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CONTRA QUOTIENT FUNCTIONS
ON GENERALIZED TOPOLOGICAL SPACES

ABSTRACT. The purpose of this paper is to study the concept of contra quotient functions on generalized topological spaces and study some of its stronger forms.

KEY WORDS: generalized topological space, λ-α-open set, λ-semi-open set, λ-preopen set, contra λ-α-irresolute function, contra λ-α-quotient function, contra λ-α*-quotient function.

AMS Mathematics Subject Classification: 57C10, 57C08, 57C05.

1. Introduction

In [2, 3, 4], Császár has introduced the theory of generalized topological spaces, and studied the extremely elementary character of these classes. Especially he introduced the notions of continuous functions on generalized topological spaces, and investigated characterizations of generalized continuous functions (= (λ, μ)-continuous functions in [4]). In [6, 7, 9], Min introduced the notions of weak (λ, μ)-continuity, almost (λ, μ)-continuity, (α, μ)-continuity, (σ, μ)-continuity, (π, μ)-continuity and (β, μ)-continuity on generalized topological spaces. In [1], Bai and Zuo introduced the notion of λ-α-irresolute functions and investigated their properties and relationships between (λ, μ)-continuity (resp. weak (λ, μ)-continuity, almost (λ, μ)-continuity, (α, μ)-continuity, (σ, μ)-continuity, (π, μ)-continuity and (β, μ)-continuity) and λ-α-irresoluteness. In [5], Jayanthi introduced the notion of contra continuous functions on generalized topological spaces.

In this paper, we introduce new classes of generalized topological functions called contra λ-α-irresolute functions, contra λ-α-quotient functions and contra λ-α*-quotient functions on generalized topological spaces. At every places the new notions have been substantiated with suitable examples.

2. Preliminaries

Definition 1 ([4]). *Let X be a nonempty set and λ be a collection of subsets of X. Then λ is called a generalized topology (briefly GT) on X if*

$\emptyset \in \lambda$ and $G_i \in \lambda$ for $i \in I \neq \emptyset$ implies $G = \cup_{i \in I} Gi \in \lambda$. We say λ is strong if $X \in \lambda$, and we call the pair (X, λ) a generalized topological space (briefly GTS) on X.

The elements of λ are called λ-open sets and their complements are called λ-closed sets. The generalized closure of a subset S of X, denoted by $c_\lambda(S)$, is the intersection of λ-closed sets containing S. And the interior of S, denoted by $i_\lambda(S)$, is the union of λ-open sets contained in S.

Definition 2 ([3]). *Let (X, λ) be a generalized topological space and $A \subset X$. Then A is said to be*
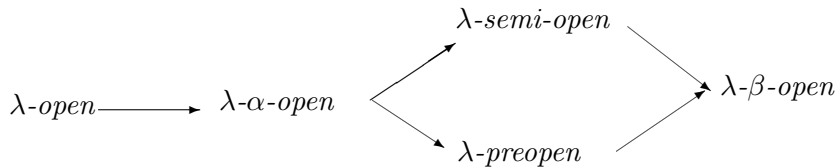
(a) *λ-semi-open if $A \subset c_\lambda(i_\lambda(A))$,*
(b) *λ-preopen if $A \subset i_\lambda(c_\lambda(A))$,*
(c) *λ-α-open if $A \subset i_\lambda(c_\lambda(i_\lambda(A)))$,*
(d) *λ-β-open if $A \subset c_\lambda(i_\lambda(c_\lambda(A)))$.*

The complement of λ-semi-open (resp. λ-preopen, λ-α-open, λ-β-open) is said to be λ-semi-closed (resp. λ-preclosed, λ-α-closed, λ-β-closed).

Let us denote by $\sigma(\lambda_X)$ (briefly σ_X or σ) the class of all λ-semi-open sets on X, by $\pi(\lambda_X)$ (briefly π_X or π) the class of all λ-preopen sets on X, by $\alpha(\lambda_X)$ (briefly α_X or α) the class of all λ-α-open sets on X, by $\beta(\lambda_X)$ (briefly β_X or β) the class of all λ-β-open sets on X.

