DE GRUYTER OPEN
F A S C I C U L I   M A T H E M A T I C I
Nr 58 2017
DOI:10.1515/fascmath-2017-0004

M. Caldas and S. Jafari

# ON SEMI $\delta$s-IRRESOLUTE FUNCTIONS

Abstract. The concept of semi $\delta$s-irresolute function in topological spaces is introduced and studied. Some of their characteristic properties are considered. Also we investigate the relationships between these classes of functions and other classes of noncontinuous functions.

Key words: topological spaces, $\delta$-semiopen sets, $\delta$-closed sets, semi $\delta$s-irresolute and $\delta$-irresolute functions.

*AMS Mathematics Subject Classification*: 54C10, 54D10.

## 1. Introduction and preliminaries

Levine, [10] defined semiopen sets which are weaker than open sets in topological spaces. Since, the advent of Levine's semiopen sets, many researchers offered different and interesting new modifications of open sets which showed to be fruitful. In 1968, Veličko [19] introduced $\delta$-open sets, which are stronger than open sets, in order to investigate the characterizations of $H$-closed spaces. In 1997, Park et al. [18] introduced the notion of $\delta$-semiopen sets which are stronger than semiopen sets but weaker than $\delta$-open sets and investigated the relationships between several types of these open sets. Recently, Caldas et al. [3], [4] and Ekici [7] further investigated this class of sets and also studied some of its applications.

The purpose of the present paper is to introduce and investigate a new class of functions, namely semi $\delta$s-irresolute function and give several of its characterizations and properties. Relations between this class of functions and other classes of functions are obtained.

In what follows $(X,\tau)$ and $(Y,\sigma)$ (or $X$ and $Y$) denote topological spaces. Let $A$ be a subset of $X$. We denote the interior, the closure and the complement of a set $A$ by $Int(A)$, $Cl(A)$ and $X\backslash A$ respectively.

A subset $A$ of a topological space $X$ is said to be a semiopen [10] (resp. preopen [11], $\alpha$-open [14], $\beta$-open [1]) set if $A \subset Cl(Int(A))$ (resp. $A \subset Int(Cl(A))$, $A \subset Int(Cl(Int(A)))$, $A \subset Cl(Int(Cl(A)))$). A point $x \in X$ is called the $\delta$-cluster point of $A$ if $A \cap Int(Cl(U)) \neq \emptyset$ for every open set $U$ of $X$ containing $x$. The set of all $\delta$-cluster points of $A$ is called the $\delta$-closure of $A$ and denoted by $Cl_\delta(A)$. A subset $A$ of $X$ is called $\delta$-closed if $A = Cl_\delta(A)$. The complement of a $\delta$-closed set is called $\delta$-open. A subset $A$ is said to be a $\delta$-semiopen [18] if there exists a $\delta$-open set $U$ of $X$ such that $U \subset A \subset Cl(U)$. The complement of a $\delta$-semiopen (resp. semiopen) set is called a $\delta$-semiclosed (resp. semiclosed). A point $x \in X$ is called the $\delta$-semicluster point of $A$ if $A \cap U \neq \emptyset$ for every $\delta$-semiopen set $U$ of $X$ containing $x$. The set of all $\delta$-semicluster points of $A$ is called the $\delta$-semiclosure of $A$, denoted by $\delta Cl_S(A)$. We denote the collection of all $\delta$-semiopen (resp. $\delta$-open) sets by $\delta\mathrm{SO}(X)$ (resp. $\delta O(X)$). We set $\delta SO(X,x) = \{U : x \in U \in \delta\mathrm{SO}(X)\}$, and $\delta O(X,x) = \{U : x \in U \in \delta O(X)\}$.

