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ON ITERATE MINIMAL STRUCTURES AND M-ITERATE CONTINUOUS FUNCTIONS

Abstract. We introduce the notion of mIT-structures determined by operators mInt and mCl on an m-space (X, m_X). By using mIT-structures, we introduce and investigate a function $f : (X, mIT) \to (Y, m_Y)$ called MIT-continuous. As special cases of MIT-continuity, we obtain M-semicontinuity [21] and M-precontinuity [23].

Key words: m-structure, M-continuous, m-semiopen, m-preopen, mIT-structure, MIT-continuous.

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1. Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets and b-open sets play an important role in the researches of generalizations of continuity in topological spaces. By using these sets, several authors introduced and studied various types of non-continuous functions. Certain of these non-continuous functions have properties similar to those of continuous functions and they hold, in many part, parallel to the theory of continuous functions.

In [26] and [27], the present authors introduced and studied the notions of minimal structures, m-spaces, m-continuity and M-continuity. Quite recently, in [19], [20] and [22], Min and Kim introduced the notions of m-semiopen sets, m-preopen sets and αm-open sets which generalize the notion of m-open sets and also M-semicontinuity, M-precontinuity and αM-continuity which generalize the notion of M-continuity. Rosas et al. [30] also introduced the notions of m-semiopen sets and m-preopen sets. The notion of βm-open sets is introduced by Boonpok [5].

The notions of m-semiopen sets, m-preopen sets, αm-open sets and βm-open sets are defined by using the m-interior mInt and the m-closure mCl on an m-space (X, m_X). The each family of m-semiopen sets, m-preopen sets, αm-open sets or βm-open sets becomes an m-structure with property $\mathcal{B}$, that is, it is closed under arbitrary union. The purpose of the present paper is to obtain the unified theory of M-semicontinuity, M-precontinuity, αM-continuity, βM-continuity and M-b-continuity.

2. Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. We recall some generalized open sets in topological spaces.

Definition 1. *Let (X, τ) be a topological space. A subset A of X is said to be*
(*a*) *α-open [24] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,*
(*b*) *semi-open [11] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,*
(*c*) *preopen [16] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,*
(*d*) *b-open [4] or γ-open [9] if $A \subset \mathrm{Int}(\mathrm{Cl}(A)) \cup \mathrm{Cl}(\mathrm{Int}(A))$,*
(*e*) *β-open [1] or semi-preopen [3] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$.*

The family of all α-open (resp. semi-open, preopen, b-open, β-open) sets in (X, τ) is denoted by $\alpha(X)$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\mathrm{BO}(X)$, $\beta(X)$).

Definition 2. *Let (X, τ) be a topological space. A subset A of X is said to be α-closed [18] (resp. semi-closed [6], preclosed [16], b-closed [4], β-closed [1]) if the complement of A is α-open (resp. semi-open, preopen, b-open, β-open).*

Definition 3. *Let (X, τ) be a topological space and A a subset of X. The intersection of all α-closed (resp. semi-closed, preclosed, b-closed, β-closed) sets of X containing A is called the α-closure [18] (resp. semi-closure [6], preclosure [10], b-closure [4], β-closure [2]) of A and is denoted by $\alpha\mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\mathrm{bCl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$).*

Definition 4. *Let (X, τ) be a topological space and A a subset of X. The union of all α-open (resp. semi-open, preopen, b-open, β-open) sets of X contained in A is called the α-interior [18] (resp. semi-interior [6], preinterior [10], b-interior [4], β-interior [2]) of A and is denoted by $\alpha\mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\mathrm{bInt}(A)$, ${}_{\beta}\mathrm{Int}(A)$).*