Let us denote by $\sigma'(\lambda_X)$ (briefly σ'_X or σ') the class of all λ-semi-closed sets on X, by $\pi'(\lambda_X)$ (briefly π'_X or π') the class of all λ-preclosed sets on X, by $\alpha'(\lambda_X)$ (briefly α'_X or α') the class of all λ-α-closed sets on X, by $\beta'(\lambda_X)$ (briefly β'_X or β') the class of all λ-β-closed sets on X.

Obviously in [9] $\lambda_X \subset \alpha(\lambda_X) \subset \sigma(\lambda_X) \subset \beta(\lambda_X)$ and $\lambda_X \subset \alpha(\lambda_X) \subset \pi(\lambda_X) \subset \beta(\lambda_X)$. Namely,

$$\lambda\text{-open} \longrightarrow \lambda\text{-}\alpha\text{-open} \nearrow \begin{matrix} \lambda\text{-semi-open} \\ \\ \lambda\text{-preopen} \end{matrix} \searrow \lambda\text{-}\beta\text{-open}$$

Lemma 1 ([3]). *Let (X, λ) be a generalized topological space and $A \subset X$. Then A is λ-α-open if and only if it is λ-semi-open and λ-preopen.*

Definition 3. *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be*

(a) *(λ, μ)-continuous [4] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-open in X,*
(b) *(α, μ)-continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-α-open in X,*
(c) *(σ, μ)-continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-semi-open in X,*

(d) (π, μ)-*continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-preopen in X,*

(e) (β, μ)-*continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-β-open in X.*

Definition 4 ([5])**.** *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be*

(a) *contra (λ, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-closed in X,*

(b) *contra (α, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-α-closed in X,*

(c) *contra (σ, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-semi-closed in X,*

(d) *contra (π, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-preclosed in X,*

(e) *contra (β, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-β-closed in X.*

Remark 1 ([5])**.** Let $f : (X, \lambda) \to (Y, \mu)$ be a function between $GTS's$ (X, λ) and (Y, μ). Then we have the following implications:
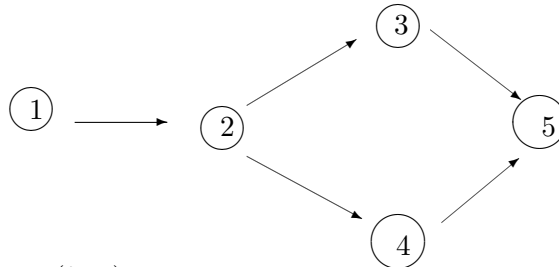
where: (1) = contra (λ, μ)-continuous,

(2) = contra (α, μ)-continuous,   (3) = contra (σ, μ)-continuous

(4) = contra (π, μ)-continuous,   (5) = contra (β, μ)-continuous.

Theorem 1 ([5])**.** *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is contra (α, μ)-continuous if and only if it is both contra (π, μ)-continuous and contra (σ, μ)-continuous.*

Definition 5 ([8])**.** *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be (λ, μ)-open if the image of each λ-open set in X is an μ-open set of Y.*

Definition 6 ([1])**.** *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be λ-α-irresolute if the inverse image of every μ-α-open set in Y is an λ-α-open set in X.*

Definition 7 ([10])**.** *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be λ-semi-irresolute (resp. λ-pre-irresolute, λ-β-irresolute) if the inverse image of every μ-semi-open (resp. μ-preopen, μ-β-open) set in Y is a λ-semi-open (resp. λ-preopen, λ-β-open) set in X.*

Definition 8 ([10])**.** *Let (X, λ) and (Y, μ) be GTS's and $f : (X, \lambda) \to (Y, \mu)$ be a surjective function. Then f is said to be*

(a) *λ-quotient provided a subset S of Y is μ-open in Y if and only if $f^{-1}(S)$ is λ-open in X.*

(b) *λ-α-quotient if f is (α, μ)-continuous and $f^{-1}(V)$ is λ-open in X implies V is an μ-α-open set in Y.*