**Lemma 1** (Park et al. [18]). *The intersection (resp. union) of arbitrary collection of $\delta$-semiclosed (resp. $\delta$-semiopen) sets in $(X,\tau)$ is $\delta$-semiclosed (resp. $\delta$-semiopen).*

**Corollary 1.** *Let $A$ be a subset of a topological space $(X,\tau)$, then $\delta Cl_S(A) = \cap\{F \in \delta SC(X,\tau) : A \subset F\}$.*

**Lemma 2** (Park et al. [18]). *Let $A$, $B$ and $A_i$ $(i \in I)$ be subsets of a space $(X,\tau)$, the following properties hold:*
(1) *$A \subset \delta Cl_S(A)$,*
(2) *if $A \subset B$, then $\delta Cl_S(A) \subset \delta Cl_S(B)$,*
(3) *if $\delta Cl_S(A)$ is $\delta$-semiclosed,*
(4) *$\delta Cl_S(\delta Cl_S(A)) = \delta Cl_S(A)$,*
(5) *$A$ is $\delta$-semiclosed if and only $A = \delta Cl_S(A)$.*

Recall that a function $f : X \to Y$ is said to be:
(1) semicontinuous [10] If $f^{-1}(V)$ is semiopen in $X$ for each open set $V$ of $Y$.
(2) almost semicontinuous [13] If $f^{-1}(V)$ is semiopen in $X$ for each regular open set $V$ of $Y$.
(3) irresolute [5] if $f^{-1}(V)$ is semiopen in $X$ for each semiopen set $V$ of $Y$.
(4) semi $\alpha$-irresolute [2] if $f^{-1}(V)$ is semiopen in $X$ for each $\alpha$-open set $V$ of $Y$.
(5) almost irresolute [6] if $f^{-1}(V)$ is $\beta$-open in $X$ for each semiopen set $V$ of $Y$.
(6) $\delta$-semiirresolute [3] if $f^{-1}(V)$ is $\delta$-semiopen in $X$ for each $\delta$-semiopen set $V$ of $Y$.

## 2. Semi $\delta$s-irresolute functions

**Definition 1.** *A function $f : X \to Y$ is said to be semi $\delta s$-irresolute at $x \in X$ if for each $\delta$-semiopen set $V$ of $Y$ containing $f(x)$, there exists a semiopen set $U$ in $X$ containing $x$ such that $f(U) \subset V$. If $f$ is semi $\delta s$-irresolute at every point of $X$, then it is called semi $\delta s$-irresolute.*

**Theorem 1.** *For a function $f : X \to Y$, the following are equivalent:*
(1) *$f$ is semi $\delta s$-irresolute;*
(2) *$f^{-1}(V)$ is semiopen in $X$ for each $\delta$-semiopen set $V$ of $Y$;*
(3) *$f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ for every $\delta$-semiopen set $V$ of $Y$;*
(4) *$f^{-1}(F)$ is semiclosed in $X$ for every $\delta$-semiclosed set $F$ of $Y$;*
(5) *$Int(Cl(f^{-1}(B))) \subset f^{-1}(\delta Cl_S(B))$ for every subset $B$ of $Y$;*
(6) *$f(Int(Cl(A))) \subset \delta Cl_S(f(A))$ for every subset $A$ of $X$.*

**Proof.** (1) $\Rightarrow$ (2): Let $V$ be an arbitrary $\delta$-semiopen set in $Y$. We are going to prove that $f^{-1}(V)$ is semiopen in $X$. For this purpose, let $x$ be any point in $f^{-1}(V)$. Then $f(x) \in V$. Since $f$ is semi $\delta$s-irresolute, there exists a semiopen set $U$ of $X$ containing $x$ such that $f(U) \subset V$. This implies $x \in U \subset f^{-1}(V)$. It follows that $f^{-1}(V)$ is a semiopen set in $X$.

(2) $\Rightarrow$ (1): Let $x \in X$ and $V$ be any $\delta$-semiopen set of $Y$ containing $f(x)$. By (2), $f^{-1}(V)$ is semiopen in $X$ and $x \in f^{-1}(V)$. Set $U = f^{-1}(V)$, then $U$ is a semiopen set of $X$ containing $x$ such that $f(U) \subset V$.

