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Nr 52 2014

Ahmad Al-Omari and Takashi Noiri

$\wedge_w$-SETS AND $\vee_w$-SETS IN WEAK STRUCTURES SPACE DUE TO CSÁSZÁR

Abstract. In this paper we introduce the concepts of $\wedge_w$-sets and $\vee_w$-sets in a weak structure space due to Császár. It is shown that many results in previous papers can be considered as special cases of our results.

Key words: weak structure, $\wedge_w$-set, $\vee_w$-set, w-T_1, w-$T^*_{1/2}$.

AMS Mathematics Subject Classification: 54A05, 54D10.

1. Introduction

The notion of $\wedge$-sets was introduced by Maki [7] in 1986. A subset A of a topological space is called a $\wedge$-set if it is the intersection of all open sets containing A. Recently many authors have introduced and studied modifications of $\wedge$-sets. By using a minimal structure, Cammaroto and Noiri [3] introduced the notions of $\wedge_m$-sets and $\vee_m$-sets as unified forms of these modifications. Furthermore, recently Ekici and Roy [6] have introduced and investigated the notions of $\wedge_\mu$-sets and $\vee_\mu$-sets on a generalized topological space (X, μ) due to Császár [4]. Quite recently, Császár [5] has introduced the notion of weak structures and obtained several fundamental properties of weak structures, moreover see [8].

In this paper, we introduce the notions of $\wedge_w$-sets and $\vee_w$-sets on a weak structure space (X, w) and investigate the properties of sets and spaces related to $\wedge_w$-sets and $\vee_w$-sets.

2. Preliminaries

Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X. A subfamily w of $\mathcal{P}(X)$ is called a weak structure (briefly, WS) [5] if $\phi \in w$. The pair (X, w) is called a weak structure (WS) space. Each member of a WS w is said to be w-open [5] and the complement of a w-open set is said to be w-closed. Let A be a subset of X. The union of all w-open sets contained in A is called the w-interior of A and is denoted by $i_w(A)$ [5]. The intersection of all w-closed sets containing A is called the w-closure of A and is denoted by $c_w(A)$.

For the w-interior and the w-closure, the following lemmas are useful in the sequel.

Lemma 1 ([5]). *Let w be a WS on X and A, B subsets of X, then*
(1) $i_w(A) \subseteq A \subseteq c_w(A)$.
(2) *If* $A \subseteq B$ *implies that* $i_w(A) \subseteq i_w(B)$ *and* $c_w(A) \subseteq c_w(B)$.
(3) $i_w(i_w(A)) = i_w(A)$ *and* $c_w(c_w(A)) = c_w(A)$.
(4) $i_w(X - A) = X - c_w(A)$ *and* $c_w(X - A) = X - i_w(A)$.

Lemma 2 ([5]). *Let w be a WS on X, then*
(1) $x \in i_w(A)$ *if and only if there exists* $W \in w$ *such that* $x \in W \subseteq A$.
(2) $x \in c_w(A)$ *if and only if* $W \cap A \neq \emptyset$ *whenever* $x \in W \in w$.
(3) *If* $A \in w$, *then* $A = i_w(A)$ *and if* A *is w-closed, then* $A = c_w(A)$.

Remark 1. If w is a WS on X, then
(1) $i_w(\emptyset) = \emptyset$ and $c_w(X) = X$.
(2) $i_w(X)$ is the union of all w-open sets in X.
(3) $c_w(\emptyset)$ is the intersection of all w-closed sets in X.

We call a class $\mu \subseteq \mathcal{P}(X)$ a generalized topology [4] (briefly, GT) if $\phi \in \mu$ and the arbitrary union of elements of μ belongs to μ. A set X with a GT μ on it is called a generalized topological space (briefly, GTS) and is denoted by (X, μ).

3. $\wedge_w$-sets and $\vee_w$-sets

Definition 1. *Let* w *be a WS on a set X and $A \subseteq X$. Then the subsets* $\wedge_{\mathrm{w}}(A)$ *and* $\vee_{\mathrm{w}}(A)$ *are defined as follows:*

$$\wedge_{\mathrm{w}}(A) = \begin{cases} \cap\{G : A \subseteq G, G \in \mathrm{w}\}, & \text{if there exists } G \in \mathrm{w} \\ & \text{such that } A \subseteq G; \\ X, & \text{otherwise} \end{cases}$$

and

$$\vee_{\mathrm{w}}(A) = \begin{cases} \cup\{H : H \subseteq A, X - H \in \mathrm{w}\}, & \text{if there exists } H \text{ such that} \\ & X - H \in \mathrm{w} \text{ and } H \subseteq A; \\ \phi, & \text{otherwise} \end{cases}$$

Proposition 1. *Let A, B and $\{C_\alpha : \alpha \in \Delta\}$ be subsets of a WS on X. Then the following properties hold:*

(1) $B \subseteq \wedge_w(B)$.

