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PROPERTIES OF γ-SEMI-REGULAR-OPEN SETS AND γ-s-CLOSED SPACES

Abstract. The concept of γ-semi-regular-open sets and γ-s-closed spaces have been introduced and explored in [6]. In this paper, we continue studying the properties and characterizations of γ-semi-regular open sets and γ-s-closed spaces.

Key words: γ-semi-closed (open), γ-semi-interior (closure), γ-semi-regular-open (closed), γ-θ-semi-open (closed), γ-semi-extremally disconnected, γ-SR-converge, γ-SR-accumulate, γ-s-closed spaces.

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1. Introduction

Topology is the branch of mathematics, whose concepts exist not only in all branches of mathematics, but also in many real life applications. A lot of researchers working on the different structures of topological spaces. N. Levine [11] initiated and explored the notion of semi-open sets. S. Kasahara [10] introduced and discussed an operation γ of a topology τ into the power set $P(X)$ of a space X. H. Ogata [13] introduced the concept of γ-open sets and investigated the related topological properties of the associated topology τ_γ and τ by using operation γ.

S. Hussain and B. Ahmad [1-9] continued studying the properties of γ-operations on topological spaces and investigated many interesting results. S. Hussain, B. Ahmad and T. Noiri [8] and B. Ahmad and S. Hussain [4] defined and discussed γ-semi-open sets in topological spaces. They explored many interesting properties of γ-semi-open sets. It is interesting to mention that γ-semi-open sets generalized γ-open sets introduced by H. Ogata [13].

In 1987, G. Di. Maio and T. Noiri [12] introduced the notion of s-closed space. S. Hussain and B. Ahmad [6] introduced a class of topological spaces called γ-s-closed space by utilizing γ-semi-closure[1]. It is shown that the concept of γ-s-closed space generalized s-closed space [12]. It is observed that γ-s-closedness is the generalization of γ_0-compactness defined and investigated in [3]. In [5], they also defined and characterized sets γ-closed relative to a space X.

The purpose of this paper is to continue studying the properties and characterizations of γ-semi-regular open sets and γ-s-closed spaces defined and discussed in [6].

First, we recall some definitions and results used in this paper. Hereafter, we shall write a space in place of a topological space.

2. Preliminaries

Throughout the present paper, X denotes topological spaces.

Definition 1 ([10]). *An operation $\gamma : \tau \to P(X)$ is a function from τ to the power set of X such that $V \subseteq V^\gamma$, for each $V \in \tau$, where V^γ denotes the value of γ at V. The operations defined by $\gamma(G) = G$, $\gamma(G) = cl(G)$ and $\gamma(G) = intcl(G)$ are examples of operation γ.*

Definition 2 ([10]). *Let $A \subseteq X$. A point $x \in A$ is said to be γ-interior point of A, if there exists an open nbd N of x such that $N^\gamma \subseteq A$ and we denote the set of all such points by $int_\gamma(A)$. Thus*

$$int_\gamma(A) = \{x \in A : x \in N \in \tau \text{ and } N^\gamma \subseteq A\} \subseteq A.$$

Note that A is γ-open [13] iff $A = int_\gamma(A)$. A set A is called γ-closed [1] iff $X - A$ is γ-open.

Definition 3 ([1]). *A point $x \in X$ is called a γ-closure point of $A \subseteq X$, if $U^\gamma \cap A \neq \phi$, for each open nbd U of x. The set of all γ-closure points of A is called γ-closure of A and is denoted by $cl_\gamma(A)$. A subset A of X is called γ-closed, if $cl_\gamma(A) \subseteq A$. Note that $cl_\gamma(A)$ is contained in every γ-closed superset of A.*

Definition 4 ([8]). *A subset A of a space X is said to be a γ-semi-open set, if there exists a γ-open set O such that $O \subseteq A \subseteq cl_\gamma(O)$. The set of all γ-semi-open sets is denoted by $SO_\gamma(X)$. A is γ-semi-closed if and only if $X - A$ is γ-semi-open in X. Note that A is γ-semi-closed if and only if $int_\gamma cl_\gamma(A) \subseteq A$.*