Definition 5. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be irresolute [7] (resp. preirresolute [29] or M-preirresolute [17], α-irresolute [13] or strongly feebly continuous [12], γ-irresolute ($=$ b-irresolute) [8], β-irresolute [14]) at $x \in X$ if for each semi-open (resp. preopen, α-open, γ-open, β-open) set V containing $f(x)$, there exists a semi-open (resp. preopen, α-open, γ-open, β-open) set U of X containing x such that $f(U) \subset V$. The function f is said to be irresolute (resp. preirresolute, α-irresolute, γ-irresolute, β-irresolute) if it has this property at each point $x \in X$.*

3. Minimal structures and M-continuity

Definition 6. *Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X. A subfamily m_X of $\mathcal{P}(X)$ is called a minimal structure (briefly m-structure) on X [26], [27] if $\emptyset \in m_X$ and $X \in m_X$.*

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and call it an m-space. Each member of m_X is said to be m_X-*open* (briefly m-open) and the complement of an m_X-open set is said to be m_X-*closed* (briefly m-closed).

Remark 1. Let (X, τ) be a topological space. The families τ, $\alpha(X)$, $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\mathrm{BO}(X)$ and $\beta(X)$ are all minimal structures on X.

Definition 7. *Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X-closure of A and the m_X-interior of A are defined in [15] as follows:*
(a) $\mathrm{mCl}(A) = \cap\{F : A \subset F, X \setminus F \in m_X\}$,
(b) $\mathrm{mInt}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Remark 2. Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\mathrm{BO}(X)$, $\beta(X)$), then we have
(a) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, $\mathrm{bCl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$),
(b) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, $\mathrm{bInt}(A)$, ${}_{\beta}\mathrm{Int}(A)$).

Lemma 1 (Maki et al. [15]). *Let X be a nonempty set and m_X a minimal structure on X. For subsets A and B of X, the following properties hold:*
(a) $\mathrm{mCl}(X \setminus A) = X \setminus \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X \setminus A) = X \setminus \mathrm{mCl}(A)$,
(b) *If* $(X \setminus A) \in m_X$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m_X$, *then* $\mathrm{mInt}(A) = A$,
(c) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,
(d) *If* $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,
(e) $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,
(f) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Lemma 2 (Popa and Noiri [26]). *Let (X, m_X) be an m-space and A a subset of X. Then $x \in \mathrm{mCl}(A)$ if and only if $U \cap A \neq \emptyset$ for each $U \in m_X$ containing x.*

Definition 8. *A minimal structure m_X on a nonempty set X is said to have* property $\mathcal{B}$ *[15] if the union of any family of subsets belonging to m_X belongs to m_X.*

Remark 3. If (X, τ) is a topological space, then the m-structures $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\mathrm{BO}(X)$ and $\beta(X)$ have property $\mathcal{B}$.

Lemma 3 (Popa and Noiri [28]). *Let X be a nonempty set and m_X an m-structure on X satisfying property $\mathcal{B}$. For a subset A of X, the following properties hold:*
(a) $A \in m_X$ *if and only if* $\mathrm{mInt}(A) = A$,
(b) A *is* m_X*-closed if and only if* $\mathrm{mCl}(A) = A$,
(c) $\mathrm{mInt}(A) \in m_X$ *and* $\mathrm{mCl}(A)$ *is* m_X*-closed.*

Definition 9. *A function $f : (X, m_X) \to (Y, m_Y)$ is said to be M-continuous at $x \in X$ [26] if for each m_Y-open set V containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$. The function f is M-continuous if it has this property at each $x \in X$.*

Theorem 1 (Popa and Noiri [26]). *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties are equivalent:*
(a) f *is* M*-continuous*;
(b) $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$ *for every* m*-open set* V *of* Y;
(c) $f^{-1}(F) = \mathrm{mCl}(f^{-1}(F))$ *for every* m*-closed set* F *of* Y;
(d) $\mathrm{mCl}(f^{-1}(B)) \subset f^{-1}(\mathrm{mCl}(B))$ *for every subset* B *of* Y;
(e) $f(\mathrm{mCl}(A)) \subset \mathrm{mCl}(f(A))$ *for every subset* A *of* X;
(f) $f^{-1}(\mathrm{mInt}(B)) \subset \mathrm{mInt}(f^{-1}(B))$ *for every subset* B *of* Y.

