FASCICULI MATHEMATICI

Nr 62 2019

DOI: 10.21008/j.0044-4413.2019.0005

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FRACTIONAL INTEGRAL INEQUALITIES FOR COMPOSITE AND k-COMPOSITE PREINVEX FUNCTIONS

Abstract. In this article we found some Ostrowski inequalities for composite and **k**-composite preinvex functions via fractional integrals. Also some special cases will be given.

Key words: Ostrowski inequality, Minkowski inequality, Hölder inequality, power mean inequality, Riemann-Liouville integrals.

AMS Mathematics Subject Classification: 26A33, 26A51, 26D15, 26D10, 26D07.

1. Introduction

Theorem 1. *[33]Let $f : I \longrightarrow \mathbb{R}$ be a differentiable mapping on I° and $a_1, a_2 \in I^\circ$ with $a_1 < a_2$. If $|f'(x)| \leq L$, $\forall x \in [a_1, a_2]$, then*

$$\left| f(x) - \frac{1}{a_2 - a_1} \int_{a_1}^{a_2} f(t)dt \right| \leq L(a_2 - a_1) \left[\frac{1}{4} + \frac{\left(x - \frac{a_1+a_2}{2}\right)^2}{(a_2 - a_1)^2} \right], \tag{1}$$

$\forall x \in [a_1, a_2]$.

For other recent results concerning Ostrowski type inequalities, see [2]-[4], [14]-[18], [23], [26], [27], [31]-[34], [36]-[38], [41], [43], [45], [46], [48], [50]. Ostrowski inequality is playing a key role in all the fields of mathematics, see [11]-[13]. In recent years, a number of integral inequalities are introduced by many authors involving various fractional operators like, Katugampola, conformable fractional integral operators etc., see [1], [6]-[10], [28]-[30], [40], [42], [44]. Ostrowski inequality provides the bounds for many numerical quadrature rules, see [20],[21]. In recent decades Ostrowski, Hermite-Hadamard and Simpson type inequalities are studied in fractional calculus and generalized invexity analysis point of view by many mathematicians, see [5], [19], [22], [24], [25], [35], [39], [47], [49].

Now, let us evoke some basic definitions.

Definition 1. *Let $f \in L[a_1, a_2]$. The Riemann-Liouville integrals $J^{\alpha}_{a_1+}f$ and $J^{\alpha}_{a_2-}f$ where $\alpha > 0$ with $a_1 \geq 0$ are*

$$J^{\alpha}_{a_1+}f(x) = \frac{1}{\Gamma(\alpha)} \int_{a_1}^{x} (x-t)^{\alpha-1} f(t)dt, \quad x > a_1$$

and

$$J^{\alpha}_{a_2-}f(x) = \frac{1}{\Gamma(\alpha)} \int_{x}^{a_2} (t-x)^{\alpha-1} f(t)dt, \quad a_2 > x.$$

Here $J^{0}_{a_1+}f(x) = J^{0}_{a_2-}f(x) = f(x)$.

Definition 2 ([5])**.** *A set $P \subseteq \mathbb{R}^n$ is called invex with $\varsigma : P \times P \longrightarrow \mathbb{R}^n$, if $x + t\varsigma(y, x) \in P$ for every $x, y \in P$ and $t \in [0, 1]$.*

Convex set is invex for $\varsigma(y, x) = y - x$, see [5], [47].

Definition 3 ([39])**.** *A function f defined on the invex set $P \subseteq \mathbb{R}^n$ is called preinvex with ς, if for all $x, y \in P$ and $t \in [0, 1]$, we have that*

$$f\left(x + t\varsigma(y, x)\right) \leq (1-t)f(x) + tf(y).$$

Let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a *continuous strictly increasing function* and differentiable on (a_1, a_2).

Definition 4 ([19])**.** *A function $f : [a_1, a_2] \longrightarrow \mathbb{R}$ will be called composite-g^{-1} convex (concave) on $[a_1, a_2]$ if the composite function $f \circ g^{-1} : [g(a_1), g(a_2)] \longrightarrow \mathbb{R}$ is convex (concave) in the usual sense on $[g(a_1), g(a_2)]$.*

In this way, any concept of convexity (log-convexity, harmonic convexity, trigonometric convexity, hyperbolic convexity, h-convexity, quasi-convexity, s-convexity, s-Godunova-Levin convexity etc.) can be extended to the corresponding *composite-g^{-1} convexity.* The details however will not be presented here.