(1) $\Rightarrow$ (3): Let $V$ be any $\delta$-semiopen set of $Y$ and $x \in f^{-1}(V)$. By (1), there exists a semiopen set $U$ of $X$ containing $x$ such that $f(U) \subset V$. Thus we have $x \in U \subset Cl(Int(U)) \subset Cl(Int(f^{-1}(V)))$ and hence $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$.

(3) $\Rightarrow$ (4): Let $F$ be any $\delta$-semiclosed subset of $Y$. Set $V = Y \backslash F$, then $V$ is $\delta$-semiopen in $Y$. By (3), we obtain $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ and hence $f^{-1}(F) = X \backslash f^{-1}(Y \backslash F) = X \backslash f^{-1}(V)$ is semiclosed in $X$.

(4) $\Rightarrow$ (5): Let $B$ be any subset of $Y$. Since $\delta Cl_S(B)$ is a $\delta$-semiclosed subset of $Y$, $f^{-1}(\delta Cl_S(B))$ is semiclosed in $X$ and hence $Int(Cl(f^{-1}(\delta Cl_S(B)))) \subset f^{-1}(\delta Cl_S(B))$. Therefore, we obtain $Int(Cl(f^{-1}(B))) \subset f^{-1}(\delta Cl_S(B))$.

(5) $\Rightarrow$ (6): Let $A$ be any subset of $X$. By (5), we have $Int(Cl(A)) \subset Int(Cl(f^{-1}(f(A)))) \subset f^{-1}(\delta Cl_S(f(A)))$ and hence $f(Int(Cl(A))) \subset \delta Cl_S (f(A))$.

(6) $\Rightarrow$ (2): Let $V$ be any $\delta$-semiopen subset of $Y$. Since $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is a subset of X and by (6), we obtain $f(Int(Cl(f^{-1}(Y \backslash V)))) \subset \delta Cl_S(f(f^{-1}(Y \backslash V))) \subset \delta Cl_S(Y \backslash V) = Y \backslash \delta Int_S(V) = Y \backslash V$ and hence $X \backslash Cl(Int(f^{-1}(V))) = Int(Cl(X \backslash f^{-1}(V))) = Int(Cl(f^{-1}(Y \backslash V))) \subset f^{-1}(f(Int (Cl(f^{-1}(Y \backslash V))))) \subset f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$. ■

Therefore, we have $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ and hence $f^{-1}(V)$ is semiopen in $X$.

**Remark 1.** From the above definitions, we have the diagram following:

**DIAGRAM**

$$\begin{array}{ccc} \textit{irresoluteness} & \Rightarrow & \textit{semi } \delta s\textit{-irresoluteness} \\ \Downarrow & & \Downarrow \\ \textit{semicontinuity} & \Rightarrow & \textit{almost semicontinuity} \end{array}$$

By the following examples the converse implications in the above diagram are not true in general:

**Example 1.** Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \tau)$ be defined by $f(a) = b$, $f(b) = a$ and $f(c) = a$. Then $f$ is almost semicontinuous but not semicontinuous.

**Example 2** ([7], Example 12)**.** Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Let $f : (X, \tau) \to (X, \tau)$ be defined by $f(a) = a$, $f(b) = d$, $f(c) = c$ and $f(d) = d$. Then $f$ is almost semicontinuous but not almost $\delta$-semicontinuous.

**Example 3.** Semicomtinuity does not imply irresoluteness: It follows from the fact that every irresolute function is almost irresolute and every semi $\alpha$-irresolute function is semicontinuous (see [2], Example 3.3).