(c) *λ-semi-quotient if f is (σ, μ)-continuous and $f^{-1}(V)$ is λ-open in X implies V is a μ-semi-open set in Y.*

(d) *λ-pre-quotient if f is (π, μ)-continuous and $f^{-1}(V)$ is λ-open in X implies V is a μ-preopen set in Y.*

Definition 9 ([10])**.** *Let (X, λ) and (Y, μ) be GTS's, and $f : (X, \lambda) \to (Y, \mu)$ be a surjective function. Then f is said to be*

(a) *λ-α*-quotient if f is λ-α-irresolute and $f^{-1}(S)$ is λ-α-open set in X implies S is μ-open set in Y.*

(b) *λ-semi-*quotient if f is λ-semi-irresolute and $f^{-1}(S)$ is λ-semi-open set in X implies S is μ-open set in Y.*

(c) *λ-pre-*quotient if f is λ-pre-irresolute and $f^{-1}(S)$ is λ-preopen set in X implies S is μ-open set in Y.*

3. Contra λ-α-irresolute functions

Definition 10. *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be contra (λ, μ)-open (resp. contra (λ, α)-open, contra (λ, σ)-open, contra (λ, π)-open, contra (λ, β)-open) if the image of each λ-open set in X is an μ-closed (resp. μ-α-closed, μ-semi-closed, μ-preclosed, μ-β-closed) set of Y.*

Definition 11. *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be contra λ-α-irresolute (resp. contra λ-semi-irre-solute, contra λ-pre-irresolute, contra λ-β-irresolute) if the inverse image of every μ-α-open (resp. μ-semi-open, μ-preopen, μ-β-open) set in Y is an λ-α-closed (resp. λ-semi-closed, λ-preclosed, λ-β-closed) set in X.*

Theorem 2. *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-semi-irresolute if and only if for every μ-semi-closed subset A of Y, $f^{-1}(A)$ is λ-semi-open in X.*

Proof. If f is contra λ-semi-irresolute, then for every μ-semi-open subset B of Y, $f^{-1}(B)$ is λ-semi-closed in X. If A is any μ-semi-closed subset of Y, then $Y - A$ is μ-semi-open. Thus $f^{-1}(Y - A)$ is λ-semi-closed but $f^{-1}(Y - A) = X - f^{-1}(A)$ so that $f^{-1}(A)$ is λ-semi-open in X.

Conversely, if, for all μ-semi-closed subsets A of Y, $f^{-1}(A)$ is λ-semi-open in X and if B is any μ-semi-open subset of Y, then $Y - B$ is μ-semi-closed. Also $f^{-1}(Y - B) = X - f^{-1}(B)$ is λ-semi-open. Thus $f^{-1}(B)$ is λ-semi-closed in X. Hence f is contra λ-semi-irresolute. ■

Theorem 3. *Let* (X, λ), (Y, μ) *and* (Z, γ) *be* $GTS's$, *and* f *and* g *be two functions. If* $f : (X, \lambda) \to (Y, \mu)$ *is contra* λ-*semi-irresolute and* $g : (Y, \mu) \to (Z, \gamma)$ *is contra* μ-*semi-irresolute then* $gof : (X, \lambda) \to (Z, \gamma)$ *is* λ-*semi-irresolute.*

Proof. If $A \subseteq Z$ is γ-semi-open, then $g^{-1}(A)$ is μ-semi-closed set in Y because g is contra μ-semi-irresolute. Consequently since f is contra λ-semi-irresolute, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is λ-semi-open set in X. Hence gof is λ-semi-irresolute. ■

Corollary 1. *Let* (X, λ), (Y, μ) *and* (Z, γ) *be* $GTS's$. *If the function* $f : (X, \lambda) \to (Y, \mu)$ *is contra* λ-α-*irresolute and the function* $g : (Y, \mu) \to (Z, \gamma)$ *is contra* μ-α-*irresolute then* $gof : (X, \lambda) \to (Z, \gamma)$ *is* λ-α-*irresolute.*

