets. More importantly, they also suggest several new properties of topological spaces. As generalization of closed sets, $\widetilde{g}$-closed sets were introduced and studied by Jafari et al [2]. This notion was further studied by Rajesh and Ekici [7, 5]. In this paper, we introduce a new class of functions called weakly $\widetilde{g}$-continuous functions and investigate some of their fundamental properties.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$ and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

We recall the following definitions, which are useful in the sequel.

Definition 1. *A subset A of a space (X, τ) is called semi-open [3] if $A \subset Cl(Int(A))$. The complement of semi-open set is called semi-closed [1].*

The semi-closure [1] of a subset A of X, denoted by $s\mathrm{Cl}(A)$ is defined to be the intersection of all semi-closed sets of (X, τ) containing A.

Definition 2. *A subset A of a space (X, τ) is called:*

(*i*) *$\widehat{g}$-closed [14] (ω-closed [12]) if $Cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in X. The complement of $\widehat{g}$-closed set is called $\widehat{g}$-open.*

(*ii*) *$^*g$-closed [13] if $Cl(A) \subset U$ whenever $A \subset U$ and U is $\widehat{g}$-open in X. The complement of $^*g$-closed set is called $^*g$-open.*

(*iii*) *$^{\#}g$-semi-closed [15] if $sCl(A) \subset U$ whenever $A \subset U$ and U is $^*g$-open in X. The complement of $^{\#}g$-semi-closed is called $^{\#}g$-semi-open.*

(*iv*) *$\widetilde{g}$-closed [2] if $Cl(A) \subset U$ whenever $A \subset U$ and U is $^{\#}g$-semi-open in X. The complement of $\widetilde{g}$-closed set is called $\widetilde{g}$-open.*

The family of all $\widetilde{g}$-open (resp. $\widetilde{g}$-closed) sets of X is denoted by $\widetilde{G}(\tau)$ (resp. $\widetilde{G}C(X)$). We set $\widetilde{G}O(X, x) = \{U | U \in \widetilde{G}(\tau) \text{ and } x \in U\}$. In [2] shown that the set $\widetilde{G}(\tau)$ forms a topology, which is finer than τ.

Definition 3. *The intersection (union) of all $\widetilde{g}$-closed ($\widetilde{g}$-open) sets containing (contained in) A is called the $\widetilde{g}$-closure ($\widetilde{g}$-interior) [5] of A and is denoted by $\widetilde{g}$-$Cl(A)$ ($\widetilde{g}$-$Int(A)$).*

A set A is $\widetilde{g}$-closed ($\widetilde{g}$-open) if and only if $\widetilde{g}$-$Cl(A) = A$ ($\widetilde{g}$-$Int(A) = A$). [5].

Lemma 1 ([5])**.** *Let A be a subset of a topological space (X, τ). Then $x \in \widetilde{g}$-$Cl(A)$ if and only if $U \cap A \neq \varnothing$ for every $U \in \widetilde{G}O(X, x)$.*

Definition 4. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\widetilde{g}$-continuous [7] (resp. $\widetilde{g}$-irresolute [6]) if $f^{-1}(V) \in \widetilde{G}(\tau)$ for every open set V of Y (resp. $V \in \widetilde{G}(\sigma)$).*

Definition 5. *A topological space (X, τ) is said to be $\widetilde{g}$-regular [8] if for each closed set F and each $x \notin F$, there exist disjoint $\widetilde{g}$-open sets U and V such that $x \in U$ and $F \subset V$.*

Lemma 2 ([8])**.** *For a topological space (X, τ), the following are equivalent:*

(*i*) *X is $\widetilde{g}$-regular;*

(*ii*) *for each open set U and each $x \in U$, there exists $V \in \widetilde{G}(\tau)$ such that $x \in V \subset \widetilde{g}$-$Cl(V) \subset U$.*

3. Weakly $\widetilde{g}$-continuous functions

Definition 6. *A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly $\widetilde{g}$-continuous if for each $x \in X$ and each open set V containing $f(x)$ there exists $U \in \widetilde{G}O(X, x)$ such that $f(U) \subseteq \widetilde{g}$-$Cl(V)$.*

It is clear that every $\widetilde{g}$-continuous function is weakly $\widetilde{g}$-continuous but not converse.