Corollary 1 (Popa and Noiri [26]). *For a function $f : (X, m_X) \to (Y, m_Y)$, where m_X has property $\mathcal{B}$, the following properties are equivalent:*
(a) f *is* M*-continuous*;
(b) $f^{-1}(V)$ *is* m*-open in* X *for every* m*-open set* V *of* Y*;*
(c) $f^{-1}(F)$ *is* m*-closed in* X *for every* m*-closed set* F *of* Y.

For a function $f : (X, m_X) \to (Y, m_Y)$, we define $D_M(f)$ as follows:

$$D_M(f) = \{x \in X : f \text{ is not } M\text{-continuous at } x\}.$$

Theorem 2 (Noiri and Popa [25]). *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties hold:*

$$\begin{aligned} D_M(f) &= \textstyle\bigcup_{G \in m_Y} \{f^{-1}(G)\text{-}\mathrm{mInt}(f^{-1}(G))\} \\ &= \textstyle\bigcup_{B \in \mathcal{P}(Y)} \{f^{-1}(\mathrm{mInt}(B))\text{-}\mathrm{mInt}(f^{-1}(B))\} \\ &= \textstyle\bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{mCl}(f^{-1}(B)) - f^{-1}(\mathrm{mCl}(B))\} \\ &= \textstyle\bigcup_{A \in \mathcal{P}(X)} \{\mathrm{mCl}(A) - f^{-1}(\mathrm{mCl}(f(A)))\} \\ &= \textstyle\bigcup_{F \in \mathcal{F}} \{\mathrm{mCl}(f^{-1}(F)) - f^{-1}(F)\}, \end{aligned}$$

where $\mathcal{F}$ is the family of m-closed sets of (Y, m_Y).

4. m-Iterate structures and M-iterate continuity

Definition 10. *Let (X, m_X) be an m-space. A subset A of X is said to be*

(a) *αm-open [20] if* $A \subset \mathrm{mInt}(\mathrm{mCl}(\mathrm{mInt}(A)))$,
b) *m-semiopen [19] if* $A \subset \mathrm{mCl}(\mathrm{mInt}(A))$,
(c) *m-preopen [22] if* $A \subset \mathrm{mInt}(\mathrm{mCl}(A))$,
(d) *βm-open [5] if* $A \subset \mathrm{mCl}(\mathrm{mInt}(\mathrm{mCl}(A)))$,
(e) *m-b-open if* $A \subset \mathrm{mInt}(\mathrm{mCl}(A)) \cup \mathrm{mCl}(\mathrm{mInt}(A))$.

The family of all αm-open (resp. m-semiopen, m-preopen, βm-open, m-b-open) sets in (X, m_X) is denoted by $\alpha\mathrm{m}(X)$ (resp. $\mathrm{mSO}(X)$, $\mathrm{mPO}(X)$, $\beta\mathrm{m}(X)$, $\mathrm{mBO}(X)$).

Remark 4. Let (X, m_X) be an m-space.
(a) Similar definitions of m-semiopen sets, m-preopen sets, αm-open sets, βm-open sets are provided in [30].
(b) The families $\alpha\mathrm{m}(X)$, $\mathrm{mSO}(X)$, $\mathrm{mPO}(X)$, $\beta\mathrm{m}(X)$ and $\mathrm{mBO}(X)$ are all minimal structures on X.

Let (X, m_X) be an m-space. Then $\mathrm{mSO}(X)$, $\mathrm{mPO}(X)$, $\alpha\mathrm{m}(X)$, $\beta\mathrm{m}(X)$ and $\mathrm{mBO}(X)$ are determined by iterating operators mInt and mCl. Hence, they are called *m-iterate structures* and are denoted by $\mathrm{mIT}(X)$ (briefly mIT).

