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ON EXISTENCE OF MILD AND CLASSICAL SOLUTIONS OF SECOND ORDER IMPULSIVE IMPLICIT DIFFERENTIAL EQUATIONS

Abstract. In this paper, we establish the existence and uniqueness results of mild and classical solutions for impulsive implicit second order differential equations with nonlocal condition by using contraction principle. Furthermore, we give an example to illustrate our results.

Key words: nonlocal condition, impulsive, differential equation.

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1. Introduction

Impulsive differential equations have wide range of applications in real life problems as well as in many physical processes. For example, control theory, mechanics, medical sciences, economics etc. It has became a field of attraction in young researchers. In particular, the problem of change in speed of chemical reaction due to catalyst, death and birth rate in population dynamics can be modeled using impulsive differential equations because of sudden change in the state of them. Thus many researchers [5], [6], [7], [9], [11], [12] have studied impulsive differential equations and the references cited there in. For more details refer the monographs of Bainov and Simeonov [2] and V. Lakshmikantham et.al [10].

On the other hand, the real life phenomenon can be represented more precisely with nonlocal condition which gives more information at a time. Nonlocal condition firstly studied by L. Byszewski. It reduces the negative effects occurred due to single measurement taken at a time in classical condition. So study of impulsive differential equations with nonlocal condition gained much attention of researchers in the recent decade. For more details refer [4], [6]-[8].

A. Anguraj, M. M. Arjunan [1] studied the existence and uniqueness of mild of impulsive evolution equations of the type:

$$\begin{aligned}
&u'(t) = Au(t) + f(t, u(t), Tu(t), Su(t)), \quad 0 < t < T_0, \quad t \neq t_i \\
&u(0) = u_0, \\
&\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, ..., p.
\end{aligned}$$

Furthermore, they established that mild solution gives rise to classical solution.

In [5], E. Hernandez proved the existence and uniqueness of mild and classical solutions of following abstract second order impulsive Cauchy problem:

$$\begin{aligned}
&u''(t) = Au(t) + f(t, u(t), u'(t)), t \in (-T_0, T_1), \quad t \neq t_i \\
&u(0) = x_0, \quad u'(0) = y_0, \\
&\Delta u(t_i) = I_i^1(u(t_i)), \quad \Delta u'(t_i) = I_i^2(u'(t_i^+)).
\end{aligned}$$

In [4], L. Byszewski proved existence and uniqueness of mild and classical solutions of semilinear functional differential evolution equation given below:

$$\begin{aligned}
&u''(t) = Au(t) + f(t, u(t), u(a(t)), u'(t)), \quad t \in (0, T] \\
&u(0) = x_0, \\
&u'(0) + \sum_{i=1}^{p} h_i u(t_i) = x_1.
\end{aligned}$$

Motivated by the above work, we establish the existence result for mild and classical solution of implicit nonlocal second order differential equation of the type:

$$x''(t) = Ax(t) + f(t, x(t), x(b(t)), x'(t)), \quad t \in (0, T] \tag{1}$$

$$x(0) = x_0, \tag{2}$$

$$x'(0) + g(x) = y_0, \tag{3}$$

$$\Delta x(\tau_k) = I_k(x(\tau_k)), \quad \Delta x'(\tau_k) = \bar{I}_k(x'(\tau_k^+)),\ t \neq \tau_k,\ k = 1, 2, .., m \tag{4}$$

where A is the infinitesimal generator of strongly continuous cosine family of linear operators $C(t)$ on Banach space X. The functions $b : [0, T] \to [0, T]$, f, g, I_k and $\bar{I_K}$ are continuous functions satisfying some assumptions. The impulsive moments τ_k are such that $0 \leq \tau_0 < \tau_1 < \tau_2 < ... < \tau_m < \tau_{m+1} \leq T$, $m \in \mathbb{N}$, $\Delta x(\tau_k) = x(\tau_k + 0) - x(\tau_k - 0)$, $\Delta x'(\tau_k) = x'(\tau_k + 0) - x'(\tau_k - 0)$, where $x(\tau_k + 0)$ and $x(\tau_k - 0)$ are the right and the left limits of x at τ_k respectively.