Example 1. Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{b\}, X\}$ and $\sigma = \{\varnothing, \{a\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is weakly $\widetilde{g}$-continuous but not $\widetilde{g}$-continuous.

Theorem 1. *Let (X, τ) be a $\widetilde{g}$-regular space. Then $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}$-continuous if and only if it is weakly $\widetilde{g}$-continuous.*

Proof. The proof follows from Lemma 2. ■

Theorem 2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(*i*) *f is weakly $\widetilde{g}$-continuous;*

(*ii*) *$f^{-1}(V) \subset \widetilde{g}\text{-}Int(f^{-1}(\widetilde{g}\text{-}Cl(V)))$ for every open set V of Y;*

(*iii*) *$\widetilde{g}\text{-}Cl(f^{-1}(\widetilde{g}\text{-}Int(F))) \subset f^{-1}(F)$ for every closed set F of Y;*

(*iv*) *$\widetilde{g}\text{-}Cl(f^{-1}(\widetilde{g}\text{-}Int(Cl(B)))) \subset f^{-1}(Cl(B))$ for every subset B of Y;*

(*v*) *$f^{-1}(Int(B)) \subset \widetilde{g}\text{-}Int(f^{-1}(\widetilde{g}\text{-}Cl(Int(B))))$ for every subset B of Y;*

(*vi*) *$\widetilde{g}\text{-}Cl(f^{-1}(V)) \subset f^{-1}(\widetilde{g}\text{-}Cl(V))$ for every open set V of Y.*

Proof. $(i) \Rightarrow (ii)$: Let V be an open subset of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. There exists $U \in \widetilde{G}O(X, x)$ such that $f(U) \subset \widetilde{g}\text{-Cl}(V)$. Thus, we obtain $x \in U \subset f^{-1}(\widetilde{g}\text{-Cl}(V))$. This implies that $x \in \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V)))$ and consequently $f^{-1}(V) \subset \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V)))$.

$(ii) \Rightarrow (iii)$: Let F be any closed set of Y. Then $Y \backslash F$ is open in Y. By (ii), we have $\widetilde{g}\text{-Cl}(f^{-1}(\widetilde{g}\text{-Int}(F))) \subset f^{-1}(F)$.

$(iii) \Rightarrow (iv)$: Let B be any subset of Y. Then $\text{Cl}(B)$ is closed in Y and by (iii), we obtain $\widetilde{g}\text{-Cl}(f^{-1}(\widetilde{g}\text{-Int}(\text{Cl}(B)))) \subset f^{-1}(\text{Cl}(B))$.

$(iv) \Rightarrow (v)$: Let B be any subset of Y. Then we have $f^{-1}(\text{Int}(B)) = X \setminus f^{-1}(\text{Cl}(Y \backslash B)) \subset X \setminus \widetilde{g}\text{-Cl}(f^{-1}(\widetilde{g}\text{-Int}(\text{Cl}(Y \backslash B)))) = \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(\text{Int}(B))))$.

$(v) \Rightarrow (vi)$: Let V be any open subset of Y. Suppose that $x \notin f^{-1}(\widetilde{g}\text{-Cl}(V))$. Then $f(x) \notin \widetilde{g}\text{-Cl}(V)$ and there exists $U \in \widetilde{G}O(Y, f(x))$ such that $U \cap V = \varnothing$; hence $\widetilde{g}\text{-Cl}(U) \cap V = \varnothing$. By (v), we have $x \in f^{-1}(U) \subset \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(U)))$ and hence there exists $W \in \widetilde{G}O(X, x)$ such that $W \subset f^{-1}(\widetilde{g}\text{-Cl}(U))$. Since $\widetilde{g}\text{-Cl}(U) \cap V = \varnothing$, $W \cap f^{-1}(V) = \varnothing$ and by Lemma 1 $x \notin \widetilde{g}\text{-Cl}(f^{-1}(V))$. Therefore, we obtain $\widetilde{g}\text{-Cl}(f^{-1}(V)) \subset f^{-1}(\widetilde{g}\text{-Cl}(V))$.