Corollary 2. *Let* (X, λ), (Y, μ) *and* (Z, γ) *be* $GTS's$. *If the function* $f : (X, \lambda) \to (Y, \mu)$ *is contra* λ-α-*irresolute and the function* $g : (Y, \mu) \to (Z, \gamma)$ *is contra* (α, γ)-*continuous then* $gof : (X, \lambda) \to (Z, \gamma)$ *is* (α, γ)-*continuous.*

Corollary 3. *Let* (X, λ), (Y, μ) *and* (Z, γ) *be* $GTS's$, *and* $f : (X, \lambda) \to (Y, \mu)$ *and* $g : (Y, \mu) \to (Z, \gamma)$ *be two functions. Then*
  (a) *If f is contra* λ-*semi-irresolute and* g *is contra* (σ, γ)-*continuous, then* gof *is* (σ, γ)-*continuous.*
  (b) *If f is contra* λ-*pre-irresolute and* g *is contra* (π, γ)-*continuous, then* gof *is* (π, γ)-*continuous.*

Theorem 4. *Let* (X, λ) *and* (Y, μ) *be* $GTS's$. *If the function* $f : (X, \lambda) \to (Y, \mu)$ *is contra* λ-*semi-irresolute and contra* λ-*pre-irresolute then f is contra* λ-α-*irresolute.*

Proof. It is obvious from Lemma 1. ■

4. Contra λ-α-quotient functions

Definition 12. *Let* (X, λ) *and* (Y, μ) *be* $GTS's$, *and* $f : (X, \lambda) \to (Y, \mu)$ *be a surjective function. Then f is said to be contra* λ-*quotient provided a subset* S *of* Y *is* μ-*open in* Y *if and only if* $f^{-1}(S)$ *is* λ-*closed in* X.

Definition 13. *Let (X, λ) and (Y, μ) be $GTS's$. Let $f : (X, \lambda) \to (Y, \mu)$ be a surjective function. Then f is said to be*

(a) *contra λ-α-quotient if f is contra (α, μ)-continuous and $f^{-1}(V)$ is λ-open in X implies V is an μ-α-closed set in Y.*

(b) *contra λ-semi-quotient if f is contra (σ, μ)-continuous and $f^{-1}(V)$ is λ-open in X implies V is a μ-semi-closed set in Y.*

(c) *contra λ-pre-quotient if f is contra (π, μ)-continuous and $f^{-1}(V)$ is λ-open in X implies V is a μ-preclosed set in Y.*

Example 1. Let $X = \{a, b, c\}$ and $\lambda = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. We have $\alpha(\lambda_X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$; $\sigma(\lambda_X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\pi(\lambda_X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

Let $Y = \{p, q, r\}$ and $\mu = \{\emptyset, Y, \{p, r\}, \{q, r\}, \{r\}\}$. Define $f : (X, \lambda) \to (Y, \mu)$ by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Clearly f is contra (α, μ)-continuous and contra λ-α-quotient function.

Theorem 5. *Let (X, λ) and (Y, μ) be $GTS's$. If the function $f : (X, \lambda) \to (Y, \mu)$ is surjective, contra (α, μ)-continuous and contra (λ, α)-open then f is contra λ-α-quotient function.*

Proof. Suppose $f^{-1}(V)$ is any λ-open set in X. Then $f(f^{-1}(V))$ is an μ-α-closed set in Y as f is contra (λ, α)-open. Since f is surjective, $f(f^{-1}(V)) = V$. Thus V is an μ-α-closed set in Y. Hence f is contra λ-α-quotient function. ∎

Theorem 6. *Let (X, λ), (Y, μ) and (Z, γ) be $GTS's$. If the function $f : (X, \lambda) \to (Y, \mu)$ is contra (λ, μ)-open, surjective and λ-α-irresolute and the function $g : (Y, \mu) \to (Z, \gamma)$ is a contra μ-α-quotient then $gof : (X, \lambda) \to (Z, \gamma)$ is a contra λ-α-quotient function.*