If $f : [a_1, a_2] \longrightarrow \mathbb{R}$ is composite-g^{-1} convex on $[a_1, a_2]$ then we have the inequality

$$f \circ g^{-1}((1-\lambda)u_1 + \lambda v_1) \leq (1-\lambda) f \circ g^{-1}(u_1) + \lambda f \circ g^{-1}(v_1) \tag{2}$$

for any $u_1, v_1 \in [g(a_1), g(a_2)]$ and $\lambda \in [0, 1]$.

This is equivalent with condition

$$f \circ g^{-1}((1-\lambda)g(t) + \lambda g(s)) \leq (1-\lambda)f(t) + \lambda f(s) \tag{3}$$

for any $t, s \in [a_1, a_2]$ and $\lambda \in [0, 1]$.

Further, assume that $f : [a_1, a_2] \longrightarrow I$ and $\mathbf{k} : I \longrightarrow \mathbb{R}$ a continuous function on I that is *strictly increasing* (*decreasing*) on I.

Definition 5 ([19]). *A function $f : [a_1, a_2] \longrightarrow I$ is $\boldsymbol{k}$-composite convex (concave) on $[a_1, a_2]$ if $\boldsymbol{k} \circ f$ is convex (concave) on $[a_1, a_2]$.*

In this way, any concept of convexity as mentioned above can be extended to the corresponding $\boldsymbol{k}$-*composite convexity.* The details however will not be presented here.

Definition 6 ([19]). *A function $f : [a_1, a_2] \longrightarrow I$ is $\boldsymbol{k}$-composite-g^{-1} convex (concave) on $[a_1, a_2]$ if $\boldsymbol{k} \circ f \circ g^{-1}$ is convex (concave) on $[g(a_1), g(a_2)]$.*

This is equivalent with condition

$$\mathbf{k} \circ f \circ g^{-1}((1-\lambda)g(t) + \lambda g(s)) \leq (\geq)\, (1-\lambda)(\mathbf{k} \circ f)(t) + \lambda(\mathbf{k} \circ f)(s) \tag{4}$$

for any $t, s \in [a_1, a_2]$ and $\lambda \in [0, 1]$.
If $\mathbf{k} : I \longrightarrow \mathbb{R}$ is *strictly increasing* (*decreasing*) on I, then the condition (4) is equivalent to:

$$f \circ g^{-1}((1-\lambda)g(t) + \lambda g(s)) \leq (\geq)\, \mathbf{k}^{-1}\big[(1-\lambda)(\mathbf{k} \circ f)(t) + \lambda(\mathbf{k} \circ f)(s)\big] \tag{5}$$

for any $t, s \in [a_1, a_2]$ and $\lambda \in [0, 1]$.

The purpose of this article is to find some Ostrowski inequalities using two identities given in Section 2 for some new classes of functions called composite and **k**-composite preinvex functions via fractional integrals. Also some new special cases will be given.

2. Main results

Firstly, we give some new definitions about composite and **k**-composite preinvex functions.

Definition 7. *Let $P \subseteq \mathbb{R}$ be an open invex set and $\varsigma : P \times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1, a_2) and $h_1, h_2 : [0, 1] \longrightarrow [0, +\infty)$ are continuous. A function $f : P \longrightarrow \mathbb{R}$ is called composite-g^{-1}-(h_1, h_2)-preinvex, if*

$$f \circ g^{-1}(g(t) + \lambda\varsigma(g(s), g(t))) \leq h_1(\lambda) f(t) + h_2(\lambda) f(s) \tag{6}$$

for any $t, s \in [a_1, a_2]$ and $\lambda \in [0, 1]$.

Remark 1. In Definition 7, if $h_1(\lambda) = 1 - \lambda$, $h_2(\lambda) = \lambda$ and $\varsigma(g(s), g(t)) = g(s) - g(t)$, we get inequality (3).

Definition 8. *Let $P \subseteq \mathbb{R}$ be an open invex set and $\varsigma : P \times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing* *function that is differentiable on* (a_1, a_2) *and* $h_1, h_2 : [0,1] \longrightarrow [0,+\infty)$ *are continuous. Further, assume that* $f : P \longrightarrow I$ *and* $\boldsymbol{k} : I \longrightarrow \mathbb{R}$ *a continuous function on* I *that is strictly increasing on* I*. A function* f *is* $\boldsymbol{k}$*-composite-*g^{-1}*-*(h_1, h_2)*-preinvex on* P *if* $\boldsymbol{k} \circ f \circ g^{-1}$ *is preinvex on* P*.*

This is equivalent with condition

$$\text{(7)} \quad \mathbf{k} \circ f \circ g^{-1}(g(t) + \lambda\varsigma(g(s), g(t))) \leq h_1(\lambda)(\mathbf{k} \circ f)(t) + h_2(\lambda)(\mathbf{k} \circ f)(s)$$

for any $t, s \in [a_1, a_2]$ and $\lambda \in [0,1]$.