**Example 4.** Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \tau)$ be defined by $f(a) = b$, $f(b) = a$ and $f(c) = a$. Then $f$ is semi $\delta$s-irresolute but not irresolute

**Lemma 3** ([15] and [9])**.** *Let $\{X_i : i \in \Omega\}$ be any family of nonempty topological spaces and $A_{i_j}$ be a nonempty subset of $X_{i_j}$ for each $j = 1, 2, ..., n$. Then $A = \prod_{i \neq i_j} X_i \times \prod_{j=1}^{n} A_{i_j}$ is a nonempty semiopen [15] (resp. $\delta$-semiopen [9]) subset of $\prod X_i$ if and only if $A_{i_j}$ is semiopen (resp. $\delta$-semiopen) in $X_{i_j}$ for each $j = 1, 2, ..., n$.*

**Theorem 2.** *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then $f$ is semi $\delta s$-irresolute if $g$ is semi $\delta s$-irresolute.*

**Proof.** Let $x \in X$ and $V$ be any $\delta$-semiopen set of $Y$ containing $f(x)$. Then, by Lemma 3, $X \times V$ is a $\delta$-semiopen set of $X \times Y$ containing $g(x)$. Since $g$ is semi $\delta$s-irresolute, there exists a semiopen set $U$ of $X$ containing $x$ such that $g(U) \subset X \times V$ and hence $f(U) \subset V$. Thus $f$ is semi $\delta$s-irresolute. ■

**Theorem 3.** *If the product function $f : \prod X_i \to \prod Y_i$ is semi $\delta s$-irresolute, then $f_i : X_i \to Y_i$ is semi $\delta s$-irresolute for each $i \in \Omega$.*

**Proof.** Let $i_0 \in \Omega$ be an arbitrary fixed index and $V_{i_0}$ be any $\delta$-semiopen in $Y_{i_0}$. Then $\prod Y_j \times V_{i_0}$ is $\delta$-semiopen in $\prod Y_i$ by Lemma 3, where $i_0 \neq j \in \Omega$. Since $f$ is semi $\delta$s-irresolute, $f^{-1}(\prod Y_j \times V_{i_0}) = \prod X_j \times f_{i_0}^{-1}(V_{i_0})$ is semiopen in $\prod X_i$ and hence, by Lemma 3, $f_{i_0}^{-1}(V_{i_0})$ is semiopen in $X_{i_0}$. This implies that $f_{i_0}$ is semi $\delta$s-irresolute. ■

**Lemma 4** ([11]). *Let $A$ and $B$ be subsets of $(X, \tau)$. If $A \in PO(X)$ and $B \in SO(X)$, then $A \cap B \in SO(A)$.*

**Theorem 4.** *If $f : (X, \tau) \to (Y, \sigma)$ is semi $\delta$s-irresolute and $A$ is a preopen subset of $X$, then the restriction $f_A : A \to Y$ is semi $\delta$s-irresolute.*

**Proof.** Let $V$ be a $\delta$-semiopen set of $Y$. Since $f$ is semi $\delta$s-irresolute, $f^{-1}(V)$ is semiopen in $X$. By Lemma 4, $(f_A)^{-1}(V) = A \cap f^{-1}(V)$ is semiopen in $A$ and hence $f_A$ is semi $\delta$s-irresolute. ■

**Lemma 5** ([17]). *Let $A$ and $B$ be subsets of $(X, \tau)$ such that $B \subset A$ and $A \in SO(X)$. Then $B \in SO(X)$ if and only if $B \in SO(A)$.*

**Theorem 5.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $\{A_i : i \in \Omega\}$ be a cover of $X$ by semiopen sets of $(X, \tau)$. Then $f$ is semi $\delta$s-irresolute, if $f_{A_i} : A_i \to Y$ is semi $\delta$s-irresolute for each $i \in \Omega$.*

**Proof.** Let $V$ be any $\delta$-semiopen set of $Y$. Since $f_{A_i}$ is semi $\delta$s-irresolute, $(f_{A_i})^{-1}(V) = f^{-1}(V) \cap A_i$ is semiopen in $A_i$ and hence, by Lemma 5, $(f_{A_i})^{-1}(V)$ is semiopen in $X$ for each $i \in \Omega$. Therefore $f^{-1}(V) = X \cap f^{-1}(V) = \cup\{A_i \cap f^{-1}(V) : i \in \Omega\} = \cup\{f_{A_i}^{-1}(V) : i \in \Omega\}$ is semiopen in $X$. Hence $f$ is semi $\delta$s-irresolute. ■

Recall that a function $f : X \to Y$ is said to be preopen if the image of every open subset of $X$ is preopen in $Y$.

