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Nr 52 2014

N. Rajesh and V. Vijayabharathi

ON ALMOST (γ,γ')-(β,β')-s-CONTINUOUS FUNCTIONS

Abstract. The aim of this paper is to introduce and study a new class of functions called almost (γ,γ')-(β,β')-s-continuous functions in topological spaces by using (γ,γ')-semiopen sets.

Key words: topological spaces, (γ,γ')-open set, (γ,γ')-semiopen set.

AMS Mathematics Subject Classification: 54A40.

1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real Analysis concerns the various modified forms of continuity, seperation axioms etc. by utilizing generalized open sets. Kasahara [3] defined the concept of an operation on topological spaces. Umehara et. al. [5] introduced the notion of $\tau_{(\gamma,\gamma')}$ which is the collection of all (γ,γ')-open sets in a topological space (X,τ). In [1] the authors, introduced the notion of (γ,γ')-semiopeness and investigated its fundamental properties. The aim of this paper is to introduce and study a new class of functions called almost (γ,γ')-(β,β')-s-continuous functions in topological spaces by using (γ,γ')-semiopen sets.

2. Preliminaries

Definition 1 ([3])**.** *Let (X,τ) be a topological space. An operation γ on the topology τ is a function from τ in to power set $\mathcal{P}(X)$ of X such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.*

Definition 2 ([5])**.** *A subset A of a topological space (X,τ) is said to be a (γ,γ')-open set if for each $x \in A$ there exist open neighborhoods U and V of x such that $U^\gamma \cup V^{\gamma'} \subset A$. The complement of a (γ,γ')-open set is called a (γ,γ')-closed set. Also $\tau_{(\gamma,\gamma')}$ denotes set of all (γ,γ')-open sets in (X,τ).*

Definition 3 ([5]). *Let A be a subset of a topological space (X,τ). A point $x \in A$ is said to be the (γ,γ')-interior point of A if there exist open neighborhods U and V of x such that $U^{\gamma} \cup V^{\gamma'} \subset A$ and we denote the set of all such points by $\mathrm{Int}_{(\gamma,\gamma')}(A)$. Thus $\mathrm{Int}_{(\gamma,\gamma')}(A) = \{x \in A : x \in U \in \tau, V \in \tau$ and $U^{\gamma} \cup V^{\gamma'} \subset A$. Note that A is (γ,γ')-open if and only if $A = \mathrm{Int}_{(\gamma,\gamma')}(A)$. A subset $A \subset X$ is called (γ,γ')-closed if and only if $X \backslash A$ is (γ,γ')-open.*

Definition 4 ([5]). *A point $x \in X$ is called the (γ,γ')-closure point of $A \subset X$, if $(U^{\gamma} \cup V^{\gamma'}) \cap A \neq \emptyset$ for any open neighborhoods U and V of x. The set of all (γ,γ')-closure points of A is called (γ,γ')-closure of A and is denoted by $\mathrm{Cl}_{(\gamma,\gamma')}(A)$. A subset A of X is called (γ,γ')-closed if $\mathrm{Cl}_{(\gamma,\gamma')}(A) \subset A$.*

Definition 5 ([1]). *A subset A of a topological space (X,τ) is said to be (γ,γ')-semiopen if there exists a (γ,γ')-open set O such that $O \subset A \subset \mathrm{Cl}_{(\gamma,\gamma')}(O)$. The set of all (γ,γ')-semiopen sets is denoted by $SO_{(\gamma,\gamma')}(X)$. A is (γ,γ')-semiclosed if and only if $X \backslash A$ is (γ,γ')-semiopen in X.*

Definition 6 ([1]). *Let A be a subset of a topological space (X,τ) and γ, γ' operators on τ.*

(1) *The intersection of all (γ,γ')-semiclosed sets containing A is called the (γ,γ')-semiclosure of A and is denoted by $s\,\mathrm{Cl}_{(\gamma,\gamma')}(A)$.*

(2) *The union of all (γ,γ')-semiopen subsets of A is called (γ,γ')-semiinterior of A and is denoted by $s\,\mathrm{Int}_{(\gamma,\gamma')}(A)$.*