Remark 2. In Definition 8, if $h_1(\lambda) = 1-\lambda$, $h_2(\lambda) = \lambda$ and $\varsigma(g(s), g(t)) = g(s) - g(t)$, we get inequality (4).

Definition 9. *Let* $P \subseteq \mathbb{R}$ *be an open invex set and* $\varsigma : P \times P \longrightarrow \mathbb{R}$*. Also, let* $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ *be a continuous strictly increasing function that is differentiable on* (a_1, a_2) *and* $h_1, h_2 : [0,1] \longrightarrow [0,+\infty)$ *are continuous. A function* $f : P \longrightarrow (0,+\infty)$ *is called composite-*g^{-1}*-*$(r; h_1, h_2)$*-preinvex, if*

$$\text{(8)} \qquad f \circ g^{-1}(g(t) + \lambda\varsigma(g(s), g(t))) \leq \left[h_1(\lambda) f^r(t) + h_2(\lambda) f^r(s)\right]^{\frac{1}{r}}$$

for any $t, s \in [a_1, a_2]$*,* $\lambda \in [0,1]$ *and* $r > 0$*.*

Remark 3. In Definition 9 if $r = 1$, we get Definition 7.

Definition 10. *Let* $P \subseteq \mathbb{R}$ *be an open invex set and* $\varsigma : P \times P \longrightarrow \mathbb{R}$*. Also, let* $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ *be a continuous strictly increasing function that is differentiable on* (a_1, a_2) *and* $h_1, h_2 : [0,1] \longrightarrow [0,+\infty)$ *are continuous. Further, assume that* $f : P \longrightarrow (0,+\infty)$ *and* $\boldsymbol{k} : (0,+\infty) \longrightarrow (0,+\infty)$ *a continuous function on* $(0,+\infty)$ *that is strictly increasing on* $(0,+\infty)$*. A function* f *is* $\boldsymbol{k}$*-composite-*g^{-1}*-*$(r; h_1, h_2)$*-preinvex on* P *if* $\boldsymbol{k} \circ f \circ g^{-1}$ *is preinvex on* P*.*

This is equivalent with condition

$$\text{(9)} \quad \mathbf{k} \circ f \circ g^{-1}(g(t) + \lambda\varsigma(g(s), g(t))) \leq \left[h_1(\lambda)(\mathbf{k} \circ f)^r(t) + h_2(\lambda)(\mathbf{k} \circ f)^r(s)\right]^{\frac{1}{r}}$$

for any $t, s \in [a_1, a_2]$, $\lambda \in [0,1]$ and $r > 0$.

Remark 4. In Definition 10, if we choose $r = 1$, we get Definition 8.

In order to give some Ostrowski inequalities for composite and **k**-composite preinvex functions using fractional integrals, we claim two identities.

Lemma 1. *Suppose* $P \subseteq \mathbb{R}$ *be an open invex subset and* $\varsigma : P \times P \longrightarrow \mathbb{R}$*. Also, let* $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ *be a continuous strictly increasing* *function that is differentiable on* (a_1, a_2). *Let* $f : P \longrightarrow \mathbb{R}$ *be a differentiable on* P° *and* $(f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, *where* $\varsigma(g(a_2), g(a_1)) > 0$. *Then, for* $\alpha > 0$ *we have the following equality for fractional integrals:*

$$
\begin{aligned}
(10)\quad & \left(\frac{\varsigma^\alpha(g(a_2), g(x)) - (\varsigma(g(a_2), g(a_1)) - \varsigma(g(a_2), g(x)))^\alpha}{\varsigma^\alpha(g(a_2), g(a_1))}\right) \\
& \times \left(f \circ g^{-1}\right)(g(a_1) + \varsigma(g(a_2), g(x))) \\
& - \frac{\Gamma(\alpha+1)}{\varsigma^\alpha(g(a_2), g(a_1))}\Big[J^\alpha_{(g(a_1)+\varsigma(g(a_2),g(x)))^-} f(g(a_1)) \\
& - J^\alpha_{(g(a_1)+\varsigma(g(a_2),g(x)))^+} f(g(a_1) + \varsigma(g(a_2), g(a_1)))\Big] \\
& = \varsigma(g(a_2), g(a_1)) \int_0^1 p(t) \left(f \circ g^{-1}\right)'(g(a_1) + t\varsigma(g(a_2), g(a_1)))dt,
\end{aligned}
$$

where $\varpi_x = \frac{\varsigma(g(a_2), g(x))}{\varsigma(g(a_2), g(a_1))}$, *and*

$$
p(t) := \begin{cases} t^\alpha, & \text{if} \quad t \in [0, \varpi_x]; \\ (1-t)^\alpha, & \text{if} \quad t \in (\varpi_x, 1]. \end{cases}
$$