$(vi) \Rightarrow (i)$: Let $x \in X$ and V any open subset of Y containing $f(x)$. By (vi), we have $x \in f^{-1}(V) \subset f^{-1}(\text{Int}(\widetilde{g}\text{-Cl}(V))) \subset f^{-1}(\widetilde{g}\text{-Int}(\widetilde{g}\text{-Cl}(V))) = X \setminus f^{-1}(\widetilde{g}\text{-Cl}(Y \backslash \widetilde{g}\text{-Cl}(V))) \subset X \setminus \widetilde{g}\text{-Cl}(f^{-1}(Y \backslash \widetilde{g}\text{-Cl}(V))) = \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V)))$. Therefore, there exists $U \in \widetilde{G}O(X, x)$ such that $U \subset \widetilde{g}\text{-Cl}(V)$. This shows that f is weakly $\widetilde{g}$-continuous. ■

Definition 7. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a strongly $\widetilde{g}$-closed graph [9] if for $(x, y) \in (X \times Y) \setminus G(f)$, there exists $U \in \widetilde{G}O(X, x)$ and an open set V of Y containing y such that $(U \times V) \cap G(f) = \varnothing$.*

Lemma 3 ([9])**.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then its graph $G(f)$ is strongly $\widetilde{g}$-closed in $X \times Y$ if and only if for each point $(x, y) \in (X \times Y) \setminus G(f)$, there exist a $\widetilde{g}$-open set U of X and an open set V of Y, containing x and y, respectively, such that $f(U) \cap V = \varnothing$.*

Theorem 3. *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $\widetilde{g}$-continuous function and (Y, σ) is a Hausdorff space, then the graph $G(f)$ is a $\widetilde{g}$-closed set of $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$. Then, we have $y \neq f(x)$. Since (Y, σ) is Hausdorff, there exist disjoint open sets W and V such that $f(x) \in W$ and $y \in V$. Since f is weakly $\widetilde{g}$-continuous, there exists a $\widetilde{g}$-open set U containing x such that $f(U) \subseteq \widetilde{g}\text{-Cl}(W)$. Since W and V are disjoint subsets of Y, we have $V \cap \widetilde{g}\text{-Cl}(W) = \varnothing$. This shows that $(U \times V) \cap G(f) = \varnothing$ and hence by Lemma 3 $G(f)$ is $\widetilde{g}$-closed. ■

Definition 8. *By a weakly $\widetilde{g}$-continuous retraction, we mean a weakly $\widetilde{g}$-continuous function $f : X \to A$, where $A \subset X$ and $f|A$ is the identity function on A.*

Theorem 4. *Let A be a subset of X and $f : (X, \tau) \to (Y, \sigma)$ be a weakly $\widetilde{g}$-continuous restraction of X onto A. If (X, τ) is a Hausdorff space, then A is a $\widetilde{g}$-closed set in X.*

Proof. Supoose that A is not $\widetilde{g}$-closed in X. Then there exists a point $x \in \widetilde{g}\text{-Cl}(A)\backslash A$. Since f is weakly $\widetilde{g}$-continuous restraction, we have $f(x) \neq x$. Since X is Hausdorff, there exist disjoint open sets U and V of X such that $x \in U$ and $f(x) \in V$. Thus, we get $U \cap \widetilde{g}\text{-Cl}(V) = \varnothing$. Now, let $W \in \widetilde{G}O(X, x)$. Then $U \cap W \in \widetilde{G}O(X, x)$ and hence $(U \cap W) \cap A \neq \varnothing$, because $x \in \widetilde{g}\text{-Cl}(A)$. Let $y \in (U \cap W) \cap A$. Since $y \in A$, $f(y) \in U$ and hence $f(y) \notin \widetilde{g}\text{-Cl}(V)$. This gives that $f(W) \nsubseteq \widetilde{g}\text{-Cl}(V)$. This contradicts that f is weakly $\widetilde{g}$-continuous. Hence A is $\widetilde{g}$-closed in X. ■