Our aim of the paper is to generalise the results given in [2] and [5]. The paper is organised as follows:

Section 2 contains preliminaries and hypotheses. In Section 3, we prove the existence of mild and classical solutions of impulsive problem (1) – (4). An example is given in Section 4.

2. Preliminaries and hypotheses

Let X be a Banach space with the supremum norm $\|.\|$. Let $PC([0,T],X)$ denote the set $\{u:[0,T]\to X|u(t)$ is piecewise continuous at $t\neq\tau_k$, left continuous at $t=\tau_k$, and the right limit $u(\tau_k+0)$ exists for $k=1,2,...,m\}$. Note that $PC([0,T],X)$ is a Banach space with the supremum norm $\|u\|_1=\sup\{\|u(t)\|+\|u'(t)\|:t\in[0,T]\}$.

Definition 1 ([8]). *A one parameter family $\{C(t):t\in\mathbb{R}\}$ of bounded linear operators in the Banach space X is called strongly continuous cosine family if and only if*

1. *$C(0)=I$ is the identity operator*
2. *$C(t+s)+C(t-s)=2C(t)C(s)\quad\forall\quad t,s\in\mathbb{R}$*
3. *The map $t\mapsto C(t)(x)$ is strongly continuous for each $x\in X$.*

The associated sine function is the family $\{S(t)\}_{t\in\mathbb{R}}$ of operators defined by $S(t)x=\int_0^t C(s)xds$, for $x\in X$, $t\in\mathbb{R}$.

Here,

$$E=\{x\in X:C(t)x\ \text{ is once continuously differentiable}\}$$

$$D(A)=\{x\in X:C(t)x\ \text{ is twice continuously differentiable}\}$$

Proposition 1 ([5]). *Let $\{C(t):t\in\mathbb{R}\}$ be strongly continuous cosine family with infinitesimal generator A and associated sine family $S(t)$, $t\in\mathbb{R}$. The following are true:*

- *If $x\in X, S(t)\in E$*
- *If $x\in E, C(t)x\in E$*
- *If $x\in E, S(t)\in D(A)$ and $\frac{d}{dt}C(t)x=AS(t)x$*
- *If $x\in E, S(t)x\in D(A)$ and $\frac{d}{dt}C(t)x=AS(t)x$ and $\frac{d^2}{dt^2}S(t)x=AS(t)x$*
- *If $x\in D(A), C(t)x\in D(A)$ and $\frac{d^2}{dt^2}C(t)x=AC(t)x=C(t)Ax$*
- *If $x\in E, S(t)x\in D(A)$ and $\frac{d^2}{dt^2}S(t)x=AS(t)x$*

Lemma 1 ([14]). *Let a nonnegative piecewise continuous function $x(t)$ satisfy for $t\geq t_0$ the inequality*

$$x(t)\leq C+\int_{t_0}^t v(s)u(s)ds+\sum_{t_0<\tau_k<t}\beta_i u(t_i)$$

where, $C \geq 0$, $\beta_i \geq 0$, $v(t) > 0$, τ_i are the first kind discontinuity points of the function $x(t)$. Then the following estimation holds for the function $x(t)$

$$x(t) \leq C \prod_{t_0<\tau_k<t} (1+\beta_i) \exp(\int_{t_0}^{t} v(s)ds)$$

Definition 2 ([8]). *A function $x(t) \in PC([0,T], X)$ is said to be mild solution of the initial value problem* (1)-(4) *if it satisfies:*

$$\begin{aligned} x(t) &= C(t)x_0 + S(t)y_0 - S(t)g(x) \\ &+ \int_0^t S(t-s)f(s, x(s), x(b(s)), x'(s))ds \\ &+ \sum_{0<\tau_k<t} C(t-\tau_k)I_k(x(\tau_k)) + \sum_{0<\tau_k<t} S(t-\tau_k)\bar{I_k}(x'(\tau_k^+)). \end{aligned} \tag{5}$$

Definition 3 ([5]). *A function $x(t) \in PC([0,T], X)$ is said to be classical solution of the initial value problem* (1)-(4) *if $x \in C^2([0,T]/(\tau_1, \tau_2, ..., \tau_k), X)$ and x satisfies* (5).