Throughout this paper we denote

$$
(11)\quad I_1 := \varsigma(g(a_2), g(a_1)) \int_0^1 p(t) \left(f \circ g^{-1}\right)'(g(a_1) + t\varsigma(g(a_2), g(a_1)))dt.
$$

Proof.

$$
\begin{aligned}
I_1 = \; & \varsigma(g(a_2), g(a_1)) \\
& \times \Bigg[\int_0^{\varpi_x} t^\alpha \left(f \circ g^{-1}\right)'(g(a_1) + t\varsigma(g(a_2), g(a_1)))dt \\
& + \int_{\varpi_x}^1 (1-t)^\alpha \left(f \circ g^{-1}\right)'(g(a_1) + t\varsigma(g(a_2), g(a_1)))dt\Bigg].
\end{aligned}
$$

Integrating by parts, we get

$$
\begin{aligned}
I_1 = \; & \varsigma(g(a_2), g(a_1)) \Bigg[\frac{t^\alpha (f \circ g^{-1})(g(a_1) + t\varsigma(g(a_2), g(a_1)))}{\varsigma(g(a_2), g(a_1))}\Bigg|_0^{\varpi_x} \\
& - \frac{\alpha}{\varsigma(g(a_2), g(a_1))} \int_0^{\varpi_x} t^{\alpha-1} (f \circ g^{-1})(g(a_1) + t\varsigma(g(a_2), g(a_1)))dt \\
& + \frac{(1-t)^\alpha (f \circ g^{-1})(g(a_1) + t\varsigma(g(a_2), g(a_1)))}{\varsigma(g(a_2), g(a_1))}\Bigg|_{\varpi_x}^1
\end{aligned}
$$

$$+\frac{\alpha}{\varsigma(g(a_2),g(a_1))}\int_{\varpi_x}^{1}(1-t)^{\alpha-1}(f\circ g^{-1})(g(a_1)+t\varsigma(g(a_2),g(a_1)))dt\Bigg]$$

$$=\left(\frac{\varsigma^{\alpha}(g(a_2),g(x))-(\varsigma(g(a_2),g(a_1))-\varsigma(g(a_2),g(x)))^{\alpha}}{\varsigma^{\alpha}(g(a_2),g(a_1))}\right)$$
$$\times\ \left(f\circ g^{-1}\right)(g(a_1)+\varsigma(g(a_2),g(x)))$$
$$-\frac{\Gamma(\alpha+1)}{\varsigma^{\alpha}(g(a_2),g(a_1))}\Big[J^{\alpha}_{(g(a_1)+\varsigma(g(a_2),g(x)))^-}f(g(a_1))$$
$$-J^{\alpha}_{(g(a_1)+\varsigma(g(a_2),g(x)))^+}f(g(a_1)+\varsigma(g(a_2),g(a_1)))\Big].$$

The proof of Lemma 1 is completed. ■

Lemma 2. *Let $P\subseteq\mathbb{R}$ be an open invex subset and $\varsigma:P\times P\longrightarrow\mathbb{R}$. Also, let $g:[a_1,a_2]\longrightarrow[g(a_1),g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1,a_2). Suppose $f:P\longrightarrow\mathbb{R}$ is a differentiable on P° and $(\boldsymbol{k}\circ f\circ g^{-1})'\in L[g(a_1),g(a_1)+\varsigma(g(a_2),g(a_1))]$, where $\varsigma(g(a_2),g(a_1))>0$ and $\boldsymbol{k}:I\longrightarrow\mathbb{R}$ a continuous function on I that is strictly increasing on I. Then, for $\alpha>0$*