Definition 7 ([1]). *A point $x \in X$ is said to be (γ,γ')-semi-θ-adherent point of a subset A of X if $s\,\mathrm{Cl}_{(\gamma,\gamma')}(U) \cap A \neq \emptyset$ for every $U \in SO_{(\gamma,\gamma')}(X)$. The set of all (γ,γ')-semi-θ-adherent points of A is called the (γ,γ')-semi-θ-closure of A and is denoted by $s_{(\gamma,\gamma')}\,\mathrm{Cl}_{\theta}(A)$. A subset A is called (γ,γ')-semi-θ-closed if $s_{(\gamma,\gamma')}\,\mathrm{Cl}_{\theta}(A) = A$. A subset A is called (γ,γ')-semi-θ-open if and only if $X \backslash A$ is (γ,γ')-semi-θ-closed.*

Definition 8 ([1]). *A subset A of a topological space (X,τ) is said to be (γ,γ')-semiregular, if it is both (γ,γ')-semiopen and (γ,γ')-semiclosed. The class of all (γ,γ')-semiregular sets of X is denoted by $SR_{(\gamma,\gamma')}(A)$.*

Definition 9 ([4]). *An operation γ on τ is said to be regular if for any open neighborhoods U, V of $x \in X$, there exists an open neighborhood W of x such that $U^{\gamma} \cap V^{\gamma} \supset W^{\gamma}$.*

Definition 10 ([4]). *An operation γ on τ is said to be open if for every neighborhood U of $x \in X$, there exists a γ-open set B such that $x \in B$ and $U^{\gamma} \supset B$.*

Definition 11 ([2]). *A subset A of a topological space (X,τ) is said to be (γ,γ')-s-closed if for every covery $\{V_{\alpha} : \alpha \in I\}$ of X by (γ,γ')-semiopen sets* *of X, there exists a finite subset I_0 of I such that $A \subset \bigcup_{\alpha\in I_0} s\operatorname{Cl}_{(\gamma,\gamma')}(V_\alpha)$. If $A = X$, the topological space (X,τ) is called a (γ,γ')-s-closed space.*

Proposition 1 ([2]). *For any space X, the following are equivalent:*
(1) *X is (γ,γ')-s-closed.*
(2) *Every cover of X by (γ,γ')-semiregular sets has a finite subcover.*

Definition 12 ([1]). *A function $f : (X,\tau) \to (Y,\tau)$ is said to be $((\gamma,\gamma'),(\beta,\beta'))$-semicontinuous if for any (β,β')-open set B in Y, $f^{-1}(B)$ is (γ,γ')-semiopen in X.*

Definition 13. *An operation $\gamma : \tau \to P(X)$ is said to be γ-open, if V^γ is γ-open for each $V \in \tau$.*

Lemma 1 ([1]). *Let A be a subset of a space X. Then we have*
(1) *If $A \in SO_{(\gamma,\gamma')}(X)$, then $s\operatorname{Cl}_{(\gamma,\gamma')}(A) = s_{(\gamma,\gamma')}\operatorname{Cl}_\theta(A)$.*
(2) *If $A \in SR_{(\gamma,\gamma')}(X)$, then A is (γ,γ')-semi-θ-closed.*

Proof. (1) Clearly $s\operatorname{Cl}_{(\gamma,\gamma')}(A) \subset s_{(\gamma,\gamma')}\operatorname{Cl}_\theta(A)$. Suppose that $x \notin s\operatorname{Cl}_{(\gamma,\gamma')}(A)$. Then, for some (γ,γ')-semiopen set U, $A\cap U = \emptyset$ and hence $A\cap s\operatorname{Cl}_{(\gamma,\gamma')}(U) = \emptyset$, since $A \in SO_{(\gamma,\gamma')}(X)$. This shows that $x \notin s_{(\gamma,\gamma')}\operatorname{Cl}_\theta(A)$. Therefore $s\operatorname{Cl}_{(\gamma,\gamma')}(A) = s_{(\gamma,\gamma')}\operatorname{Cl}_\theta(A)$.
(2) This follows from (1). ■