Definition 9. *A space (X, τ) is called $\widetilde{g}$-connected [7] if X cannot be written as the disjoint union of two nonempty $\widetilde{g}$-open sets.*

Theorem 5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a weakly $\widetilde{g}$-continuous surjective function. If X is $\widetilde{g}$-connected, then Y is connected.*

Proof. Suppose that (Y, σ) is not connected. Then there exist nonempty disjoint open sets V_1 and V_2 in Y such that $V_1 \cup V_2 = Y$. Since f is surjective,

$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty disjoint subsets of X such that $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. By Theorem 2, we have $f^{-1}(V_i) \subseteq \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V_i)))$, $i = 1, 2$. Since V_i is open and closed and every closed set is $\widetilde{g}$-closed, we obtain $f^{-1}(V_i) \subseteq \widetilde{g}\text{-Int}(f^{-1}(V_i))$ and hence $f^{-1}(V_i)$ is $\widetilde{g}$-open for $i = 1, 2$. This implies that (X, τ) is not $\widetilde{g}$-connected. ■

Definition 10. *A space (X, τ) is said to be ultra $\widetilde{g}$-Urysohn if for each pair of distinct points x and y in X, there exist open sets U, V containing x, y respectively such that $\widetilde{g}$-Cl$(U) \cap \widetilde{g}$-Cl$(V) = \varnothing$.*

Definition 11. *A space (X, τ) is said to be $\widetilde{g}$-T_2 [9] if for each pair of distinct points x and y in X, there exist $U \in \widetilde{GO}(X, x)$ and $V \in \widetilde{GO}(X, y)$ such that $U \cap V = \varnothing$.*

Theorem 6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a weakly $\widetilde{g}$-continuous injective function. If Y is ultra $\widetilde{g}$-Urysohn, then X is $\widetilde{g}$-T_2.*

Proof. Let x_1 and x_2 be any two distinct points of X. Since f is injective, $f(x_1) \neq f(x_2)$. Since (Y, σ) is ultra $\widetilde{g}$-Urysohn, there exist $V_1, V_2 \in \sigma$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $\widetilde{g}\text{Cl}\ (V_1) \cap \widetilde{g}\text{-Cl}(V_2) = \varnothing$. This gives $f^{-1}(\widetilde{g}\text{-Cl}(V_1)) \cap f^{-1}(\widetilde{g}\text{-Cl}(V_2)) = \varnothing$ and hence $\widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V_1))) \cap \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{Cl}(V_2))) = \varnothing$. Since f is weakly $\widetilde{g}$-continuous, $x_i \in f^{-1}(V_i) \subset \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V_i)))$, $i = 1, 2$. By Theorem 2 and this indicates that the space (X, τ) is $\widetilde{g}$-T_2. ■

4. Additional properties

Definition 12. *For a function $f : (X, \tau) \to (Y, \sigma)$, the graph $G(f)$ is said to be ultra $\widetilde{g}$-closed if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in \widetilde{GO}(X, x)$, $V \in \widetilde{GO}(Y, y)$ such that $(U \times \widetilde{g}$-Cl$(V)) \cap G(f) = \varnothing$.*

Lemma 4. *The function $f : (X, \tau) \to (Y, \sigma)$ has a ultra $\widetilde{g}$-closed graph if and only if for every $(x, y) \in (X \times Y) \backslash G(f)$ there exist $U \in \widetilde{GO}(X, x)$, $V \in \widetilde{GO}(Y, y)$ and $f(U) \cap \widetilde{g}$-Cl$(V) = \varnothing$.*