Proof. Let V be any γ-open set in Z. Since g is contra (α, γ)-continuous, $g^{-1}(V) \in \alpha'(\mu_Y)$. Since f is λ-α-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in \alpha'(\lambda_X)$. Thus gof is contra (α, γ)-continuous. Also suppose $f^{-1}(g^{-1}(V))$ is λ-open set in X. Since f is contra (λ, μ)-open, $f(f^{-1}(g^{-1}(V)))$ is μ-open set in Y. Since f is surjective, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ and since g is contra μ-α-quotient, $V \in \alpha'(\gamma_Z)$. Hence gof is contra λ-α-quotient. ∎

Corollary 4. *Let (X, λ), (Y, μ) and (Z, γ) be $GTS's$. If the function $f : (X, \lambda) \to (Y, \mu)$ is contra (λ, μ)-open, surjective and λ-semi-[λ-pre-] irresolute and the function $g : (Y, \mu) \to (Z, \gamma)$ is contra μ-semi-[contra μ-pre-] quotient then $gof : (X, \lambda) \to (Z, \gamma)$ is a contra λ-semi-[λ-pre-] quotient function.*

Theorem 7. *Let (X, λ) and (Y, μ) be GTS's. A surjective function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-α-quotient if and only if it is both contra λ-semi-quotient and contra λ-pre-quotient.*

Proof. Let V be any μ-open set in Y. Since f is contra (α, μ)-continuous, $f^{-1}(V) \in \alpha'(\lambda_X) = \sigma'(\lambda_X) \cap \pi'(\lambda_X)$. Thus f is both contra (σ, μ)-continuous and contra (π, μ)-continuous. Also suppose $f^{-1}(V)$ is an λ-open set in X. Since f is contra λ-α-quotient, $V \in \alpha'(\mu_Y) = \sigma'(\mu_Y) \cap \pi'(\mu_Y)$. Thus V is both μ-semi-closed set and μ-preclosed set in Y. Hence f is both contra λ-semi-quotient and contra λ-pre-quotient. ∎

Conversely, since f is both contra λ-semi-quotient and contra λ-pre-quotient, f is contra (σ, μ)-continuous and contra (π, μ)-continuous. Hence by Theorem 1, f is contra (α, μ)-continuous. Also suppose $f^{-1}(V)$ is an λ-open set in X. By Definition 13, $V \in \sigma'(\mu_Y) \cap \pi'(\mu_Y) = \alpha'(\mu_Y)$. Thus f is contra λ-α-quotient.

Definition 14. (a) *Let (X, λ) and (Y, μ) be GTS's, and $f : (X, \lambda) \to (Y, \mu)$ be a surjective and contra (α, μ)-continuous function. Then f is said to be strongly contra λ-α-quotient if $f^{-1}(S)$ is an λ-α-closed set in X iff S is a μ-open set in Y.*
(b) *Let (X, λ) and (Y, μ) be GTS's, and $f : (X, \lambda) \to (Y, \mu)$ be a surjective and contra (σ, μ)-continuous function. Then f is said to be strongly contra λ-semi-quotient if $f^{-1}(S)$ is a λ-semi-closed set in X iff S is a μ-open set in Y.*
(c) *Let (X, λ) and (Y, μ) be GTS's, and $f : (X, \lambda) \to (Y, \mu)$ be a surjective and contra (π, μ)-continuous function. Then f is said to be strongly contra λ-pre-quotient if $f^{-1}(S)$ is a λ-preclosed set in X iff S is a μ-open set in Y.*

Theorem 8. *Let (X, λ) and (Y, μ) be GTS's. If the surjective function $f : (X, \lambda) \to (Y, \mu)$ is strongly contra λ-semi-quotient and strongly contra λ-pre-quotient then f is strongly contra λ-α-quotient.*

Proof. Let $f^{-1}(V) \in \alpha'(\lambda_X)$. Then $\alpha'(\lambda_X) = \sigma'(\lambda_X) \cap \pi'(\lambda_X)$. Since f is strongly contra λ-semi-quotient and strongly contra λ-pre-quotient, V is μ-open set in Y. Hence f is strongly contra λ-α-quotient. ∎