Remark 5. (a) It easily follows from Lemma 3.1(3)(4) that $\mathrm{mSO}(X)$, $\mathrm{mPO}(X)$, $\alpha\mathrm{m}(X)$, $\beta\mathrm{m}(X)$ and $\mathrm{mBO}(X)$ are minimal structures with property $\mathcal{B}$. They are also shown in Theorem 3.5 of [19], Theorem 3.4 of [22] and Theorem 3.4 of [20] for $\mathrm{mSO}(X)$, $\mathrm{mPO}(X)$ and $\alpha\mathrm{m}(X)$, respectively.
(b) Let (X, m_X) be an m-space and $\mathrm{mIT}(X)$ an m-iterate structure on X. If $\mathrm{mIT}(X) = \mathrm{mSO}(X)$ (resp. $\mathrm{mPO}(X)$, $\alpha\mathrm{m}(X)$, $\beta\mathrm{m}(X)$), $\mathrm{mBO}(X)$), then we obtain the following definitions (for $\mathrm{mSO}(X)$, $\mathrm{mPO}(X)$ and $\alpha\mathrm{m}(X)$, they are provided in [19], [23] and [20], respectively):
$\mathrm{mITCl}(A) = \mathrm{msCl}(A)$ (resp. $\mathrm{mpCl}(A)$, $\alpha\mathrm{mCl}(A)$, $\beta\mathrm{mCl}(A)$, $\mathrm{mbCl}(A)$),
$\mathrm{mITInt}(A) = \mathrm{msInt}(A)$ (resp. $\mathrm{mpInt}(A)$, $\alpha\mathrm{mInt}(A)$, $\beta\mathrm{mInt}(A)$, $\mathrm{mbInt}(A)$).

Remark 6. (1) By Lemmas 1 and 3, we obtain Theorem 3.9 of [19], Theorems 2.3 and 2.4 of [23] and Theroems 3.8 and 3.9 of [20].
(b) By Lemma 2, we obtain Theorem 3.10 of [19], Lemma 3.9 of [22] and Theorem 3.10 of [20].

Definition 11. *A function $f : (X, m_X) \to (Y, m_Y)$ is said to be M-semicontinuous [19]* (*resp. M-precontinuous [22], αM-continuous [20], βM-continuous, M-b-continuous*) *at $x \in X$ if for each m-open set V containing $f(x)$, there exists m-semiopen set* (*resp. m-preopen, αm-open, βm-open,*

m-b-open) set U of X containing x such that $f(U) \subset V$. The function f is said to be M-semicontinuous (resp. M-precontinuous, αM-continuous, βM-continuous, M-b-continuous) if it has this property at each $x \in X$.

Remark 7. By Definition 11 and Remark 5, it follows that a function $f : (X, m_X) \to (Y, m_Y)$ is M-semicontinuous if a function $f : (X, \mathrm{mSO}(X)) \to (Y, m_Y)$ is M-continuous.

Definition 12. *A function $f : (X, m_X) \to (Y, m_Y)$ is said to be MIT-continuous at $x \in X$ (on X) if $f : (X, mIT(X)) \to (Y, m_Y)$ is M-continuous at $x \in X$ (on X).*

Remark 8. Let (X, m_X) be a minimal space. If $\mathrm{mIT}(X) = \mathrm{mSO}(X)$ (resp. $\mathrm{mPO}(X)$, $\alpha\mathrm{m}(X)$, $\beta\mathrm{m}(X)$, $\mathrm{mBO}(X)$) and $f : (X, m_X) \to (Y, m_Y)$ is MIT-continuous, then f is M-semicontinuous (resp. M-precontinuous, αM-continuous, βM-continuous, M-b-continuous).

Since $\mathrm{mIT}(X)$ has property $\mathcal{B}$, by Theorems 1 and 2 and Corollary 1 we have the following theorems.