Definition 5 ([4]). *Let A be a subset of a space X. The intersection of all γ-semi-closed sets containing A is called γ-semi-closure of A and is denoted by $scl_\gamma(A)$. Note that A is γ-semi-closed if and only if $scl_\gamma(A) = A$. The set of all γ-semi-closed subsets of A is denoted by $SC_\gamma(A)$.*

Definition 6 ([4]). *Let A be a subset of a space X. The union of all γ-semi-open subsets contained in A is called γ-semi-interior of A and is denoted by $sint_\gamma(A)$.*

Definition 7 ([4]). *An operation γ on τ is said to be semi open, if for any semi open set U of each $x \in X$, there exists semi-open set B such that $x \in B$ and $U^\gamma \supseteq B$.*

3. γ-semi-regular-open sets

Definition 8 ([6]). *A subset A of X is said to be γ-semi-regular-open (respt. γ-semi-regular-closed), if $A = sint_\gamma(scl_\gamma(A))$ (respt. $A = scl_\gamma(s\, int_\gamma(A))$). The set of all γ-semi-regular open sets is denoted by $SRO_\gamma(X,\tau)$.*

It is clear that $SRO_\gamma(X,\tau) \subseteq SO_\gamma(X,\tau)$ and A is γ-semi-regular open if and only if $X - A$ is γ-semi-regular closed.

The following example shows that the converse of above inclusion is not true in general.

Example 1. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. For $b \in X$, define an operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} A, & \text{if} \quad b \in A \\ cl(A), & \text{if} \quad b \notin A \end{cases}$$

The calculations shows that: $\{a,b\}$, $\{a,c\}$, $\{b\}$, X , $\emptyset$ are γ-open sets, $\{a,b\}$, $\{a,c\}$, $\{b\}$, X, $\emptyset$ are γ-semi-open sets and $\{a,c\}$, $\{b\}$, X, $\emptyset$ are γ-semi-regular-open sets. Clearly the set $\{a,b\}$ is γ-semi-open but not γ-semi-regular-open.

Definition 9. *A space X is called γ-semi-extremally disconnected, if for all γ-semi-open subset U of X, $scl_\gamma(U)$ is a γ-semi-open subset of X.*

Example 2. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{a,b\}\}$ be a topology on X. Define an operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} A, & \text{if} \quad b \in A \\ X, & \text{if} \quad b \notin A \end{cases}$$

The calculations shows that $\{a,b\}, X, \emptyset$ are γ-semi open sets, in consequence X is a γ-semi-extremally disconnected space.

Example 3. In Example 1, calculations shows that the space X is γ-semi-extremally disconnected space, because $scl_\gamma(\{a,b\}) = X$, $scl_\gamma(\{a,c\}) = \{a,c\}$, $scl_\gamma(\{b\}) = \{b\}$, $scl_\gamma(X) = X$ and $scl_\gamma(\emptyset) = \emptyset$ are all γ-semi open sets.

Proposition 1. *If A is a γ-semi-clopen set in X, then A is a γ-semi-regular-open set. Moreover, if X is γ-semi-extremally disconnected then the converse holds.*

Proof. If A is a γ-semi-clopen set, then $A = scl_\gamma(A)$ and $A = sint_\gamma(A)$, and so we have $A = sint_\gamma(scl_\gamma(A))$. Hence A is γ-semi-regular-open.

Suppose that X is a γ-semi-extremally disconnected space and A is a γ-semi-regular-open set in X. Then A is γ-semi-open and so $scl_\gamma(A)$ is a γ-semi-open set. Follows $A = sint_\gamma(scl_\gamma(A)) = scl_\gamma(A)$ and hence A is γ-semi-closed set. This completes the proof. ■

The following example shows that space X to be γ-semi-extremally disconnected is necessary in the converse of above Proposition.

Example 4. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. For $b \in X$, define an operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} cl(A), & \text{if} \quad a \in A \\ int(cl(A)), & \text{if} \quad a \notin A \end{cases}$$

Calculations shows that $\{a\}$, $\{a, c\}$, $\{b\}$, X, ϕ are γ-semi-open. Clearly X is not γ-extremally disconnected space. Here $\{a\}$ is a γ-semi-regular-open set but not γ-semi-clopan set.