Proof. It is an immediate consequence of Definition 12. ■

Theorem 7. *Let $f : (X, \tau) \to (Y, \sigma)$ be a weakly $\widetilde{g}$-continuous function. If (Y, σ) is ultra $\widetilde{g}$-Urysohn, then the graph $G(f)$ is ultra $\widetilde{g}$-closed.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$. Then $y \neq f(x)$. Since Y is ultra $\widetilde{g}$-Urysohn, there exist open sets V and W containing x and y, repectively such that $\widetilde{g}\text{-Cl}(V) \cap \widetilde{g}\text{-Cl}(W) = \varnothing$. Since f is weakly $\widetilde{g}$-continuous, there exist $U \in \widetilde{GO}(X, x)$ such that $f(U) \subset \widetilde{g}\text{-Cl}(U)$. This implies that $f(U) \cap \widetilde{g}\text{-Cl}(W) = \varnothing$. So, by Lemma 4 $G(f)$ is ultra $\widetilde{g}$-closed. ■

Theorem 8. *If $f : (X, \tau) \to (Y, \sigma)$ is an injective weakly $\widetilde{g}$-continuous function with a ultra $\widetilde{g}$-closed graph, then the space (X, τ) is $\widetilde{g}$-T_2.*

Proof. Let x and y be any distinct points of X. Then, since f is injective, we have $f(x) \neq f(y)$. Then we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. Since $G(f)$ is ultra $\widetilde{g}$-closed, by Lemma 4 there exist $U \in \widetilde{G}O(X, x)$ and an open set V of Y containing $f(y)$ such that $f(U) \cap \widetilde{g}\text{-Cl}(V) = \varnothing$. Since f is weakly $\widetilde{g}$-continuous, there exists $W \in \widetilde{G}O(X, y)$ such that $f(W) \subset \widetilde{g}\text{-Cl}(V)$. Therefore, we have $f(U) \cap G(f) = \varnothing$. Since f is injective, we obtain $U \cap W = \varnothing$. This shows that (X, τ) is a $\widetilde{g}$-T_2 space. ■

Theorem 9. *If $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}$-continuous function and (Y, σ) is a T_2 space, then the graph $G(f)$ is ultra-$\widetilde{g}$-closed.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$. The T_2 ness of Y gives the existence of an open set V containing y such that $f(x) \notin \text{Cl}(V)$. Now $\text{Cl}(V)$ is a closed set in Y. So, $Y \backslash \text{Cl}(V)$ is an open set in Y containing $f(x)$. Therefore, by the $\widetilde{g}$-continuity of f there exist $U \in \widetilde{G}O(X, x)$ such that $f(U) \subseteq Y \backslash \text{Cl}(V)$, hence $f(U) \cap \text{Cl}(V) = \varnothing$. Since $\widetilde{g}\text{-Cl}(A) \subseteq \text{Cl}(A)$ for every subset A of X, once obtain $f(U) \cap \widetilde{g}\text{-Cl}(V) = \varnothing$. By Lemma 4, $G(f)$ is ultra $\widetilde{g}$-closed. ■

Theorem 10. *If $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}$-irresolute function and (Y, σ) is a $\widetilde{g}$-T_2 space, then the graph $G(f)$ is ultra $\widetilde{g}$-closed.*

Proof. Similar proof of Theorem 9. ■

Definition 13. *A space (X, τ) is said to be*

(*i*) *$\widetilde{g}$-compact [7] if every cover of X by $\widetilde{g}$-open sets has a finite subcover;*

(*ii*) *$\widetilde{g}$-closed compact [4] if every cover of X by $\widetilde{g}$-open sets has a finite subcover whose $\widetilde{g}$-closure cover X.*

Definition 14. *A subset A of a space X is said to be $\widetilde{g}$-closed relative to X [7] if for every cover $\{V_\alpha\colon \alpha \in \Lambda\}$ of A by $\widetilde{g}$-open sets of X, there exists a finite subset Λ_0 of Λ such that $A \subset \bigcup \{\widetilde{g}\text{-}Cl(V_\alpha) \mid \alpha \in \Lambda_0\}$.*