Theorem 3. *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties are equivalent:*
(a) *f is MIT-continuous*;
(b) *$f^{-1}(V)$ is mIT-open for every m-open set V of Y*;
(c) *$f^{-1}(F)$ is mIT-closed for every m-closed set F of Y;*
(d) $\mathrm{mITCl}(f^{-1}(B)) \subset f^{-1}(\mathrm{mCl}(B))$ *for every subset B of Y*;
(e) $f(\mathrm{mITCl}(A)) \subset \mathrm{mCl}(f(A))$ *for every subset A of X*;
(f) $f^{-1}(\mathrm{mInt}(B)) \subset \mathrm{mITInt}(f^{-1}(B))$ *for every subset B of Y.*

For a function $f : (X, m_X) \to (Y, m_Y)$, we define $D_{MIT}(f)$ as follows:

$$D_{MIT}(f) = \{x \in X : f \text{ is not } MIT\text{-continuous at } x\}.$$

Theorem 4. *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties hold:*

$$\begin{aligned} D_{MIT}(f) &= \textstyle\bigcup_{G \in m_Y} \{f^{-1}(G)\text{-}\mathrm{mITInt}(f^{-1}(G))\} \\ &= \textstyle\bigcup_{B \in \mathcal{P}(Y)} \{f^{-1}(\mathrm{mInt}(B))\text{-}\mathrm{mITInt}(f^{-1}(B))\} \\ &= \textstyle\bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{mITCl}(f^{-1}(B)) - f^{-1}(\mathrm{mCl}(B))\} \\ &= \textstyle\bigcup_{A \in \mathcal{P}(X)} \{\mathrm{mITCl}(A) - f^{-1}(\mathrm{mCl}(f(A)))\} \\ &= \textstyle\bigcup_{F \in \mathcal{F}} \{\mathrm{mITCl}(f^{-1}(F)) - f^{-1}(F)\}, \end{aligned}$$

where $\mathcal{F}$ is the family of m-closed sets of (Y, m_Y).

Remark 9. (a) If $\mathrm{mIT}(X) = \mathrm{mSO}(X)$ (resp. $\mathrm{mPO}(X)$, $\alpha\mathrm{m}(X)$, $\beta\mathrm{m}(X)$, $\mathrm{mBO}(X)$) and $f : (X, m_X) \to (Y, m_Y)$ is MIT-continuous, then by Theorems 3 and 4 we obtain characterizations of M-semicontinuous (resp. M-precontinuous, αM-continuous, βM-continuous, M-b-continuous) functions.

(b) If $\mathrm{mIT}(X) = \mathrm{mSO}(X)$ (resp. $\mathrm{mPO}(X)$, $\alpha\mathrm{m}(X)$), then by Theorem 3 we obtain Theorem 3.15 of [19] (resp. Theorem 3.12 of [22], Theorem 3.14 of [20])).

For example, for $\mathrm{mIT}(X) = \beta\mathrm{m}(X)$ and $m_Y = \beta(Y)$, we obtain the following characterizations.

Corollary 2. *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties are equivalent:*

(a) *f is βM-continuous*;

(b) *$f^{-1}(V)$ is βm-open for every β-open set V of Y*;

(c) *$f^{-1}(F)$ is βm-closed for every β-closed set F of Y*;

(d) *$\beta\mathrm{mCl}(f^{-1}(B)) \subset f^{-1}({}_{\beta}\mathrm{Cl}(B))$ for every subset B of Y*;

(d) *$f(\beta\mathrm{mCl}(A)) \subset {}_{\beta}\mathrm{Cl}(f(A))$ for every subset A of X*;

(e) *$f^{-1}({}_{\beta}\mathrm{Int}(B)) \subset \beta\mathrm{mInt}(f^{-1}(B))$ for every subset B of Y.*

5. Some properties of MIT-continuous functions

Since the study of MIT-continuity is reduced from the study of M-continuity, the properties of MIT-continuous functions follow from the properties of M-continuous functions in [26].