Theorem 1. *Let $A \subseteq X$, then $(a) \Rightarrow (b) \Rightarrow (c)$, where:*
(a) A is γ-semi-clopen.
(b) $A = scl_\gamma(sint_\gamma(A))$.
(c) $X - A$ is γ-semi-regular-open.

Proof. $(a) \Rightarrow (b)$. This is obvious by definitions of γ-semi-open and γ-semi-closed set [4].
$(b) \Rightarrow (c)$. Let $A = scl_\gamma(sint_\gamma(A))$. Then by Theorem 3.14 [4], $X - A = X - scl_\gamma(sint_\gamma(A)) = sint_\gamma(X - sint_\gamma(A)) = sint_\gamma(scl_\gamma(X - A))$, and hence $X - A$ is γ-semi-regular-open set. Hence the proof. ■

Using Proposition 1, we have the following Theorem:

Theorem 2. *If X is a γ-semi-extremally disconnected space. Then $(a) \Rightarrow (b) \Rightarrow (c)$, where:*
(a) $X - A$ is γ-semi-regular-open.
(b) A is γ-semi-regular-open.
(c) A is γ-semi-clopen.

Proof. $(a) \Rightarrow (b)$. Suppose X is γ-semi-extremally disconnected space. From Proposition 1, $X - A$ is a γ-semi-open and γ-semi-closed set, and hence A is a γ-semi-open and γ-semi-closed set. Thus $A = sint_\gamma(scl_\gamma(A))$ implies A is γ-semi-regular-open set.

$(b) \Rightarrow (c)$. This directory follows from Proposition 1. This completes as required. ■

Combining Theorems 1 and 2, we have the following:

Theorem 3. *If X is a γ-semi-extremally disconnected space. Then the following statements are equivalent:*
(a) A is γ-semi-clopen.
(b) $A = scl_\gamma(sint_\gamma(A))$.
(c) $X - A$ is γ-semi-regular-open.
(d) A is γ-semi-regular-open.

The following example shows that if the $scl_\gamma(A)$ is a γ-semi-regular-open set, then A not necessarily is γ-semi-open set.

Example 5. In the Example 1, if we take $A = \{b, c\}$, the $scl_\gamma(A)$ is a γ-semi-regular-open set but A is not a γ-semi-open set.

Corollary 1. *Let X be a γ-semi-extremally disconnected space. Then for each subset A of X, the set $scl_\gamma(sint_\gamma(A))$ is a γ-semi-regular-open set.*

Definition 10. *A point $x \in X$ is said to be a γ-θ-semi-cluster point of a subset A of X, if $scl_\gamma(U) \cap A \neq \emptyset$, for every γ-semi-open set U containing x. The set of all γ-θ-semi-cluster points of A is called the γ-θ-semi-closure of A and is denoted by $\gamma scl_\theta(A)$.*

Definition 11. *A subset A of X is said to be γ-θ-semi-closed, if $\gamma scl_\theta(A) = A$. The complement of γ-θ-semi-closed set is called γ-θ-semi-open set. Clearly a γ-θ-semi-closed (γ-θ-semi-open) is γ-semi-closed (γ-semi-open) set.*

Proposition 2. *Let A and B be subsets of a space X. Then the following properties hold:*
(a) If $A \subseteq B$, then $\gamma scl_\theta(A) \subseteq \gamma scl_\theta(B)$.
(b) If A_i is γ-θ-semi-closed in X, for each $i \in I$, then $\bigcap_{i\in I} A_i$ is γ-θ-semi-closed in X.