Lemma 2 ([1]). *Let A be a subset of a topological space (X,τ):*
(1) *If $A \in SO_{(\gamma,\gamma')}(X)$, then $s\operatorname{Cl}_{(\gamma,\gamma')}(A) \in SR_{(\gamma,\gamma')}(A)$.*
(2) *If A is (γ,γ')-open in X, then $s\operatorname{Cl}_{(\gamma,\gamma')}(A) = \operatorname{Int}_{(\gamma,\gamma')}(\operatorname{Cl}_{(\gamma,\gamma')}(X))$.*

Proof. (1) Since $s\operatorname{Cl}_{(\gamma,\gamma')}(A)$ is (γ,γ')-semiclosed, we show that $s\operatorname{Cl}_{(\gamma,\gamma')}(A) \in SO_{(\gamma,\gamma')}(X)$. Since $A \in SO_{(\gamma,\gamma')}(X)$, then for (γ,γ')-open set U of X, $U \subset A \subset \operatorname{Cl}_{(\gamma,\gamma')} U$. Therefore we have, $U \subset s\operatorname{Cl}_{(\gamma,\gamma')}(U) \subset s\operatorname{Cl}_{(\gamma,\gamma')}(A) \subset s\operatorname{Cl}_{(\gamma,\gamma')}(\operatorname{Cl}_{(\gamma,\gamma')}(U)) = \operatorname{Cl}_{(\gamma,\gamma')}(U)$ or $U \subset s\operatorname{Cl}_{(\gamma,\gamma')}(A) \subset \operatorname{Cl}_{(\gamma,\gamma')}(U)$ and hence $s\operatorname{Cl}_{(\gamma,\gamma')}(A) \in SO_{(\gamma,\gamma')}(X)$. ■

3. Almost (γ,γ')-(β,β')-s-continuous functions

Definition 14. *A function $f : (X,\tau) \to (Y,\tau)$ is said to be almost (γ,γ')-(β,β')-s-continuous if for each point $x \in X$ and each $V \in SO_{(\beta,\beta')}(Y)$, there exists a (γ,γ')-open set U containing x such that $f(U) \subseteq s\operatorname{Cl}_{(\gamma,\gamma')}(V)$.*

Example 1. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$. Define the operations $\gamma : \tau \to \mathcal{P}(X)$, $\gamma' : \tau \to \mathcal{P}(X)$, $\beta : \sigma \to \mathcal{P}(X)$ and $\beta' : \sigma \to \mathcal{P}(X)$ by

$$A^\gamma = \begin{cases} A & \text{if } b \notin A \\ \operatorname{Cl}(A) & \text{if } b \in A, \end{cases} \qquad A^{\gamma'} = \begin{cases} \operatorname{Cl}(A) & \text{if } b \notin A \\ A & \text{if } b \in A, \end{cases}$$

$$A^{\beta} = \begin{cases} A & \text{if } a \in A \\ A \cup \{a\} & \text{if } a \notin A \end{cases} \quad \text{and} \quad A^{\beta'} = \begin{cases} A & \text{if } c \in A \\ A \cup \{c\} & \text{if } c \notin A. \end{cases}$$

Clearly, $\tau_{(\gamma,\gamma')} = \{\emptyset, X, \{b\}, \{a,b\}, \{a,c\}\}$ and $SO_{(\beta,\beta')}(Y) = \{\emptyset, X, \{a,c\}\}$. The the identity function $f : (X,\tau) \to (Y,\sigma)$ is almost (γ,γ')-(β,β')-s-continuous.