Theorem 11. *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $\widetilde{g}$-continuous function and A is a $\widetilde{g}$-compact subset of (X, τ), then $f(A)$ is $\widetilde{g}$-closed relative to (Y, σ).*

Proof. Let $\{V_i | i \in \Lambda\}$ be any cover of $f(K)$ by open sets of (Y, σ). For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is weakly $\widetilde{g}$-continuous, there exists $U(x) \in \widetilde{G}O(X, x)$ such that $f(U(x)) \subset \widetilde{g}\text{-Cl}(V_{\alpha(x)})$. The family $\{U(x) | x \in A\}$ is a cover of A by $\widetilde{g}$-open sets of X. Since A is $\widetilde{g}$-compact, there exists a finite number of points, say, x_1, x_2, ..., x_n in A such that $A \subset \bigcup\{U(x_k) \mid x_k \in A,\ 1 \leq K \leq n\}$. Therefore, we obtain $f(A) \subset \bigcup \{f(U(x_k)) \mid x_k \in A,\ 1 \leq K \leq n\} \subset \bigcup \{\widetilde{g}\text{-Cl}(V_{\alpha(x_k)}) \mid x_k \in A,\ 1 \leq K \leq n\}$. This shows that $f(A)$ is $\widetilde{g}$-closed relative to (Y, σ). ■

Corollary. *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $\widetilde{g}$-continuous surjective function and the space (X, τ) is $\widetilde{g}$-compact, then (Y, σ) is a $\widetilde{g}$-closed space.*

Definition 15. *Let A be a subset of a topological space (X, τ). Then the $\widetilde{g}$-frontier [4] of A, denoted by $\widetilde{g}$-$Fr(A)$ is defined as $\widetilde{g}$-$Fr(A) = \widetilde{g}$-$Cl(A) \cap \widetilde{g}$-$Cl(X \backslash A)$.*

Theorem 12. *The set of all points $x \in X$ at which a function $f : (X, \tau) \to (Y, \sigma)$ is not weakly $\widetilde{g}$-continuous if and only if the union of $\widetilde{g}$-frontier of the inverse images of the closure of open sets containing $f(x)$.*

Proof. *Necessity.* Suppose that f is not weakly $\widetilde{g}$-continuous at $x \in X$. Then there exists an open set V of Y containing $f(x)$ such that $f(U) \nsubseteq \widetilde{g}\text{-Cl}(V)$ for every $U \in \widetilde{G}O(X, x)$. Then $U \cap (X \backslash f^{-1}(\widetilde{g}\text{-Cl}(V))) \neq \varnothing$ for every $U \in \widetilde{G}O(X, x)$ and hence by Lemma 1 $x \in \widetilde{g}\text{-Cl}(X \backslash f^{-1}(\widetilde{g}\text{-Cl}(V)))$. On the other hand, we have $x \in f^{-1}(V) \subset \widetilde{g}\text{-Cl}(f^{-1}(\widetilde{g}\text{-Cl}(V)))$ and hence $x \in \widetilde{g}\text{-}Fr(f^{-1}(\widetilde{g}\text{-Cl}(V)))$.

Sufficiency. Suppose that f is weakly $\widetilde{g}$-continuous at $x \in X$ and let V be any open set of Y containing $f(x)$. Then by Theorem 2, we have $x \in f^{-1}(V) \subset \widetilde{g}\text{-Int}(f^{-1}(\widetilde{g}\text{-Cl}(V)))$. Therefore, $x \in \widetilde{g}\text{-}Fr(f^{-1}(\text{Cl}(V)))$ for each open set V of Y containing $f(x)$. ■

Acknowledgement. The author thank the referee for his valuable comments and suggestions.

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Received on 04.06.2011 and, in revised form, on 24.10.2011.