Proof. (a) This is obvious.
(b) Let A_i be a γ-θ-semi-closed in X for each $i \in I$. Then $A_i = \gamma scl_\theta(A_i)$ for each $i \in I$. Thus we have

$$\gamma scl_\theta(\bigcap_{i\in I} A_i) \subseteq \bigcap_{i\in I} \gamma scl_\theta(A_i) = \bigcap_{i\in I} A_i \subseteq \gamma scl_\theta(\bigcap_{i\in I} A_i).$$

Therefore, we have $\gamma scl_\theta(\bigcap_{i\in I} A_i) = \bigcap_{i\in I} A_i$ and hence $\bigcap_{i\in I} A_i$ is γ-θ-semi-closed. Hence the proof. ■

Theorem 4. *For any subset A of a γ-semi-extremally disconnected space X, the following hold:*

$$\begin{aligned}\gamma scl_\theta(A) &= \bigcap\{V : A \subseteq V \ \ and \ \ V \ \ is \ \ \gamma\text{-}\theta\text{-}semi\text{-}closed\} \\ &= \bigcap\{V : A \subseteq V \ \ and \ \ V \ \ is \ \ \gamma\text{-}semi\text{-}regular\text{-}open\}.\end{aligned}$$

Proof. Let $x \notin \gamma scl_\theta(A)$. Then there is a γ-semi-open set V with $x \in V$ such that $scl_\gamma(V) \cap A = \emptyset$. $X - scl_\gamma(V)$ is γ-semi-regular-open and hence $X - scl_\gamma(V)$ is a γ-θ-semi-closed set containing A and $x \notin X - \gamma scl_\theta(V)$. Thus we have $x \notin \bigcap\{V : A \subseteq V$ and V is γ-θ-semi-closed$\}$.

Conversely, suppose that $x \notin \bigcap\{V : A \subseteq V$ and V is γ-θ-semi-closed$\}$. Then there exists a γ-θ-semi-closed set V such that $A \subseteq V$ and $x \notin V$, and so there exists a γ-semi-open set U with $x \in U$ such that $U \subseteq scl_\gamma(U) \subseteq X - V$. Thus we have $scl_\gamma(U) \cap A \subseteq scl_\gamma(U) \cap V = \emptyset$ implies $x \notin \gamma scl_\theta(A)$. The proof of the second equation follows similarly.

This completes the proof. ■

Theorem 5. *If X is a γ-semi-extremally disconnected space and $A \subseteq X$. Then the following holds*:

(a) *$x \in \gamma scl_\theta(A)$ if and only if $V \cap A \neq \emptyset$, for each γ-semi-regular-open set V with $x \in V$.*

(b) *A is γ-θ-semi-open if and only if for each $x \in A$ there exists a γ-semi-regular-open set V with $x \in V$ such that $V \subseteq A$.*

(c) *A is a γ-semi-regular-open set if and only if A is γ-θ-semi-clopen.*

Proof. (a) and (b) follows directly from Theorem 3.

(c) Let A be a γ-semi-regular-open set. Then A is a γ-semi-open set and so $A = scl_\gamma(A) = \gamma scl_\theta(A)$ and hence A is γ-θ-semi-closed. Since $X - A$ is a γ-semi-regular-open set, by the argument above, $X - A$ is γ-θ-semi-closed and A is γ-θ-semi-open. The converse is obvious. Hence the proof. ■

It is obvious that γ-θ-semi-open $\Rightarrow$ γ-semi-regular-open $\Rightarrow$ γ-semi-open. But the converses are not necessarily true as the following example shows.

Example 6. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. For $b \in X$, define an operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} A, & \text{if} \quad b \in A\,, \\ cl(A), & \text{if} \quad b \notin A\,. \end{cases}$$

Calculations show that $\{a, b\}, \{a, c\}, \{b\}, X, \emptyset$ are γ-semi-open sets, $\{b\}, X, \emptyset$ are γ-θ-semi-open sets and γ-semi-regular-open sets are $\{a, c\}, \{b\}, X, \emptyset$. Then the subset $\{a, c\}$ is γ-semi-regular-open but not γ-θ-open set. Also $\{a, b\}$ is γ-semi-open set which is neither γ-θ-semi-open nor γ-semi-regular-open.