Theorem 1. *The following statements are equivalent for a function $f : (X,\tau) \to (Y,\sigma)$:*

(1) *f is almost (γ,γ')-(β,β')-s-continuous.*

(2) *For each $x \in X$ and $V \in SR_{(\beta,\beta')}(Y)$, there exists a (γ,γ')-open set U containing x such that $f(U) \subseteq V$.*

(3) *$f^{-1}(V)$ is (γ,γ')-clopen ($= (\gamma,\gamma')$-open as well as (γ,γ')-closed) in X for every $V \in SR_{(\beta,\beta')}(Y)$.*

(4) *$f^{-1}(V) \subseteq \mathrm{Int}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V)))$ for every $V \in SO_{(\beta,\beta')}(Y)$.*

(5) *$\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Int}_{(\beta,\beta')}(V))) \subseteq f^{-1}(V)$ for every (β,β')-semiclosed set V of Y.*

(6) *$\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(V)) \subseteq f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V))$ for every $V \in SO_{(\beta,\beta')}(Y)$, where γ and γ' are open.*

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and $V \in SR_{(\beta,\beta')}(Y)$. There exists a (γ,γ')-open set U containing x such that $f(U) \subseteq s\,\mathrm{Cl}_{(\beta,\beta')}(V) = V$.

(2) $\Rightarrow$ (3): Let $V \in SR_{(\beta,\beta')}(Y)$ and $x \in f^{-1}(V)$. Then $f(U) \subseteq V$ for some (γ,γ')-open set U of X containing x hence $x \in U \subseteq f^{-1}(V)$. This shows that $f^{-1}(V)$ is (γ,γ')-open in X. Since $Y \backslash V \in SR_{(\beta,\beta')}(Y)$, $f^{-1}(Y \backslash V)$ is also (γ,γ')-open and hence $f^{-1}(V)$ is (γ,γ')-clopen in X.

(3) $\Rightarrow$ (4): Let $V \in SO_{(\beta,\beta')}(Y)$. Then by Lemma 2, $V \subseteq s\,\mathrm{Cl}_{(\beta,\beta')}(V)$ and $s\,\mathrm{Cl}_{(\beta,\beta')}(V) \in SR_{(\beta,\beta')}(Y)$. By (3), we have $f^{-1}(V) \subseteq f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V))$ and $f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V))$ is (γ,γ')-open in X. Therefore, we obtain $f^{-1}(V) \subseteq \mathrm{Int}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V)))$.

(4) $\Rightarrow$ (5): Let V be a (β,β')-semiclosed subset of Y. By (4), we have $f^{-1}(Y \backslash V) \subseteq \mathrm{Int}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(Y \backslash V))) = \mathrm{Int}_{(\gamma,\gamma')}(f^{-1}(Y \backslash s\,\mathrm{Int}_{(\beta,\beta')}(V))) = X \backslash \mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Int}_{(\beta,\beta')}(V)))$. Therefore, we obtain $\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Int}_{(\beta,\beta')}(V))) \subseteq f^{-1}(V)$.

(5) $\Rightarrow$ (6): Let $V \in SO_{(\beta,\beta')}(Y)$. Then $s\,\mathrm{Cl}_{(\beta,\beta')}(V) \in SR_{(\beta,\beta')}(Y)$. By Lemma 2 and (5) we obtain $\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(V)) \subseteq \mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V))) \subseteq f^{-1}(s\,\mathrm{Cl}_{(\beta,\beta')}(V))$.

(6) $\Rightarrow$ (1): Let $x \in X$ and $V \in SO_{(\beta,\beta')}(Y)$. By Lemma 2, we have $s\,\mathrm{Cl}_{(\gamma,\gamma')}(X) \in SR_{(\gamma,\gamma')}(X)$ and $f(x) \notin Y \backslash s\,\mathrm{Cl}_{(\beta,\beta')}(V) = s\,\mathrm{Cl}_{(\beta,\beta')}(Y \backslash s\,\mathrm{Cl}_{(\beta,\beta')}(V))$. Thus, by (6) we obtain $x \notin \mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(Y \backslash s\,\mathrm{Cl}_{(\beta,\beta')}(V)))$. There exists a (γ,γ')-open set U of x such that $U \cap f^{-1}(Y \backslash s\,\mathrm{Cl}_{(\beta,\beta')}(V)) = \emptyset$. Therefore, we have $f(U) \cap (Y \backslash s\,\mathrm{Cl}_{(\beta,\beta')}(V)) = \emptyset$ and hence $f(U) \subseteq s\,\mathrm{Cl}_{(\beta,\beta')}(V)$. This shows that f is almost (γ,γ')-(β,β')-s-continuous. ∎