(2) *If* $A \subseteq B$*, then* $\wedge_w(A) \subseteq \wedge_w(B)$.
(3) $\wedge_w(\wedge_w(B)) = \wedge_w(B)$.
(4) $\cup_{\alpha\in\Delta}(\wedge_w(C_\alpha)) \subseteq \wedge_w(\cup_{\alpha\in\Delta} C_\alpha)$.
(5) $\wedge_w(\cap_{\alpha\in\Delta} C_\alpha) \subseteq \cap_{\alpha\in\Delta}(\wedge_w(C_\alpha))$.
(6) *If* $A \in w$*, then* $A = \wedge_w(A)$.
(7) $\wedge_w(X - B) = X - \vee_w(B)$.
(8) $\vee_w(B) \subseteq B$.
(9) *If* $X - B \in w$*, then* $B = \vee_w(B)$.
(10) *If* $A \subseteq B$*, then* $\vee_w(A) \subseteq \vee_w(B)$.
(11) $\vee_w(\cup_{\alpha\in\Delta} C_\alpha) \supseteq \cup_{\alpha\in\Delta}(\vee_w(C_\alpha))$.

Proof. (1), (6) and (8) are clear.

(2) If there does not exist any $U \in w$ such that $B \subseteq U$ then the proof is trivial. Suppose there exist $V \in w$ such that $B \subseteq V$ and that $x \notin \wedge_w(B)$. Then there exist a subset $U \in w$ such that $B \subseteq U$ with $x \notin U$. Since $A \subseteq B$, then $x \notin \wedge_w(A)$ and thus $\wedge_w(A) \subseteq \wedge_w(B)$.

(3) By (1), we have $\wedge_w(\wedge_w(B)) \supseteq \wedge_w(B)$. Suppose that $x \notin \wedge_w(B)$. Then there exists $U \in w$ such that $B \subseteq U$ and $x \notin U$. Since $B \subseteq \wedge_w(B) \subseteq U$, we have $x \notin \wedge_w(\wedge_w(B))$ and hence $\wedge_w(\wedge_w(B)) \subseteq \wedge_w(B)$.

(4) The proof follows from (2).

(5) Suppose that $x \notin \cap_{\alpha\in\Delta}(\wedge_w(C_\alpha))$. There exists $\alpha_0 \in \Delta$ such that $x \notin \wedge_w(C_{\alpha_0})$ and there exists a w-open set U such that $x \notin U$ and $C_{\alpha_0} \subseteq U$. Since $\cap_{\alpha\in\Delta} C_{\alpha_0} \subseteq C_{\alpha_0}$ we have $x \notin \wedge_w(\cap_{\alpha\in\Delta} C_\alpha)$ and hence $\wedge_w(\cap_{\alpha\in\Delta} C_\alpha) \subseteq \cap_{\alpha\in\Delta}(\wedge_w(C_\alpha))$.

(7) $X - \vee_w(B) = \cap\{X - F : X - B \subseteq X - F, X - F \in w\} = \wedge_w(X - B)$.

(9) If $X - B \in w$, then by (6) and (7) $X - B = \wedge_w(X - B) = X - \vee_w(B)$. Hence $B = \vee_w(B)$.

(10) This follows from (2) and (7).

(11) This follows from (10). ■

In (4), (5) and (11) of Proposition 1, the equality does not necessarily hold as shown in the next example.

Example 1. (1) Let $X = \{a, b, c\}$. Consider the WS w=$\{\phi, \{a\}, \{b\}\}$ on X. Let $A = \{a, b\}$ and $B = \{a, c\}$. Then $\wedge_w(A) = X$, $\wedge_w(B) = X$ and $\wedge_w(A \cap B) = \{a\}$. Thus $\wedge_w(A \cap B) \neq \wedge_w(A) \cap \wedge_w(B)$.