Definition 13. *An m-space (X, m_X) is said to be m-T_2 [26] if for each distinct points $x, y \in X$, there exist $U, V \in m_X$ containing x and y, respectively, such that $U \cap V = \emptyset$.*

Definition 14. *An m-space (X, m_X) is said to be mIT-T_2 if the m-space $(X, mIT(X))$ is m-T_2.*

Hence, an m-space (X, m_X) is mIT-T_2 if for each distinct points $x, y \in X$, there exist $U, V \in mIT(X)$ containing x and y, respectively, such that $U \cap V = \emptyset$.

Remark 10. Let (X, m_X) be an m-space. If $\mathrm{mIT}(X) = \mathrm{mSO}(X)$ (resp. $\mathrm{mPO}(X)$), then by Definition 14 we obtain the definition of m-semi-T_2 spaces in [21] (resp. m-pre-T_2-spaces in [23]).

Lemma 4 (Popa and Noiri [26])**.** *If $f : (X, m_X) \to (Y, m_Y)$ is an M-continuous injection and (Y, m_Y) is m-T_2, then (X, m_X) is m-T_2.*

Theorem 5. *If $f : (X, m_X) \to (Y, m_Y)$ is an MIT-continuous injection and (Y, m_Y) is m-T_2, then X is mIT-T_2.*

Proof. The proof follows from Definition 14 and Lemma 4. ∎

Definition 15. *An m-space (X, m_X) is said to be m-compact [26] if every cover of X by m_X-open sets of X has a finite subcover.*

A subset K of an m-space (X, m_X) is said to be *m-compact* [26] if every cover of K by m_X-open sets of X has a finite subcover.

Definition 16. *An m-space (X, m_X) is said to be mIT-compact if the m-space $(X, mIT(X))$ is m-compact.*

A subset K of an m-space (X, m_X) is said to be *mIT-compact* if every cover of K by mIT-open sets of X has a finite subcover.

Remark 11. Let (X, m_X) be an m-space. If $\mathrm{mIT}(X) = \mathrm{mSO}(X)$ (resp. $\mathrm{mPO}(X)$), then by Definition 16 we obtain the definition of m-semicompact spaces in [21] (resp. m-precompact spaces in [23]).

Lemma 5 (Popa and Noiri [26])**.** *Let $f : (X, m_X) \to (Y, m_Y)$ be an M-continuous function. If K is an m-compact set of X, then $f(K)$ is m-compact.*

Theorem 6. *If $f : (X, m_X) \to (Y, m_Y)$ is an MIT-continuous function and K is an mIT-compact set of X, then $f(K)$ is m-compact.*

Proof. The proof follows from Definition 16 and Lemma 5. ■

Definition 17. *A function $f : (X, m_X) \to (Y, m_Y)$ is said to have a strongly m-closed graph (resp. m-closed graph) [26] if for each $(x, y) \in (X \times Y) - \mathrm{G}(f)$, there exist $U \in m_X$ containing x and $V \in m_Y$ containing y such that $[U \times \mathrm{mCl}(V)] \cap \mathrm{G}(f) = \emptyset$ (resp. $[U \times V] \cap \mathrm{G}(f) = \emptyset$).*

Definition 18. *A function $f : (X, m_X) \to (Y, m_Y)$ is said to have a strongly mIT-closed graph (resp. mIT-closed graph) if a function $f : (X, mIT(X)) \to (Y, m_Y)$ has a strongly m-closed graph (resp. m-closed graph).*

Hence, a function $f : (X, m_X) \to (Y, m_Y)$ has a strongly mIT-closed graph (resp. mIT-closed graph) if for each $(x, y) \in (X \times Y) - \mathrm{G}(f)$, there exist $U \in mIT(X)$ containing x and $V \in m_Y$ containing y such that $[U \times \mathrm{mCl}(V)] \cap \mathrm{G}(f) = \emptyset$ (resp. $[U \times V] \cap \mathrm{G}(f) = \emptyset$).