Now we list some hypotheses which will be useful for proving our results:

(H_1) *Let $f : [0,T] \times X \times X \times X \to X$ be a continuous function such that there exists a positive constant L_1 such that*

$$\|f(t,x,y,z) - f(t,x',y',z')\| \leq L_1(\|x-x'\| + \|y-y'\| + \|z-z'\|)$$

for every $x, y, z, x', y', z' \in X$.

(H_2) *Let I_k and $\bar{I_k} : X \to X$ be continuous functions such that there exists a positive constants L_k and $\bar{L_k}$ satisfying*

$$\begin{aligned} \|I_k(x) - I_k(y)\| &\leq L_k\|x-y\| \\ \|\bar{I_k}(x) - \bar{I_k}(y)\| &\leq \bar{L_k}\|x-y\| \end{aligned}$$

for every $x, y \in X$.

(H_3) *Let $g : PC([0,T], X) \to PC([0,T], X)$ be a continuous function such that there exists a positive constant G satisfying*

$$\|g(x) - g(y)\| \leq G\|x-y\|,$$

for every $x, y \in X$.

Also we denote,

$$L_k^* = \max\{L_k, \bar{L_k}\}$$

and

$$C = \sup\{\|C'(t)\| + \|S(t)\| + \|S'(t)\|, \;\; t \in (0,T]\|.$$

3. Main results

3.1. Existence results for mild solution

Theorem 1. *Let b be function of class C^1 on $[0,T]$, $x_0 \in E$ and $y_0 \in X$. Assume that the hypotheses $[H_1]-[H_3]$ holds. Then there exists mild solution for impulsive implicit problem* (1) − (4) *if* $2C(G+2TL_1+2L_k^*)<1$.

Proof. We define an operator $F : PC([0,T],X) \to PC([0,T],X)$ as follows:

$$\begin{aligned}(Fx)(t) &= C(t)x_0 + S(t)y_0 - S(t)g(x)\\ &\quad + \int_0^t S(t-s)f(s,x(s),x(b(s)),x'(s))ds\\ &\quad + \sum_{0<\tau_k<t} C(t-\tau_k)I_k(x(\tau_k)) + \sum_{0<\tau_k<t} S(t-\tau_k)\bar{I}_k(x'(\tau_k^+)),\quad t\in[0,T].\end{aligned}$$

Note that $PC([0,T],X)$ is a Banach space with the norm $\|x\|_1 = \sup\{\|x(t)\| + \|x'(t)\| : t \in [0,T]\}$. We prove that F is a contraction on the Banach space $PC([0,T],X)$ with the norm $\|x\|_1$. Consider,

$$\begin{aligned}(6)\quad &\|(Fx)(t)-(Fv)(t)\| = \|S(t)(g(x)-g(v))\\ &+ \int_0^t S(t-s)\big[f(s,x(s),x(b(s)),x'(s)) - f(s,v(s),v(b(s)),v'(s))\big]ds\\ &+ \sum_{0<\tau_k<t} C(t-\tau_k)[I_k(x(\tau_k)) - I_k(v(\tau_k))]\\ &+ \sum_{0<\tau_k<t} S(t-\tau_k)[\bar{I}_k(x'(\tau_k^+)) - \bar{I}_k(v'(\tau_k^+))]\|\\ &\le CG\|x-v\| + \int_0^t \|S(t-s)\|L_1(\|x(s)-v(s)\|\\ &\qquad\qquad + \|x(b(s))-v(b(s))\| + \|x'(s)-v'(s)\|)ds\\ &+ \sum_{0<\tau_k<t} \|C(t-\tau_k)\|\|[I_k(x(\tau_k)) - I_k(v(\tau_k))]\|\\ &+ \sum_{0<\tau_k<t} \|S(t-\tau_k)\|\|[\bar{I}_k(x'(\tau_k^+)) - \bar{I}_k(v'(\tau_k^+))]\|\\ &\le CG\|x-v\|_1 + CL_1\int_0^t 2\|x-v\|_1 ds\\ &+ \sum_{0<\tau_k<t} CL_k\|x(\tau_k)-v(\tau_k)\| + \sum_{0<\tau_k<t} C\bar{L}_k\|x'(\tau_k)-v'(\tau_k)\|\\ &\le CG\|x-v\|_1 + 2CL_1t\|x-v\|_1 + CL_k^*\|x-v\|_1 + CL_k^*\|x-v\|_1\\ &\le C(G+2TL_1+2L_k^*)\|x-v\|_1\end{aligned}$$