4. γ-s-closed spaces

Definition 12 ([6]). *A filterbase Γ in X is said to be γ-SR-converges to $x_0 \in X$, if for each γ-semi-regular-open set A with $x_0 \in A$, there exists $F \in \Gamma$ such that $F \subseteq A$.*

Definition 13 ([6]). *A filterbase Γ in X is said to be γ-SR-accumulates to $x_0 \in X$, if for each γ-semi-regular-open set A with $x_0 \in A$ and each $F \in \Gamma$, $F \cap A \neq \phi$.*

The following Theorems directly follow from the above definitions.

Theorem 6. *If a filterbase Γ in X, γ-SR-converges to $x_0 \in X$, then Γ γ-SR-accumulates to x_0.*

Theorem 7. *If Γ_1 and Γ_2 are filterbases in X such that Γ_2 subordinate to Γ_1 and Γ_2 γ-SR-accumulates to x_0, then Γ_1 γ-SR-accumulates to x_0.*

Theorem 8. *If Γ is a maximal filterbase in X, then Γ γ-SR-accumulates to x_0 if and only if Γ γ-SR-converges to x_0.*

Definition 14 ([6]). *A space X is said to be γ-s-closed, if every cover $\{V_\alpha : \alpha \in I\}$ of X by γ-semi-open sets has a finite subset I_0 of I such that $X = \bigcup_{\alpha \in I} scl_\gamma(V_\alpha)$.*

Proposition 3. *If γ is a semi open operation, Then the following are equivalent:*
(*a*) *X is γ-s-closed.*
(*b*) *For each family $\{A_\alpha : \alpha \in I\}$ of γ-semi-closed subsets of X such that $\bigcap_{\alpha \in I} A_\alpha = \phi$, there exists a finite subset I_0 of I such that $\bigcap_{\alpha \in I_0} sint_\gamma(A_\alpha) = \phi$.*
(*c*) *For each family $\{A_\alpha : \alpha \in I\}$ of γ-semi-closed subsets of X, if $\bigcap_{\alpha \in I_0} sint_\gamma(A_\alpha) \neq \phi$, for every finite subset I_0 of I, then $\bigcap_{\alpha \in I} A_\alpha \neq \phi$.*
(*d*) *Every filterbase Γ in X γ-SR-accumulates to $x_0 \in X$.*
(*e*) *Every maximal filterbase Γ in X γ-SR-converges to $x_0 \in X$.*

Proof. $(b) \Leftrightarrow (c)$. This is obvious.

$(b) \Rightarrow (a)$. Let $\{A_\alpha : \alpha \in I\}$ be a family of γ-semi-open subsets of X such hat $X = \bigcup_{\alpha \in I} A_\alpha$. Then each $X - A_\alpha$ is a γ-semi-closed subset of X and $\bigcap_{\alpha \in I}(X - A_\alpha) = \emptyset$, and so there exists a finite subset I_0 of I such that $\bigcap_{\alpha \in I_0} sint_\gamma(X - A_\alpha) = \emptyset$, and hence $X = \bigcup_{\alpha \in I_0}(X - sint_\gamma(X - A_\alpha)) = \bigcup_{\alpha \in I_0} scl_\gamma(A_\alpha)$. Therefore X is γ-s-closed, since γ is semi open.

$(d) \Rightarrow (b)$. Let $\{A_\alpha : \alpha \in I\}$ be a family of γ-semi-closed subsets of X such that $\bigcap_{\alpha \in I} A_\alpha = \emptyset$. Suppose that for every finite subfamily $\{A_{\alpha_i} : i = 1, 2, \ldots, n\}$, $\bigcap_{i=1}^{n} sint_\gamma(A_{\alpha_i}) \neq \emptyset$. Then $\bigcap_{i=1}^{n}(A_{\alpha_i}) \neq \emptyset$ and $\Gamma = \{\bigcap_{i=1}^{n} A_{\alpha_i} : n \in N, \alpha_i \in I\}$ forms a filterbase in X. By (4), Γ γ-SR-accumulates to some $x_0 \in X$. Thus for every γ-semi-open set A with $x_0 \in A$ and every $F \in \Gamma$, $F \cap scl_\gamma(A) \neq \emptyset$. Since $\bigcap_{F\in\Gamma} F = \emptyset$, there exists a $F \in \Gamma$ such that $x_0 \notin F$, and so there exists $\alpha_0 \in I$ such that $x_0 \notin A_{\alpha_0}$ and hence $x_0 \in X - A_{\alpha_0}$ and $X - A_{\alpha_0}$ is a γ-semi-open set. Thus $x_0 \notin sint_\gamma(A_{\alpha_0})$ and $x_0 \in X - sint_\gamma(A_{\alpha_0})$, and hence $F_0 \cap (X - sint_\gamma(A_{\alpha_0})) = F_0 \cap scl_\gamma(X - A_{\alpha_0}) = \emptyset$, which is a contradiction to our hypothesis.