Theorem 2. *The following statements are equivalent for a function $f : (X,\tau) \to (Y,\tau)$:*

(1) *f is almost (γ,γ')-(β,β')-s-continuous.*

(2) *For each $x \in X$ and each $V \in SR_{(\beta,\beta')}(Y)$, there exists a (γ,γ')-clopen set U containing x such that $f(U) \subseteq V$.*

(3) *For each $x \in X$ and each $V \in SO_{(\beta,\beta')}(Y)$, there exists a (γ,γ')-open set U containing x such that $f(\mathrm{Cl}_{(\gamma,\gamma')}(U)) \subseteq s\,\mathrm{Cl}_{(\beta,\beta')}(V)$.*

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and $V \in SR_{(\beta,\beta')}(Y)$. By Theorem 1. $f^{-1}(V)$ is (γ,γ')-clopen in X. Put $U = f^{-1}(V)$, then $x \in U$ and $f(U) \subseteq V$. The proof of the other implications are obvious. ■

Theorem 3. *The following statements are equivalent for a function $f : (X,\tau) \to (Y,\tau)$:*

(1) *f is almost (γ,γ')-(β,β')-s-continuous.*

(2) *$f(\mathrm{Cl}_{(\gamma,\gamma')}(A)) \subseteq s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(f(A))$ for every subset A of X.*

(3) *$\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(B)) \subseteq f^{-1}(s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(B))$ for every subset B of Y.*

(4) *$f^{-1}(F)$ is (γ,γ')-closed in X for every (β,β')-semi-θ-closed set F of Y.*

(5) *$f^{-1}(V)$ is (γ,γ')-open in X for every (β,β')-semi-θ-open set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y and $x \notin f^{-1}(s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(B))$. Then $f(x) \notin s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(B)$ and there exists $V \in SO_{(\beta,\beta')}(Y, f(x))$ such that $s_{(\beta,\beta')}\,\mathrm{Cl}(V) \cap B = \varnothing$. By (1), there exists a (γ,γ')-open set U containing x such that $f(U) \subset s_{(\beta,\beta')}\,\mathrm{Cl}(V)$. Hence $f(U) \cap B = \emptyset$ and $U \cap f^{-1}(B) = \emptyset$. Consequently, we obtain $x \notin \mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(B))$.

(2) $\Rightarrow$ (3): Let A be any subset of X. By (2), we have $\mathrm{Cl}_{(\gamma,\gamma')}(A) \subset \mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(f(A))) \subset f^{-1}(s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(f(A)))$ and hence $f(\mathrm{Cl}_{(\gamma,\gamma')}(A)) \subset s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(f(A))$.

(3) $\Rightarrow$ (4): Let F be any (β,β')-semi-θ-closed set of Y. Then, by (3), we have $f(\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(F))) \subset s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(f(f^{-1}(F))) \subset s_{(\beta,\beta')}\,\mathrm{Cl}_\theta(F) = F$. Therefore, we have $\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(F)) \subset f^{-1}(F)$ and hence $\mathrm{Cl}_{(\gamma,\gamma')}(f^{-1}(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is (γ,γ')-closed set in X.

(4) $\Rightarrow$ (5): This is obvious.

(5)$\Rightarrow$(1): Let $x \in X$ and $V \in SO_{(\beta,\beta')}(Y, f(x))$. By Lemmas 1 and 2, $s_{(\beta,\beta')}\,\mathrm{Cl}(V)$ is (β,β')-θ-open in Y. Put $U = f^{-1}(s_{(\beta,\beta')}\,\mathrm{Cl}(V))$. Then by (5), U is (γ,γ')-open containing x and $f(U) \subset s_{(\beta,\beta')}\,\mathrm{Cl}(V)$. Thus, f is almost (γ,γ')-(β,β')-s-continuous. ■

Definition 15. *A point $x \in X$ is said to be a (γ,γ')-θ-adherent point of a subset A of X if $\mathrm{Cl}_{(\gamma,\gamma')}(U) \cap A \neq \emptyset$ for every (γ,γ')-open set U containing x. The set of all (γ,γ')-θ-adherent points of A is called the (γ,γ')-θ-closure of A and is dentoed by $\mathrm{Cl}_{(\gamma,\gamma')\theta}(A)$. Note that a subset A is called (γ,γ')-θ-closed* *if* $\mathrm{Cl}_{(\gamma,\gamma')\theta}(A) = A$. *The complement of a* (γ,γ')-θ-*closed set is called a* (γ,γ')-θ-*open set.*

The proof of the following theorem is similar to Theorem 3 and thus omitted.