Now,

$$
\begin{aligned}
(7)\quad \|(Fx)'(t)-(Fv)'(t)\| &= \|C(t)(g(x)-g(v)) \\
&+\int_0^t C(t-s)\big[f(s,x(s),x(b(s)),x'(s))-f(s,v(s),v(b(s)),v'(s))\big]ds \\
&+\sum_{0<\tau_k<t} AS(t-\tau_k)[I_k(x(\tau_k))-I_k(v(\tau_k))] \\
&+\sum_{0<\tau_k<t} C(t-\tau_k)[\bar{I}_k(x'(\tau_k^+))-\bar{I}_k(v'(\tau_k^+))]\| \\
&\le CG\|x-v\|+\int_0^t \|S(t-s)\|L_1(\|x(s)-v(s)\| \\
&\qquad +\|x(b(s))-v(b(s))\|+\|x'(s)-v'(s)\|)ds \\
&+\sum_{0<\tau_k<t} \|C(t-\tau_k)\|\|[I_k(x(\tau_k))-I_k(v(\tau_k))]\| \\
&+\sum_{0<\tau_k<t} \|S(t-\tau_k)\|\|[\bar{I}_k(x'(\tau_k^+))-\bar{I}_k(v'(\tau_k^+))]\| \\
&\le CG\|x-v\|+CL_1\int_0^t 2\|x-v\|_1 ds \\
&+\sum_{0<\tau_k<t} CL_k\|x(\tau_k)-v(\tau_k)\|+\sum_{0<\tau_k<t} C\bar{L}_k\|x'(\tau_k)-v'(\tau_k)\| \\
&\le CG\|x-v\|_1+2CL_1t\|x-v\|_1+CL_k^*\|x-v\|_1+CL_k^*\|x-v\|_1 \\
&\le C(G+2TL_1+2L_k^*)\|x-v\|_1
\end{aligned}
$$

from (6) and (7), we get,

$$\|Fx-Fv\|_1 \le 2C(G+2TL_1+2L_k^*)\|x-v\|_1, \quad forall;\ x,v\in PC([0,T],X).$$

Therefore, F is contraction on $PC([0,T],X)$. Hence by Banach contraction principle there exists a unique fixed point of F which is mild solution of impulsive implicit problem $(1)-(4)$. ■

3.2. Existence result for classical solution

Let us assume f is continuously differentiable function with $N=\sup\{f(s, x(s),x(b(s)),x'(s),s\in[0,T])\}$. Also there exists constants $M\ge 1$ and $\omega\ge 0$ such that $\|C(t)\|\le Me^{\omega t}$ and $\|S(t)\|\le Me^{\omega t}$ for $t\in\mathbb{R}$. We will use Bielecki's norm to prove existence result of the classical solution since its an efficient way to use impulsive inequality. Bielecki's norm is weighted spremum norm weighted by exponential function.

Theorem 2. *Let, $x_0\in D(A)$, $y_0\in E$, $x(\tau_k^-)\in D(A)$, $x'(\tau_{k-}^-)\in E$, $I_k(x(\tau_k^-))\in D(A)$, $\bar{I}_kx'(\tau_k^-)\in E$, $\forall k=1,2,...,n$ and $x\in PC([0,T],X)\cap$*

$C^2([0,T]/(\tau_1,\tau_2,...,\tau_k),X)$. *Assume that the hypotheses* $[H_1]-[H_3]$ *holds. Then there exists classical solution for impulsive implicit problem* $(1)-(4)$.

Proof. Let t and $t+h$ be any two points belonging to $[0,T]$. Then

$$\begin{aligned}
&\|x(t+h)-x(t)\| = \|C(t+h)x_0 + S(t+h)y_0\\
&\quad - s(t+h)g(x) + \int_0^{t+h} S(t+h-s)f(s,x(s),x(b(s)),x'(s))ds\\
&\quad + \sum_{0<\tau_k<t+h} C(t+h-\tau_k)I_k(x(\tau_k)) + \sum_{0<\tau_k<t+h} S(t+h-\tau_k)\bar{I}_k(x'(\tau_k^+))\\
&\quad - \big(C(t)x_0 + S(t)y_0 - s(t)g(x) + \int_0^t S(t-s)f(s,x(s),x(b(s)),x'(s))ds.\\
&\quad + \sum_{0<\tau_k<t} C(t-\tau_k)I_k(x(\tau_k)) + \sum_{0<\tau_k<t} S(t-\tau_k)\bar{I}_k(x'(\tau_k^+))\big)\|
\end{aligned}$$