5. Contra λ-α*-quotient functions

Definition 15. *Let (X, λ) and (Y, μ) be GTS's, and $f : (X, \lambda) \to (Y, \mu)$ be a surjective function. Then f is said to be*
(a) *contra λ-α*-quotient if f is contra λ-α-irresolute and $f^{-1}(S)$ is λ-α-closed set in X implies S is μ-open set in Y.*

(b) *contra λ-semi-*quotient if f is contra λ-semi-irresolute and $f^{-1}(S)$ is λ-semi-closed set in X implies S is μ-open set in Y.*

(c) *contra λ-pre-*quotient if f is contra λ-pre-irresolute and $f^{-1}(S)$ is λ-pre- closed set in X implies S is μ-open set in Y.*

Definition 16. *Let (X, λ) and (Y, μ) be $GTS's$, and $f : (X, \lambda) \to (Y, \mu)$ be a function. Then f is said to be strongly contra (λ, α)-open if the image of every λ-α-open set in X is an μ-α-closed set in Y.*

Example 2. Consider the Example 1. Clearly f is contra λ-α-irresolute and contra λ-α*-quotient.

Example 3. Consider the Example 1. Clearly f is strongly contra (λ, α)-open.

Theorem 9. *Let (X, λ), (Y, μ) and (Z, γ) be $GTS's$, and the function $f : (X, \lambda) \to (Y, \mu)$ be surjective, strongly contra (λ, α)-open and contra λ-α-irresolute, and the function $g : (Y, \mu) \to (Z, \gamma)$ be a contra μ-α *-quotient. Then $gof : (X, \lambda) \to (Z, \gamma)$ is an λ-α *-quotient function.*

Proof. Let V be any γ-α-open set in Z. Then $g^{-1}(V)$ is an μ-α-closed set in Y as g is a contra μ-α-irresolute function. Then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is an λ-α-open set in X as f is contra λ-α-irresolute. This shows that gof is λ-α-irresolute. Also suppose $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is any λ-α-open set in X. Since f is strongly contra (λ, α)-open, $f(f^{-1}(g^{-1}(V)))$ is an μ-α-closed set in Y. Since f is surjective, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is an μ-α-closed set in Y. Since g is a contra μ-α*-quotient function, V is γ-open in Z. Thus gof is λ-α*-quotient. ∎

Theorem 10. *Let (X, λ) and (Y, μ) be $GTS's$. If the surjective function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-semi-*quotient and contra λ-pre-*quotient then f is contra λ-α *-quotient.*

Proof. Since f is both contra λ-semi-*quotient and contra λ-pre-*quotient, f is contra λ-semi-irresolute and contra λ-pre-irresolute. By Theorem 4, f is contra λ-α-irresolute. Also suppose $f^{-1}(V) \in \alpha'(\lambda_X)$. Then $\alpha'(\lambda_X) = \sigma'(\lambda_X) \cap \pi'(\lambda_X)$. Therefore $f^{-1}(V)$ is λ-semi-closed in X and $f^{-1}(V)$ is λ-preclosed in X. Since f is contra λ-semi-*quotient and contra λ-pre-*quotient, by Definition 15, V is μ-open set in Y. Thus f is contra λ-α*-quotient. ∎

Theorem 11. *Let (X, λ), (Y, μ) and (Z, γ) be $GTS's$, and $f : (X, \lambda) \to (Y, \mu)$ be surjective, strongly contra λ-α-quotient and contra λ-α-irresolute function and $g : (Y, \mu) \to (Z, \gamma)$ be surjective, μ-α *-quotient function then $gof : (X, \lambda) \to (Z, \gamma)$ is a contra λ-α *-quotient.*