$(e) \Rightarrow (d)$. Let Γ be filterbase in X. Then there exists a maximal filterbase ξ in X such that ξ subordinate to Γ. Since ξ γ-SR-converges to x_0, so by Theorems 7 and 8, Γ γ-SR-accumulate to x_0 .

$(a) \Rightarrow (e)$. Suppose that $\Gamma = \{F_a : a \in I\}$ is a maximal filterbase in X which does not γ-SR-converge to any point in X. From Theorem 8, Γ does not γ-SR-accumulates at any point in X. Thus for every $x \in X$, there exists a γ-semi-open set A_x containing x and $F_{a_x} \in \Gamma$ such that $F_{a_x} \cap scl_\gamma(A_x) = \emptyset$. Since $\{A_x : x \in X\}$ is γ-semi-open cover of X, there exists a finite subfamily $\{A_{x_i} : i = 1, 2, \ldots, n\}$ such that $X = \bigcup_{i=1}^{n} scl_\gamma(A_{x_i})$. Because Γ is a filterbase in X, there exists $F_0 \in \Gamma$ such that $F_0 \subseteq \bigcap_{i=1}^{n} F_{a_{x_i}}$, and hence $F_0 \cap scl_\gamma(A_{x_i})) = \phi$, for all $i = 1, 2, \ldots, n$. Hence we have that, $\emptyset = F_0 \bigcap (\bigcup_{i=1}^{n} scl_\gamma(A_{x_i})) = F_0 \cap X$, and hence $F_0 = \emptyset$.

This is a contradiction. Hence the proof. ■

Definition 15. *A net $(x_i)_{i\in D}$ in a space X is said to be γ-SR-converges to $x \in X$, if for each γ-semi-open set U with $x \in U$, there exists i_0 such that $x_i \in scl_\gamma(U)$ for all $i \geqslant i_0$, where D is a directed set.*

Definition 16. *A net $(x_i)_{i\in D}$ in a space X is said to be γ-SR-accumulates to $x \in X$, if for each γ-semi-open set U with $x \in U$ and each i, $x_i \in scl_\gamma(U)$, where D is a directed set.*

The proofs of following Propositions are easy and thus are omitted:

Proposition 4. *Let $(x_i)_{i\in D}$ be a net in X. For the filterbase $F((x_i)_{i\in D}) = \{\{x_i : i \leq j\} : j \in D\}$ in X,*

(a) $F((x_i)_{i\in D})$ γ-SR-converges to x if and only if $(x_i)_{i\in D}$ γ-SR-converges to x.

(b) $F((x_i)_{i\in D})$ γ-SR-accumulates to x if and only if $(x_i)_{i\in D}$ γ-SR-accumulates to x.

Proposition 5. *Every filterbase F in X determines a net $(x_i)_{i\in D}$ in X such that*

(a) $F\gamma$-SR-converges to x if and only if $(x_i)_{i\in D}$ γ-SR-converges to x.

(b) $F\gamma$-SR-accumulates to x if and only if $(x_i)_{i\in D}$ γ-SR-accumulates to x.

From Propositions 4 and 5, filterbses and nets are equivalent in the sense of γ-SR-converges and γ-SR-accumulates. Thus we have the following Theorem:

Theorem 9. *For a space X, the following are equivalent:*
(a) *X is γ-s-closed.*
(b) *Each net $(x_i)_{i\in D}$ in X has a γ-SR-accumulation point.*
(c) *Each universal net in X γ-SR-converges.*

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