(2) Let $X = \{a, b, c\}$. Consider the $WSw = \{\phi, \{a\}, \{b\}\}$ on X. Let $A = \{a\}$ and $B = \{b\}$. Then $\wedge_w(A) = \{a\}$, $\wedge_w(B) = \{b\}$ and $\wedge_w(A \cup B) = X$. Thus $\wedge_w(A \cup B) \neq \wedge_w(A) \cup \wedge_w(B)$.

(3) Let $X = \{a, b, c\}$. Consider the $WSw = \{\phi, \{a\}, \{b, c\}\}$ on X. Let $A = \{b\}$ and $B = \{c\}$. Then $\vee_w(A) = \phi$, $\vee_w(B) = \phi$ and $\vee_w(A \cup B) = \{b, c\}$. Thus $\vee_w(A \cup B) \neq \vee_w(A) \cup \vee_w(B)$.

Definition 2. *In a WS space (X, w) a subset A is called a $\wedge_w$-set (resp. $\vee_w$-set) if $\wedge_w(A) = A$ (resp. $\vee_w(A) = A$). By $\wedge_w$ (resp. $\vee_w$), we denote the family of all $\wedge_w$-sets (resp. $\vee_w$-sets) of the WS space (X, w).*

Remark 2. It follows from Proposition 1 (6) and (9) that in a WS w if $A \in w$, then A is a $\wedge_w$-set and if $X - A \in w$ then A is a $\vee_w$-set. Also it is easy to observe form Definition 1 that, X is a $\wedge_w$-set and ϕ is a $\vee_w$-set.

Theorem 1. *If w is a WS on X, then*
(1) *ϕ and X are $\vee_w$-sets (ϕ and X are $\wedge_w$-sets).*
(2) *The union of $\vee_w$-sets is a $\vee_w$-set.*
(3) *The intersection of $\wedge_w$-sets is a $\wedge_w$-set.*

Proof. (1) This follows from Remark 2.
(2) Let $\{C_\alpha : \alpha \in \Omega\}$ be a family of $\vee_w$-sets in a WS on X. Then by Proposition 1 and Definition 2, $\cup_{\alpha\in\Omega} C_\alpha = \cup_{\alpha\in\Omega} [\vee_w(C_\alpha)] \subseteq \vee_w [\cup_{\alpha\in\Omega}(C_\alpha)] \subseteq \cup_{\alpha\in\Omega}(C_\alpha)$. Hence $\cup_{\alpha\in\Omega} C_\alpha = \vee_w [\cup_{\alpha\in\Omega}(C_\alpha)]$.
(3) Let $\{C_\alpha : \alpha \in \Omega\}$ be a family of $\wedge_w$-sets in a WS on X. Then by Proposition 1 and Definition 2, $\cap_{\alpha\in\Omega} C_\alpha = \cap_{\alpha\in\Omega} [\wedge_w(C_\alpha)] \supseteq \wedge_w [\cap_{\alpha\in\Omega}(C_\alpha)] \supseteq \cap_{\alpha\in\Omega}(C_\alpha)$. Hence $\cap_{\alpha\in\Omega} C_\alpha = \wedge_w [\cap_{\alpha\in\Omega}(C_\alpha)]$. ■

Definition 3. *A WS space (X, w) is said to be w-T_1 if for any pair of distinct points x and y of X, there exist a w-open set U of X containing x but not y and a w-open set V of X containing y but not x.*

Theorem 2. *For a WS space (X, w), the implications $(2) \Rightarrow (3) \Rightarrow (1)$ hold. If w is GT, then the following properties are equivalent:*
(1) *(X, w) is w-T_1;*
(2) *For each $x \in X$, the singleton $\{x\}$ is w-closed in (X, w);*
(3) *For each $x \in X$, the singleton $\{x\}$ is a $\wedge_w$-set.*

Proof. $(1) \Rightarrow (2)$: Let y be any point of X and $x \in X - \{y\}$. There exists $V_x \in w$ such that $x \in V_x$ and $y \notin V_x$. Hence we have $X - \{y\} = \cup_{x\in X-\{y\}} V_x$. Therefore, the singleton $\{y\}$ is w-closed in (X, w).
$(2) \Rightarrow (3)$: Let x be any point of X and $y \in X - \{x\}$. Then $x \in X - \{y\} \in w$ and $\wedge_w(\{x\}) \subseteq X - \{y\}$. Therefore, $y \notin \wedge_w(\{x\})$ and $\wedge_w(\{x\}) \subseteq \{x\}$. This shows that $\wedge_w(\{x\}) = \{x\}$. Therefore, the singleton $\{x\}$ is a $\wedge_w$-set.
$(3) \Rightarrow (1)$: Suppose that the singleton $\{x\}$ is a $\wedge_w$-set for each $x \in X$. Let x and y be any distinct points. Then $y \notin \wedge_w(\{x\})$ and there exists a w-open set U_x such that $x \in U_x$ and $y \notin U_x$. Similarly, $x \notin \wedge_w(\{y\})$ and there exists a w-open set U_y such that $y \in U_y$ and $x \notin U_y$. This shows that (X, w) is w-T_1. ■