**Lemma 6** ([8]). *If $f : X \to Y$ is semi continuous and preopen, then $f$ is irresolute.*

**Theorem 6.** *The following statements hold for functions $f : X \to Y$ and $g : Y \to Z$:*

(*i*) *If $f$ is semi $\delta$s-irresolute and $g$ is $\delta$-semiirresolute, then $g \circ f : X \to Z$ is semi $\delta$s-irresolute.*

(*ii*) *If $f$ is semicontinuous and preopen and $g$ is semi $\delta$s-irresolute, then $g \circ f : X \to Z$ is semi $\delta$s-irresolute.*

(*iii*) *If $f$ is irresolute and $g$ is semi $\delta$s-irresolute, then $g \circ f : X \to Z$ is semi $\delta$s-irresolute.*

**Proof.** $(i)$ Let $W$ be any $\delta$-semiopen subset in $Z$. Since $g$ is $\delta$-semiirresolute, $g^{-1}(W)$ is $\delta$-semiopen in $Y$. Since $f$ is semi $\delta$s-irresolute, $f^{-1}(g^{-1}((V)))$ $= (g \circ f)^{-1}(V)$ is semiopen in $X$. Therefore, $g \circ f$ is semi $\delta$s-irresolute.

$(ii)$ Let $W$ be any $\delta$-semiopen set of $Z$. Since $g$ is semi $\delta$s-irresolute, then $g^{-1}(W)$ is semiopen in $Y$. Then, by Lemma 6, we have $f^{-1}(g^{-1}((V))) =$ $(g \circ f)^{-1}(V)$ is semiopen in $X$. Therefore, $g \circ f$ is semi $\delta$s-irresolute.

$(iii)$ It follows immediately from definitions.

Recall that the semifrontier of $A$ denoted by $sfr(A)$, as $sfr(A) = Cl_S(A)\setminus Int_S(A)$, equivalently $sfr(A) = Cl_S(A) \cap Cl_S(X\setminus A)$. ■

**Theorem 7.** *The set of all points $x \in X$ at which $f : (X, \tau) \to (Y, \sigma)$ is not semi $\delta$s-irresolute is identical with the union of the semifrontiers of the inverse images of $\delta$-semiopen subsets of $Y$ containing $f(x)$.*

**Proof.** *Necessity.* Suppose that $f$ is not semi $\delta$s-irresolute at a point $x$ of $X$. Then, there exists a $\delta$-semiopen set $V \subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of $V$ for every $U \in SO(X, x)$. Hence we have $U \cap (X \setminus f^{-1}(V)) \neq \emptyset$ for every $U \in SO(X, x)$. It follows that $x \in Cl_S(X \setminus f^{-1}(V))$. We also have $x \in f^{-1}(V) \subset Cl_S(f^{-1}(V))$. This means that $x \in sfr(f^{-1}(V))$.

*Sufficiency.* Suppose that $x \in sfr(f^{-1}(V))$ for some $V \in \delta SO(Y, f(x))$ Now, we assume that $f$ is semi $\delta$s-irresolute at $x \in X$. Then there exists $U \in SO(X, x)$ such that $f(U) \subset V$. Therefore, we have $x \in U \subset f^{-1}(V)$ and hence $x \in Int_S(f^{-1}(V)) \subset X \setminus sfr(f^{-1}(V))$ . This is a contradiction. This means that $f$ is not semi $\delta$s-irresolute at $x$. ■

Recall that, a topological space $(X, \tau)$ is called semi-$T_2$ [12] (resp. $\delta$-semi $T_2$ [3]) if for any two distinct points $x$ and $y$ in $X$, there exist $U \in SO(X, x)$ and $V \in SO(X, y)$ (resp. $U \in \delta SO(X, x)$ and $V \in \delta SO(X, y)$) such that $U \cap V = \emptyset$.