Since $C(t)x_0+S(t)(y_0-g(x))$ is of class C^2 in [0,T], there exists positive constants $M_1>0$ and $M_2>0$ such that

$$\|(C(t+h)-C(t))x_0 + (S(t+h)-S(t))(y_0-g(x))\| \le M_1|h|$$

$$\|\big[(C(t+h)-C(t))x_0\big]' + \big[(S(t+h)-S(t))(y_0-g(x))\big]'\| \le M_2|h|.$$

Consider

$$\begin{aligned}
&\|x(t+h)-x(t)\| \le M_1|h|\\
&\quad + \|\int_0^t S(s)\big[f(t+h-s,x(t+h-s),x(b(t+h-s)),x'(t+h-s))\\
&\qquad\qquad - f(t-s,x(t-s),x(b(t-s)),x'(t-s))\big]ds\|\\
&\quad + \|\int_t^{t+h} S(s)\big[f(t+h-s,x(t+h-s),x(b(t+h-s)),x'(t+h-s))\big]ds\|\\
&\quad + \|\sum_{0<\tau_k<t} \big[C(t+h-\tau_k)I_k(x(\tau_k)) + S(t+h-\tau_k)\bar{I}_k x'(\tau_k^+)\big]\\
&\quad - \sum_{0<\tau_k<t} \big[C(t-\tau_k)I_k(x(\tau_k)) + S(t-\tau_k)\bar{I}_k(x'(\tau_k^+))\big]\|\\
&\quad + \|\sum_{t<\tau_k<t+h} C(t+h-\tau_k)I_k(x(\tau_k)) + \sum_{t<\tau_k<t+h} S(t+h-\tau_k)\bar{I}_k(x'(\tau_k^+))\|\\
&\le M_1|h| + \int_0^t Me^{\omega T}L_1(|h| + \|x(t+h-s)-x(t-s)\|\\
&\qquad + \|x'(b(t+h-s))-x'(b(t-s))\|\\
&\qquad + \|x'(t+h-s)-x'(t-s)\|)ds + Me^{\omega T}N|h|
\end{aligned}$$

$$+ \sum_{0<\tau_k<t} \|C(t+h-\tau_k)I_k(x(\tau_k)) - C(t-\tau_k)I_k(x(\tau_k))\|$$
$$+ \sum_{0<\tau_k<t} \|S(t+h-\tau_k)\bar{I}_k x'(\tau_k^+) - S(t-\tau_k)\bar{I}_k x'(\tau_k^+)\|$$
$$\leq M_3|h| + M_4 \int_0^t (\|x(s+h)-x(s)\| + \|x'(s+h)-x'(s)\|)ds$$
$$+ \sum_{0<t<\tau_k} (Me^{\omega|t+h-\tau_k|} + Me^{\omega|t-\tau_k|})\|I_k(x(\tau_k))\|$$
$$+ \sum_{0<t<\tau_k} (Me^{\omega|t+h-\tau_k|} + Me^{\omega|t-\tau_k|})\|\bar{I}_k(x'(\tau_k^+))\|$$
$$\leq M_3|h| + M_4 \int_0^t (\|x(s+h)-x(s)\| + \|x'(s+h)-x'(s)\|)ds$$
$$+ \sum_{0<t<\tau_k} 2Me^{\omega T} L_k \|(x(\tau_k)\| + \sum_{0<t<\tau_k} 2Me^{\omega T} \bar{L_k} \|x'(\tau_k^+)\|)$$
$$\leq M_3|h| + M_4 \int_0^t (\|x(s+h)-x(s)\| + \|x'(s+h)-x'(s)\|)ds$$
$$+ \sum_{0<t<\tau_k} 2Me^{\omega T} L_k^* (\|x(\tau_k)\| + \|x'(\tau_k^+)\|)$$
$$\leq M_3|h| + M_4 \int_0^t (\|x(s+h)-x(s)\| + \|x'(s+h)-x'(s)\|)ds$$
$$+ \sum_{0<t<\tau_k} M_5 (\|x(\tau_k)\| + \|x'(\tau_k^+)\|)\|$$