Proof. Let $V \in \alpha(\gamma_Z)$. Since g is μ-α-irresolute, $g^{-1}(V) \in \alpha(\mu_Y)$. Since f is contra λ-α-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in \alpha'(\lambda_X)$. Thus gof is contra λ-α-irresolute. Also suppose $(gof)^{-1}(V) = f^{-1}(g^{-1}(V)) \in \alpha'(\lambda_X)$. Since f is strongly contra λ-α-quotient, $g^{-1}(V)$ is μ-open set in Y. Then $g^{-1}(V) \in \alpha(\mu_Y)$. Since g is μ-α*-quotient, V is γ-open set in Z. Hence gof is contra λ-α*-quotient. ∎

6. Comparison

Theorem 12. *Let (X, λ) and (Y, μ) be $GTS's$. A surjective function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-α*-quotient if and only if it is strongly contra λ-α-quotient.*

Proof. Let f be contra λ-α*-quotient and S be any μ-open set in Y. Then $S \in \alpha(\mu_Y)$. Since f is contra λ-α-irresolute, $f^{-1}(S) \in \alpha'(\lambda_X)$. Thus f is contra (α, μ)-continuous. Also since f is contra λ-α*-quotient, $f^{-1}(S) \in \alpha'(\lambda_X)$ implies S is μ-open set in Y. Hence f is strongly contra λ-α-quotient function. ∎

Conversely, let f be strongly contra λ-α-quotient and S be any μ-open set in Y. Then $S \in \alpha(\mu_Y)$. Since f is strongly contra λ-α-quotient, $f^{-1}(S) \in \alpha'(\lambda_X)$. Thus f is contra λ-α-irresolute. Also since f is strongly contra λ-α-quotient, $f^{-1}(S) \in \alpha'(\lambda_X)$ implies S is an μ-open set in Y. Hence f is contra λ-α*-quotient function.

Theorem 13. *Let (X, λ) and (Y, μ) be $GTS's$. If the function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-quotient then it is contra λ-α-quotient.*

Proof. Let V be any μ-open set in Y. Since f is contra λ-quotient, $f^{-1}(V)$ is λ-closed set in X and $f^{-1}(V) \in \alpha'(\lambda_X)$. Hence f is contra (α, μ)-continuous. Suppose $f^{-1}(V)$ is any λ-open set in X. Since f is contra λ-quotient, V is μ-closed set in Y. Then $V \in \alpha'(\mu_Y)$. Hence f is contra λ-α-quotient function. ∎

Theorem 14. *Let (X, λ) and (Y, μ) be $GTS's$. If the function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-α-irresolute then it is contra (α, μ)-continuous.*

Proof. Let A be any μ-open set in Y. Then $A \in \alpha(\mu_Y)$. Since f is contra λ-α-irresolute, $f^{-1}(A) \in \alpha'(\lambda_X)$. It shows that f is contra (α, μ)-continuous function. ∎

Theorem 15. *Let (X, λ) and (Y, μ) be $GTS's$. If the function $f : (X, \lambda) \to (Y, \mu)$ is contra λ-α*-quotient then it is contra λ-α-quotient.*

Proof. Let f be contra λ-α*-quotient. Then f is contra λ-α-irresolute. We have f is contra (α, μ)-continuous. Also suppose $f^{-1}(V)$ is any λ-open in X. Then $f^{-1}(V) \in \alpha(\lambda_X)$. By assumption, V is μ-closed set in Y. Therefore $V \in \alpha'(\mu_Y)$. Hence f is contra λ-α-quotient. ∎

Theorem 16. *Every contra λ-α*-quotient function is contra λ-α-irresolute.*

Proof. We obtain it from Definition 15. ∎

Theorem 17. *Every contra λ-α-quotient function is contra (α, μ)-continuous.*

Proof. We obtain it from Definition 13. ∎

Remark 2. The converses of Theorems 8 and 10 are not true as seen from the following example.