Theorem 3. *For a WS space (X, w), the implications $(2) \Leftrightarrow (3) \Rightarrow (1)$ hold. If w is a GT, then the following properties are equivalent:*

(1) (X, w) *is* w-T_1.
(2) *Every subset of* X *is a* $\wedge_w$*-set.*
(3) *Every subset of* X *is a* $\vee_w$*-set.*

Proof. It is obvious that (2) $\Leftrightarrow$ (3).

(1) $\Rightarrow$ (3): Let A be any subset of X. Since $A = \cup\{\{x\} : x \in A\}$, by Theorem 2 A is the union of w-closed sets, hence A is a $\vee_w$-set (by Remark 2 and Theorem 1).

(2) $\Rightarrow$ (1): Let $x \in X$. Then by (2), $\{x\}$ is a $\wedge_w$-set. Let p, q be any two distint points of X. Then $q \notin \wedge_w(\{p\}) = \{p\}$. So by definition of $\wedge_w$-sets, there exists a w-open set U such that $p \in U$ but $q \notin U$. Similarly the other case can done. Thus (X, w) is w-T_1. ■

4. Generalized $\wedge_w$-sets and generalized $\vee_w$-sets

Definition 4. *In a* WS *space* (X, w)*, a subset* B *is called a generalized* $\wedge_w$*-set (briefly* $g.\wedge_w$*-set) if* $\wedge_w(B) \subseteq F$ *whenever* $B \subseteq F$ *and* F *is* w*-closed. The complement of a* $g.\wedge_w$*-set is called a* $g.\vee_w$*-set.*

Proposition 2. *In a* WS *space* (X, w)*, the following properties hold:*
(1) *Every* $\wedge_w$*-set is a* $g.\wedge_w$*-set;*
(2) *Every* $\vee_w$*-set is a* $g.\vee_w$*-set.*

Proof. (1) This follows from Definitions 2 and 4.

(2) Let B be a $\vee_w$-set subset of X. Then $B = \vee_w(B)$. By Proposition 1 (7), $\wedge_w(X - B) = X - \vee_w(B) = X - B$. Thus by (1) and Definition 4, B is a $g.\vee_w$-set. ■

Proposition 3. *Let* (X, w) *be a WS space. For each* $x \in X$*, the following properties hold:*
(1) $\{x\}$ *is* w*-open or* $X - \{x\}$ *is a* $g.\wedge_w$*-set.*
(2) $\{x\}$ *is* w*-open or* $\{x\}$ *is a* $g.\vee_w$*-set.*

Proof. (1) Suppose $\{x\}$ is not a w-open set. Then the only w-closed set F containing $X - \{x\}$ is X. Thus $\wedge_w(X - \{x\}) \subseteq F = X$ and thus $X - \{x\}$ is a $g.\wedge_w$-set of X.

(2) This follows from (1) and Definition 4. ■

Proposition 4. *If* A *is a* $g.\wedge_w$*-set of a WS space* (X, w) *and* $A \subseteq B \subseteq \wedge_w(A)$*, then* B *is a* $g.\wedge_w$*-set of* (X, w)*.*

Proof. Since $A \subseteq B \subseteq \wedge_w(A)$, by Proposition 1 (2), (3) $\wedge_w(A) = \wedge_w(B)$. Let F be any w-closed subset of X such that $B \subseteq F$. Then, $\wedge_w(B) = \wedge_w(A) \subseteq F$, since A is a $g.\wedge_w$-set. ■

Proposition 5. *A subset B of a WS space (X,w) is a $g.\vee_w$-set if and only if $U \subseteq \vee_w(B)$ whenever $U \subseteq B$ and $U \in w$.*