Theorem 4. *The following statements are equivalent for a function* $f : (X,\tau) \to (Y,\tau)$*:*

(1) f *is almost* (γ,γ')-(β,β')-*s-continuous.*

(2) $\mathrm{Cl}_{(\gamma,\gamma')\theta}(f^{-1}(A)) \subseteq f^{-1}(s_{(\beta,\beta')}\,\mathrm{Cl}_{\theta}(A))$ *for every subset* A *of* Y.

(3) $f(\mathrm{Cl}_{(\gamma,\gamma')\theta}(B)) \subseteq s_{(\beta,\beta')}\,\mathrm{Cl}_{\theta}(f(B))$ *for every subset* B *of* X.

(4) $f^{-1}(F)$ *is* (γ,γ')-θ-*closed in* X *for every* (β,β')-*semi*-θ-*closed set* F *of* Y.

(5) $f^{-1}(V)$ *is* (γ,γ')-θ-*open in* X *for every* (β,β')-*semi*-θ-*open set* V *of* Y.

Theorem 5. *If* $f : (X,\tau) \to (Y,\tau)$ *is almost* (γ,γ')-(β,β')-*s-continuous and* A *is* (γ,γ')-*s-closed in* X, *then* $f(A)$ *is* (β,β')-*s-closed in* Y.

Proof. Let $\{V_\alpha : \alpha \in I\}$ be any cover of $f(A)$ by (β,β')-semiregular sets of Y. By Theorem 1, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a cover of A by (γ,γ')-clopen sets of X. By Proposition 1, there exists a finite subset I_0 of I such that $A \subseteq \cup s\,\mathrm{Cl}_{(\gamma,\gamma')}\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ and hence $f(A) \subseteq \cup s\,\mathrm{Cl}_{(\beta,\beta')}\{V_\alpha : \alpha \in I_0\}$. Hence $f(A)$ is (β,β')-semiclosed relative to Y. ■

Theorem 6. *If* $f : (X,\tau) \to (Y,\tau)$ *is almost* (γ,γ')-(β,β')-*s-continuous surjection and* X *is* (γ,γ')-*s-closed, then* Y *is* (β,β')-*s-closed.*

Proof. The proof is clear. ■

Theorem 7. *Let* $f : (X,\tau) \to (Y,\tau)$ *be a function and* $x \in X$. *If there exists a* (γ,γ')-*open set* N *of* X *containing* x *such that the restriction* f_N *of* f *to* N *is almost* (γ,γ')-(β,β')-*s-continuous at* x, *then* f *is almost* (γ,γ')-(β,β')-*s-continuous at* x, *where* γ *and* γ' *are regular.*

Proof. Let U be any (γ,γ')-semiregular set containing $f(x)$. Since f_N is almost (γ,γ')-(β,β')-s-continuous at x, there is a (γ,γ')-open set V containing x such that $x \in N \cap V$ and $f(N \cap V) \subseteq s\,\mathrm{Cl}_{(\beta,\beta')}(U) = U$ or $f(N \cap V) \subseteq U$. Since γ and γ' are regular, $N \cap V$ is a (γ,γ')-open set containing x. Hence f is almost (γ,γ')-(β,β')-s-continuous at x. ■

Theorem 8. *Let* X_1, X_2 *be* (γ,γ')-*closed sets in a topological space* (X,τ) *and* $X = X_1 \cup X_2$. *If* $f : X \to Y$ *be a function and* f_{X_1} *and* f_{X_2} *are almost* (γ,γ')-(β,β')-*s-continuous functions, then* f *is almost* (γ,γ')-(β,β')-*s-continuous, where* γ *and* γ' *are regular.*