**Theorem 8.** *If $f : X \to Y$ is a semi $\delta$s-irresolute injection and $Y$ is $\delta$-semi $T_2$, then $X$ is semi-$T_2$.*

**Proof.** Suppose that $Y$ is $\delta$-semi $T_2$. Let $x$ and $y$ be distinct points of $X$. Then $f(x) \neq f(y)$. Since $Y$ is $\delta$-semi $T_2$, there exist disjoint $\delta$-semiopen sets $V$ and $W$ containing $f(x)$ and $f(y)$, respectively. Since $f$ is semi $\delta$s-irresolute, there exist semiopen sets $G$ and $H$ containing $x$ and $y$ , respectively such that $f(G) \subset V$ and $f(H) \subset W$. It follows that $G \cap H = \emptyset$. This shows that $X$ is semi-$T_2$. ■

**Lemma 7.** *If $A_i$ is a semiopen set of $X_i$ (i=1, 2), then $A_1 \times A_2$ is semiopen in $X_1 \times X_2$*

**Proof.** By Theorem 11 of [10]. ■

**Theorem 9.** *If $f : X \to Y$ is a semi $\delta s$-irresolute and $Y$ is $\delta$-semi$T_2$, then $E = \{(x, y) : f(x) = f(y)\}$ is semiclosed in $X \times X$.*

**Proof.** Suppose that $(x, y) \notin E$. Then $f(x) \neq f(y)$. Since $Y$ is $\delta$-semi $T_2$, there exist $V \in \delta SO(Y, f(x))$ and $W \in \delta SO(Y, f(y))$ such that $V \cap W = \emptyset$. Since $f$ is semi $\delta$s-irresolute, there exist $U \in SO(X, x)$ and $G \in SO(X, y)$ such that $f(U) \subset V$ and $f(G) \subset W$. Set $D = U \times G$. By Lemma 7 $(x, y) \in D \in SO(X \times X)$ and $D \cap E = \emptyset$. This means that $Cl_S(E) \subset E$ and therefore $E$ is semiclosed in $X \times X$. ■

**Definition 2.** *For a function $f : X \to Y$, the graph $G(f) = \{(x, f(x)) : x \in X\}$ is called $(\delta, s)$-closed if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in SO(X, x)$ and $V \in \delta SO(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.*

**Lemma 8.** *A function $f : X \to Y$ has a $(\delta, s)$-closed graph $G(f)$ if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in SO(X, x)$ and $V \in \delta SO(Y, y)$ such that $f(U) \cap V = \emptyset$.*

**Proof.** It is an immediate consequence of Definition 2 and the fact that for any subsets $U \subset X$ and $V \subset Y$, $(U \times V) \cap G(f) = \emptyset$ if and only if $f(U) \cap V = \emptyset$. ■

**Theorem 10.** *If $f : X \to Y$ is semi $\delta s$-irresolute and $Y$ is $\delta$-semi $T_2$, then $G(f)$ is $(\delta, s)$-closed in $X \times Y$.*

**Proof.** Let $(x, y) \in (X \times Y)\backslash G(f)$. It follows that $f(x) \neq y$. Since $Y$ is $\delta$-semi $T_2$, there exist disjoint $\delta$-semiopen sets $V$ and $W$ in $Y$ containing $f(x)$ and $y$, respectively. Since $f$ is semi $\delta$s-irresolute, there exists $U \in SO(X, x)$ such that $f(U) \subset V$. Therefore $f(U) \cap W = \emptyset$ and $G(f)$ is $(\delta, s)$-closed in $X \times Y$. ■

**Theorem 11.** *If $f : X \to Y$ is a semi $\delta s$-irresolute injection with a $(\delta, s)$-closed graph, then $X$ is semi-$T_2$*