Proof. Let U be a w-open subset of (X,w) such that $U \subseteq B$. Then since $X - U$ is w-closed and $X - B \subseteq X - U$, we have $\wedge_w(X - B) \subseteq X - U$ by Definition 4. Hence by Proposition 1 (7) $X - \vee_w(B) \subseteq X - U$. Thus $U \subseteq \vee_w(B)$. Conversely, let F be a w-closed subset of X such that $X - B \subseteq F$. Since $X - F$ is w-open and $X - F \subseteq B$, by assumption we have $X - F \subseteq \vee_w(B)$. Then $\wedge_w(X - B) = X - \vee_w(B) \subseteq F$ by Proposition 1 (7). Thus $X - B$ is a $g.\wedge_w$-set and hence B is a $g.\vee_w$-set. ■

Corollary 1. *Let B be a $g.\vee_w$-set in a WS space (X,w). Then for every w-closed set F such that $\vee_w(B) \cup (X - B) \subseteq F$, $X = F$ holds.*

Proof. The assumption $\vee_w(B) \cup (X - B) \subseteq F$ implies that $X - F \subseteq (X - \vee_w(B)) \cap B$. Since B is a $g.\vee_w$-set, then by Proposition 5, we have $X - F \subseteq \vee_w(B)$. On the other hand, $X - F \subseteq \vee_w(B) \cap (X - \vee_w(B)) = \phi$. Therefore, we have $X = F$. ■

Corollary 2. *Let B be a $g.\vee_w$-set in a WS space (X,w). Then $\vee_w(B) \cup (X - B)$ is a w-closed set if and only if B is a $\vee_w(B)$-set.*

Proof. Suppose that $\vee_w(B) = B$, then $\vee_w(B) \cup (X - B) = X$ is w-closed.

Conversely, by Corollary 1, $X = (X - B) \cup \vee_w(B)$. Thus $(X - \vee_w(B)) \cap B = \phi$. Hence by Proposition 1 (8), $\vee_w(B) = B$. ■

Definition 5. *Let* w*be a weak structure (WS) on X. Then $A \subseteq X$ is called a* w*-generalized closed set (or simply* w*g-closed set) if $c_{\mathrm{w}}(A) \subseteq U$ whenever $A \subseteq U$ and U is* w*-open. The complement of a* w*g-closed set is called a* w*-generalized open (or simply* w*g-open) set.*

Theorem 4. *Let (X,w) be a WS space such that $H \cap c_w(K)$ is w-closed for any w-closed set H and any subset K of X. Then a subset A of X is wg-closed if and only if $c_w(A) - A$ contains no nonempty w-closed sets.*

Proof. Suppose that A is wg-closed. Let F be a w-closed subset of $c_w(A) - A$. Since $A \subseteq X - F$ and A is wg-closed, $c_w(A) \subseteq X - F$ and so $F \subseteq X - c_w(A)$. Therefore, $F = \phi$. Conversely, suppose the condition holds and $A \subseteq M$ and $M \in w$. If $c_w(A) \nsubseteq M$, then $c_w(A) \cap (X - M)$ is a nonempty w-closed subset of $c_w(A) - A$. This contradicts the hypothesis. Therefore, $c_w(A) \subseteq M$ which implies that A is wg-closed. ■

Definition 6. *A WS space (X,w) is said to be* w*-$T_{\frac{1}{2}}$ if every* w*g-closed subset of X is* w*-closed.*

Theorem 5. *For a WS space (X, w), the implications (1) $\Rightarrow$ (2) hold. If w is GT, then the following statements are equivalent:*

(1) *(X, w) is w-$T_{\frac{1}{2}}$,*

(2) *For each $x \in X$ the singleton $\{x\}$ is w-closed or w-open.*

Proof. (1) $\Rightarrow$ (2). Suppose that (X, w) is w-$T_{\frac{1}{2}}$ and let $x \in X$. If $\{x\}$ is not w-closed, then $X - \{x\}$ is not w-open, and thus X is the only possible w-open set containing $X - \{x\}$. Thus $X - \{x\}$ is wg-closed. By assumption, $X - \{x\}$ is w-closed, that is $\{x\}$ is w-open.

(2) $\Rightarrow$ (1). Suppose that every singleton of X is w-open or w-closed and let A be a wg-closed subset of X. Let $x \in c_w(A)$. We discuss the following two cases:

(a) $\{x\}$ is w-open. Then $\{x\} \cap A \neq \phi$, that is $x \in A$.

(b) $\{x\}$ is w-closed. Since A is wg-closed, it follows from Theorem 4 that $x \notin c_w(A) - A$ and so $x \in A$.