Proof. Let A be a (β,β')-semiregular subset of Y. Since f_{X_1} and f_{X_2} are both almost (γ,γ')-(β,β')-s-continuous, $(f_{X_1})^{-1}(A)$ and $(f_{X_2})^{-1}(A)$ are both (γ,γ')-clopen subsets of X and $f^{-1}(A) = (f_{X_1})^{-1}(A) \cup (f_{X_1})^{-1}(A)$. Since γ and γ' are regular, $f^{-1}(A)$ is the union of two (γ,γ')-clopen sets and is thus (γ,γ')-clopen in X. Hence f is almost (γ,γ')-(β,β')-s-continuous function. ■

Remark 1. The following example shows that the regularity on γ and γ' of Theorem 8 can not be removed.

Example 2. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}\}$. Define the operations $\gamma : \tau \to \mathcal{P}(X)$, $\gamma' : \tau \to \mathcal{P}(X)$, $\beta : \sigma \to \mathcal{P}(X)$ and $\beta' : \sigma \to \mathcal{P}(X)$ by

$$A^{\gamma} = \begin{cases} A & \text{if } b \notin A \\ \mathrm{Cl}(A) & \text{if } b \in A, \end{cases} \qquad A^{\gamma'} = \begin{cases} \mathrm{Cl}(A) & \text{if } b \notin A \\ A & \text{if } b \in A, \end{cases}$$

$$A^{\beta} = \begin{cases} A & \text{if } a \in A \\ A \cup \{a\} & \text{if } a \notin A \end{cases} \quad \text{and} \quad A^{\beta'} = \begin{cases} A & \text{if } c \in A \\ A \cup \{c\} & \text{if } c \notin A. \end{cases}$$

Then, it is shown that γ' is not regular. Let $X_1 = \{b\}$, $X_2 = \{a,c\}$ and $f : (X,\tau) \to (Y,\tau)$ be an identity function. Clearly, f_{X_1} and f_{X_2} are almost (γ,γ')-(β,β')-s-continuous functions but f is almost (γ,γ')-(β,β')-s-continuous function.

Theorem 9. *If $f : X \to X$ be a function f_{X_1} and f_{X_2} are both almost (γ,γ')-(β,β')-s-continuous at a point $x \in X = X_1 \cup X_2$, then f is almost (γ,γ')-(β,β')-s-continuous at x, where γ and γ' are regular operations.*

Proof. Let U be any (γ,γ')-semiregular set containing $f(x)$. Since $x \in X_1 \cap X_2$ and f_{X_1}, f_{X_2} are both almost (γ,γ')-(β,β')-s-continuous at a point x, there exists (γ,γ')-open sets V_1 and V_2 of X, respectively containing x such that $x \in X_1 \cap V_1, f(X_1 \cap V_1) \subseteq U$ and $x \in X_2 \cap V_2, f(X_2 \cap V_2) \subseteq U$. Since $X = X_1 \cup X_2$, $f(V_1 \cap V_2) = f(X_1 \cap V_1 \cap V_2) \cup f(X_2 \cap V_1 \cap V_2) \subseteq f(X_1 \cap V_1) \cup f(X_2 \cap V_2) \subseteq U$. Since γ and γ' are regular, $V_1 \cap V_2 = V$ is a (γ,γ')-open set containing x such that $f(V) \subseteq U$ and hence f is almost (γ,γ')-(β,β')-s-continuous by Theorem 1. ■

Acknowledgement. The authors wish to thank the referee for his suggestions and corrections which helped to improve this paper.

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N. Rajesh
Department of Mathematics
Rajah Serfoji Govt. College
Thanjavur-613005, Tamilnadu, India
e-mail: nrajesh_topology@yahoo.co.in

V. Vijayabharathi
Department of Mathematics
National Institute of Technology
Tiruchirappalli, Tamilnadu, India
e-mail: vijayabharathi_v@yahoo.com

Received on 22.11.2012 and, in revised form, on 25.02.2013.