Example 4. Let $X = \{a, b, c\}$, $\lambda = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{p, q, r\}$ and $\mu = \{\emptyset, Y, \{r\}, \{p, r\}, \{q, r\}\}$. Define $f : (X, \lambda) \to (Y, \mu)$ by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Clearly f is contra (α, μ)-continuous and strongly contra λ-α-quotient. Since $f^{-1}(\{q\}) = \{b\} \in \sigma'(\lambda_X)$ and $\{q\}$ is not μ-open set in Y, f is not strongly contra λ-semi-quotient. Moreover f is contra λ-α-irresolute and contra λ-α*-quotient but not contra λ-semi-irresolute. Hence f is not contra λ-semi-*quotient.

Remark 3. The converses of Theorems 15 and 16 are not true as seen from the following example.

Example 5. Let $X = \{a, b, c\}$, $\lambda = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{p, q, r\}$ and $\mu = \{\emptyset, Y, \{q, r\}, \{r\}\}$. Define $f : (X, \lambda) \to (Y, \mu)$ by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Clearly f is contra λ-α-irresolute and contra λ-α-quotient functions. Since $f^{-1}(\{p, r\}) = \{a, c\} \in \alpha'(\lambda_X)$ and $\{p, r\}$ is not μ-open set in Y, f is neither strongly contra λ-α-quotient nor contra λ-α*-quotient.

Remark 4. The converse of Theorem 13 is not true and a strongly contra λ-α-quotient function need not be contra λ-quotient as seen from the following example.

Example 6. Let $X = \{a, b, c\}$, $\lambda = \{\emptyset, X, \{a\}\}$, $Y = \{p, q, r\}$ and $\mu = \{\emptyset, Y, \{q\}, \{r\}, \{q, r\}\}$. Define $f : (X, \lambda) \to (Y, \mu)$ by $f(a) = p$, $f(b) = q$ and $f(c) = r$. Clearly f is contra λ-α-quotient and strongly contra λ-α-quotient function. Since $f^{-1}(\{r\}) = \{c\}$ is not λ-closed in X where $\{r\}$ is μ-open in Y, f is not contra λ-quotient function.

Remark 5. A contra λ-quotient function need not be strongly contra λ-α-quotient as seen from the following example.

Example 7. Let $X = \{a, b, c\}$, $\lambda = \{\emptyset, X, \{a\}, \{a, b\}\}$, $Y = \{p, q, r\}$ and $\mu = \{\emptyset, Y, \{r\}, \{q, r\}\}$. Define $f : (X, \lambda) \to (Y, \mu)$ by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Clearly f is contra λ-quotient but not strongly contra λ-α-quotient function.

Remark 6. The converses of Theorems 14 and 17 are not true as seen from the following example.

Example 8. Let $X = \{a, b, c\}$, $\lambda = \{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$, $Y = \{p, q, r\}$ and $\mu = \{\emptyset, Y, \{p\}\}$. Define $f : (X, \lambda) \to (Y, \mu)$ by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Clearly f is contra (α, μ)-continuous. Since $f^{-1}(\{p, r\}) = \{a, c\} \notin \alpha'(\lambda_X)$ where $\{p, r\} \in \alpha(\mu_Y)$, f is not contra λ-α-irresolute. Also, since $f^{-1}(\{p, r\}) = \{a, c\}$ is λ-open in X but $\{p, r\} \notin \alpha'(\mu_Y)$, f is not contra λ-α-quotient function.

Remark 7. We obtain the following diagram from the above discussions.
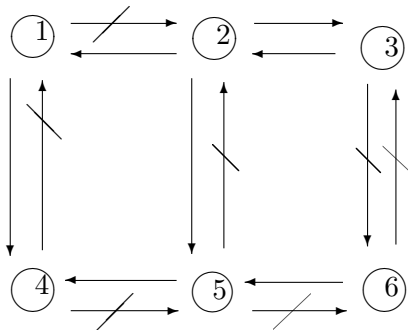
Where $A \nrightarrow B$ means that A does not necessarily imply B and, moreover,
(1) = contra λ-α-irresolute function.
(2) = contra λ-α*-quotient function.
(3) = Strongly contra λ-α-quotient function.
(4) = contra (α, μ)-continuous function.
(5) = contra λ-α-quotient function.
(6) = contra λ-quotient function.

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