Thus in both cases, $x \in A$. Therefore, $c_w(A) = A$, that is, A is w-closed. Hence, (X, w) is w-$T_{\frac{1}{2}}$. ■

Theorem 6. *For a WS space (X, w), the implications (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3) hold. If w is GT, then the following statements are equivalent:*

(1) *(X, w) is w-$T_{\frac{1}{2}}$.*

(2) *Every $g.\wedge_w$-set is a $\wedge_w$-set.*

(3) *Every $g.\vee_w$-set is a $\vee_w$-set.*

Proof. (1) $\Rightarrow$ (2). Suppose that (X, w) is w-$T_{\frac{1}{2}}$. If A is a $g.\wedge_w$-set which is not a $\wedge_w$-set, then since $A \subseteq \wedge_w(A)$, there exists $x \in \wedge_w(A)$ such that $x \notin A$. By Theorem 5, $\{x\}$ is w-open or w-closed. We discuss two cases:

(a) $\{x\}$ is w-open. Then $X - \{x\}$ is a w-closed set containing A and A is a $g.\wedge_w$-set. Hence $\wedge_w(A) \subseteq X - \{x\}$, that is, $x \notin \wedge_w(A)$. This is a contradiction.

(b) $\{x\}$ is w-closed. Then $X - \{x\}$ is a w-open set containing A, and $\wedge_w(A) \subseteq X - \{x\}$. This is contray that $x \in \wedge_w(A)$. This contradiction proves the implication (1) $\Rightarrow$ (2).

(2) $\Rightarrow$ (1). Suppose that every $g.\wedge_w$-set is a $\wedge_w$-set and let $x \in X$. We will prove that $\{x\}$ is w-open or w-closed. If $\{x\}$ is not w-open, then $X - \{x\}$ is not w-closed, and so the only w-closed set containing $X - \{x\}$ is X. Thus, $X - \{x\}$ is a $g.\wedge_w$-set. By assumption, $X - \{x\}$ is a $\wedge_w$-set. Therefore, $X - \{x\}$ is w-open, that is, $\{x\}$ is w-closed. Hence by Theorem 5, (X, w) is w-$T_{\frac{1}{2}}$.

(2) $\Leftrightarrow$ (3). This is obvious. ■

Theorem 7 ([2]). *Let $\tau(\Lambda_w)$ be the topology generated by Λ_w. That is $\tau(\Lambda_w) = \{V : V = \bigcup_{B \in \mathcal{B}}$ for any $\mathcal{B} \subseteq \Lambda_w\}$ is a topology for X.*

Definition 7 ([1]). *A WS space (X, w) is said to be* w-R_0 *if every* w-*open set contains the* w-*closure of each of its singletons.*

Definition 8 ([2]). *A WS space (X, w) is said to be*

(1) *w-T_0 if for any pair of distinct points of X, there exists a w-open set containing one of the points but not the other.*

(2) *w-T_1 if for any pair of distinct points x and y of X, there exist a w-open set U of X containing x but not y and a w-open set V of X containing y but not x.*

Theorem 8 ([2]). *For a WS space (X, w), the following properties are equivalent:*

(1) *(X, w) is w-T_1;*
(2) *(X, w) is w-T_0 and w-R_0;*
(3) *$(X, \tau(\Lambda_w))$ is T_0 and R_0;*
(4) *$(X, \tau(\Lambda_w))$ is T_1.*

5. Conclusion

The investigation enables us to obtain a unified theory of notions related to different sets for example $\wedge$-sets, $\vee$-sets, semi-$\wedge$-sets, semi-$\vee$-sets, pre-$\wedge$-sets, pre-$\vee$-sets in topological spaces, $\wedge_m$-sets and $\vee_m$-sets in m-spaces and $\wedge_\mu$-sets and $\vee_\mu$-sets in GT spaces.

Acknowledgement. The authors wish to thank the referee for useful comments and suggestions. Especially, Theorems 7 and 8 by suggestions of the referee.

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AHMAD AL-OMARI
AL AL-BAYT UNIVERSITY
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
P.O. BOX 130095
MAFRAQ 25113, JORDAN
e-mail: omarimutah1@yahoo.com

TAKASHI NOIRI
2949-1 SHIOKITA-CHO
HINAGU, YATSUSHIRO-SHI
KUMAMOTO-KEN, 869-5142 JAPAN
e-mail: t.noiri@nifty.com

Received on 22.05.2012 and, in revised form, on 26.02.2013.