Lemma 6 (Popa and Noiri [26])**.** *If $f : (X, m_X) \to (Y, m_Y)$ is an M-continuous function and (Y, m_Y) is m-T_2, then f has a strongly m-closed graph.*

Theorem 7. *If $f : (X, m_X) \to (Y, m_Y)$ is an MIT-continuous function and (Y, m_Y) is m-T_2, then f has a strongly mIT-closed graph.*

Proof. The proof follows from Definition 18 and Lemma 6. ■

Lemma 7 (Popa and Noiri [26]). *If $f : (X, m_X) \to (Y, m_Y)$ is a surjective function with a strongly m-closed graph, then (Y, m_Y) is m-T_2.*

Theorem 8. *If $f : (X, m_X) \to (Y, m_Y)$ is a surjective function with a strongly mIT-closed graph, then (Y, m_Y) is m-T_2.*

Proof. The proof follows from Definition 18 and Lemma 7. ■

Lemma 8 (Popa and Noiri [26]). *Let (X, m_X) be an m-space and m_X have property $\mathcal{B}$. If $f : (X, m_X) \to (Y, m_Y)$ is an injective M-continuous function with an m-closed graph, then X is m-T_2.*

Theorem 9. *If $f : (X, m_X) \to (Y, m_Y)$ is an injective MIT-continuous function with an mIT-closed graph, then X is mIT-T_2.*

Proof. The proof follows from Definition 18, Lemma 8 and the fact that $\mathrm{mIT}(X)$ has property $\mathcal{B}$. ■

Definition 19. *An m-space (X, m_X) is said to be m-connected [26] if X cannot be written as the union of two nonempty sets of m_X.*

Definition 20. *An m-space (X, m_X) is said to be mIT-connected if the m-space $(X, mIT(X))$ is m-connected.*

Hence, the m-space $(X, mIT(X))$ is m-connected if X cannot be written as the union of two nonempty sets of $mIT(X)$.

Lemma 9 (Popa and Noiri [26]). *Let $f : (X, m_X) \to (Y, m_Y)$ be a function, where m_X has property $\mathcal{B}$. If f is an M-continuous surjection and (X, m_X) is m-connected, then (Y, m_Y) is m-connected.*

Theorem 10. *If $f : (X, m_X) \to (Y, m_Y)$ is an mIT-continuous surjection and (X, m_X) is mIT-connected, then (Y, m_Y) is m-connected.*

Proof. The proof follows from Definition 20, Lemma 9 and the fact that $mIT(X)$ has property $\mathcal{B}$. ■

Definition 21. *Let (X, m_X) be an m-space and A a subset of X. The m-frontier of A,* $\mathrm{mFr}(A)$, *[27] is defined by* $\mathrm{mFr}(A) = \mathrm{mCl}(A) \cap \mathrm{mCl}(X \setminus A) = \mathrm{mCl}(A) \setminus \mathrm{mInt}(A)$.

Definition 22. *Let (X, m_X) be an m-space and A a subset of X. The mIT-frontier of A,* $\mathrm{mITFr}(A)$, *is defined by* $\mathrm{mITFr}(A) = \mathrm{mITCl}(A) \cap \mathrm{mITCl}(X \setminus A) = \mathrm{mITCl}(A) \setminus \mathrm{mITInt}(A)$.

Lemma 10 (Popa and Noiri [28]). *The set of all points of X at which a function $f : (X, m_X) \to (Y, m_Y)$ is not M-continuous is identical with the union of the m-frontier of the inverse images of m-open sets of (Y, m_Y) containing $f(x)$.*

Theorem 11. *The set of all points of X at which a function $f : (X, m_X) \to (Y, m_Y)$ is not MIT-continuous is identical with the union of the mIT-frontier of the inverse images of m-open sets of (Y, m_Y) containing $f(x)$.*

Proof. The proof follows from Definition 22 and Lemma 10. ■

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