Similarly, we can obtain,

$$\|x'(t+h) - x'(t)\|$$
$$\leq M_6|h| + M_7 \int_0^t (\|x(s+h)-x(s)\| + \|x'(s+h)-x'(s)\|)ds$$
$$+ \sum_{0<t<\tau_k} M_8 (\|x(\tau_k)\| + \|x'(\tau_k^+)\|)$$

Hence,

$$\|x(t+h) - x(t)\| + \|x'(t+h) - x'(t)\|$$
$$\leq M_9|h| + M_{10} \int_0^t (\|x(s+h)-x(s)\| + \|x'(s+h)-x'(s)\|)ds$$
$$+ M_{11} \sum_{0<t<\tau_k} (\|x(\tau_k)\| + \|x'(\tau_k^+)\|),$$

where $M_i's$ are some positive constants. Now applying impulsive inequality given in Lemma 1, we obtain, $\|x(t+h) - x(t)\| + \|x'(t+h) - x'(t)\| \leq$

$M^*|h|$, where M^* is some constant. Hence, the existence of classical solution proved. ■

4. Example

We give an example to illustrate our results obtained in Section 3. Consider the second order IVP given below:

$$\begin{aligned} x"(t) &= \frac{e^{-t}}{200}x(t) + \frac{e^{-t}(x(t)+x'(t))}{(8+e^t)(1+(x(t)+x'(t)))} \\ x(0) &= 0, x'(0) + \frac{x(t)}{5+x(t)} = 0 \\ \Delta x|_{t=\frac{5}{4}} &= \frac{x((\frac{5}{4})^-)}{6+x(\frac{5}{4})^-}, \quad t \neq 5/4 \\ \Delta x|_{t=\frac{5}{4}} &= \frac{x'((\frac{5}{4})^-)}{6+x'(\frac{5}{4})}. \end{aligned}$$

comparing with IVP (1)-(4), we get,

$$\begin{aligned} f(t,x(t),x(b(t)),x'(t)) &= \frac{e^{-t}(x(t)+x'(t))}{(8+e^t)(1+(x(t)+x'(t)))}, \\ g(x) &= \frac{x}{5+x}, \quad b(t) = t \\ I_k(x) &= \frac{x}{6+x}, \\ \bar{I}_k(x) &= \frac{x'}{6+x'}. \end{aligned}$$

Then

$$\begin{aligned} &\|f(t,x(t),x(b(t)),x'(t)) - f(t,v(t),v(b(t)),v'(t))\| \\ &\quad \leq |\frac{e^{-t}}{8+e^t}|\|\frac{x(t)+x'(t)}{1+[x(t)+x'(t)]} - \frac{v(t)+v'(t)}{1+[v(t)+v'(t)]}\| \\ &\quad \leq \frac{1}{9}\|\frac{x(t)+x'(t)-(v(t)+v'(t))}{(1+[x(t)+x'(t)])(1+[v(t)+v'(t)])}\| \\ &\quad \leq \frac{1}{9}\|(x(t)-v(t))+(x'(t)-v'(t))\| \\ &\quad \leq \frac{1}{9}\|(x(t)-v(t))\| + \|(x'(t)-v'(t))\| \end{aligned}$$

$$\|g(x)-g(v)\| = \|\frac{x}{5+x} - \frac{v}{5+v}\| \leq \frac{5\|x-v\|}{\|(5+x)(5+v)\|} \leq \frac{1}{5}\|x-v\|.$$

Similarly we get, $\|I_k(x) - I_k(v)\| \leq \frac{1}{6}\|x - v\|$ and $\|\bar{I}_k(x) - \bar{I}_k(v)\| \leq \frac{1}{6}\|x - v\|$. Taking, $L_1 = \frac{1}{9}, G = \frac{1}{5}, L_k = \bar{L}_k = \frac{1}{6}$,and choosing $C < \frac{45}{68}$ we get $2C(G + 2TL_1 + 2L_k^*) < 1$. Here all the conditions of Theorem 1 holds good hence there exist a mild solution for the given problem.

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