**Proof.** Let $x$ and $y$ be any distinct points of $X$. Then since $f$ is injective, we have $f(x) \neq f(y)$. Then we have $(x, f(y)) \in (X \times Y)\backslash G(f)$. Since $G(f)$ is $(\delta, s)$-closed, there exist $U \in SO(X, x)$ and $V \in \delta SO(Y, f(y))$ such that $f(U) \cap V = \emptyset$. Since $f$ is semi $\delta$s-irresolute, there exists $G \in SO(Y, y)$ such that $f(G) \subset V$. Therefore, we have $f(U) \cap f(G) = \emptyset$ and hence $U \cap G = \emptyset$. This shows that $X$ is semi-$T_2$. ■

**Acknowledgment.** The authors thank Professor Y. Beceren for providing them with reference 2.

## References

[1] Abd El-Monsef M.E., El-Deeb S.N., Mahmoud R.A., $\beta$-open sets and $\beta$-continuous mappings, *Bull. Fac. Assiut Univ.,* 12(1983), 77-90.
[2] Beceren Y., On semi-$\alpha$-irresolute functions, *J. Indian Acad. Math.,* 22 (2000), 353-362.
[3] Caldas M., Georgiou D.N., Jafari S., Noiri T., More on $\delta$-semiopen sets, *Note di Matematica,* 22(2004), 113-126.
[4] Caldas M., Ganster M., Georgiou D.N., Jafari S., Moshokoa S.P., $\delta$-semiopen sets in topology, *Topology Proceeding,* 29(2005), 369-383.
[5] Cammaroto F., Noiri T., Almost irresolute functions, *Indian J. Pute Appl. Math.,* 20(1989), 472-482.
[6] Crossley S.G., Hildebrand S.K., Semi-topological properties, *Fund. Math.,* 74(1972), 233-254.
[7] Ekici E., On $\delta$-semiopen sets and a generaligation of functions, *Bol. Soc. Paran. Mat. 3s.,* 23(2005), 73-84.
[8] Jankovic D.S., A note on mappings of extremally connected spaces, *Acta Math. Hungar.,* 46(1985), 83-92.
[9] Lee B.Y., Son M.J., Park J.H., $\delta$-semiopen sets and its applications, *Far East J. Math. Sci.* (*FJMS*)*,* 3(2001), 745-759.
[10] Levine N., Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly,* 70(1963), 36-41.
[11] Mashhour A.S., Hasanein I.A., El-Deeb S.N., A note on semi-continuity and precontinuity, *Indian J. Pure Appl. Math.,* 13(1982), 1119-1123.
[12] Maheswari S.N., Prasad R., Some new separation axioms, *Ann. Soc. Sci. Bruxelles,* 89(1975), 395-402.
[13] Munshi B.M., Bassan D.S., Almostr semicontinuous mappings, *Math. Student,* 49(1981), 239-248.
[14] Njåstad O., On some classes of nearly open sets, *Pacific J. Math.,* 15(1965), 961-970.
[15] Noiri T., On semi-continuous mappings, *Lincei Rend. Sci. Fisc. Mat. e Nat.,* 54(1973), 210-222.
[16] Noiri T., A note on semi-continuous mappings, *Lincei-Rend. Sc. Fisc. Mat. e Nat.,* 55(1973), 400-403.
[17] Noiri T., Remarks on semiopen mappings, *Bull. Calcutta Math. Soc.,* 65 (1973), 197-201.
[18] Park J.H., Lee B.Y., Son M.J., On $\delta$-semiopen sets in topological spaces, *J. Indian Acad. Math.,* 19(1997), 1-4.
[19] Veličko N.V., $H$-closed topological spaces, *Amer. Math. Soc. Transl.,* 2(78) (1968), 103-118.

M. Caldas
Departamento de Matematica Aplicada
Universidade Federal Fluminense
Rua Mario Santos Braga, s/n
24020-140, Niteroi, RJ Brasil
*e-mail:* gmamccs@vm.uff.br

S. Jafari
College of Vestsjaelland South
Herrestraede 11
4200 Slagelse, Denmark
*e-mail:* jafaripersia@gmail.com

*Received on 26.04.2016 and, in revised form, on 10.05.2017.*