$$\left(\frac{\varsigma^{\alpha}(g(a_2),g(x))-(\varsigma(g(a_2),g(a_1))-\varsigma(g(a_2),g(x)))^{\alpha}}{\varsigma^{\alpha}(g(a_2),g(a_1))}\right)\tag{12}$$
$$\times\ \left(\boldsymbol{k}\circ f\circ g^{-1}\right)(g(a_1)+\varsigma(g(a_2),g(x)))-\frac{\Gamma(\alpha+1)}{\varsigma^{\alpha}(g(a_2),g(a_1))}$$
$$\times\Big[J^{\alpha}_{(g(a_1)+\varsigma(g(a_2),g(x)))^-}\boldsymbol{k}\circ f(g(a_1))$$
$$-J^{\alpha}_{(g(a_1)+\varsigma(g(a_2),g(x)))^+}\boldsymbol{k}\circ f(g(a_1)+\varsigma(g(a_2),g(a_1)))\Big]$$
$$=\varsigma(g(a_2),g(a_1))\int_0^1 p(t)\left(\boldsymbol{k}\circ f\circ g^{-1}\right)'(g(a_1)+t\varsigma(g(a_2),g(a_1)))dt,$$

where ϖ_x and $p(t)$ are defined as in Lemma 1.

Throughout this paper we denote

$$I_2:=\varsigma(g(a_2),g(a_1))\int_0^1 p(t)\left(\boldsymbol{k}\circ f\circ g^{-1}\right)'(g(a_1)+t\varsigma(g(a_2),g(a_1)))dt.\tag{13}$$

Proof. See Lemma 1. ■

By using Lemmas 1 and 2, we get the following.

Theorem 2. *Suppose $P\subseteq\mathbb{R}$ be an open invex subset and $\varsigma:P\times P\longrightarrow\mathbb{R}$. Also, let $g:[a_1,a_2]\longrightarrow[g(a_1),g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1,a_2) and $h_1,h_2:[0,1]\longrightarrow[0,+\infty)$ are continuous. Let $f:P\longrightarrow\mathbb{R}$ be a differentiable on P° and*

$(f\circ g^{-1})' \in L[g(a_1), g(a_1)+\varsigma(g(a_2),g(a_1))]$, *where* $\varsigma(g(a_2),g(a_1))>0$. *If* $|f'|^q$ *is composite-*g^{-1}*-*(h_1,h_2)*-preinvex function,* $q>1$ *and* $p^{-1}+q^{-1}=1$*, then for* $\alpha>0$

$$(14)\quad \big|I_1\big| \le \varsigma(g(a_2),g(a_1)) \times \Bigg\{ \sqrt[p]{A}\sqrt[q]{\big|(f\circ g^{-1})'(g(a_1))\big|^q J_1 + \big|(f\circ g^{-1})'(g(a_2))\big|^q J_2} + \sqrt[p]{B}\sqrt[q]{\big|(f\circ g^{-1})'(g(a_1))\big|^q G_1 + \big|(f\circ g^{-1})'(g(a_2))\big|^q G_2}\Bigg\},$$

where

$$(15)\quad A := \int_0^{\varpi_x} t^{p\alpha}dt = \frac{\varpi_x^{p\alpha+1}}{p\alpha+1}, \quad B := \int_{\varpi_x}^1 (1-t)^{p\alpha}dt = \frac{(1-\varpi_x)^{p\alpha+1}}{p\alpha+1}$$

and

$$(16)\qquad J_i := \int_0^{\varpi_x} h_i(t)dt, \qquad G_i := \int_{\varpi_x}^1 h_i(t)dt, \ \ \forall\, i=1,2.$$

Proof. From Lemma 1, composite-g^{-1}-(h_1,h_2)-preinvexity of $|f'|^q$, Hölder inequality and properties of the modulus, we have

$$\begin{aligned}
\big|I_1\big| &\le |\varsigma(g(a_2),g(a_1))|\Bigg[\int_0^{\varpi_x} t^\alpha\big|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\big|dt \\
&\quad + \int_{\varpi_x}^1 (1-t)^\alpha\big|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\big|dt\Bigg] \\
&\le \varsigma(g(a_2),g(a_1)) \\
&\quad \times \Bigg[\left(\int_0^{\varpi_x} t^{p\alpha}dt\right)^{\frac{1}{p}}\left(\int_0^{\varpi_x}\big|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\big|^q dt\right)^{\frac{1}{q}} \\
&\quad + \left(\int_{\varpi_x}^1 (1-t)^{p\alpha}dt\right)^{\frac{1}{p}}\left(\int_{\varpi_x}^1\big|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\big|^q dt\right)^{\frac{1}{q}}\Bigg] \\
&\le \varsigma(g(a_2),g(a_1)) \\
&\quad \times \Bigg[\sqrt[p]{A}\left(\int_0^{\varpi_x}\Big[\big|(f\circ g^{-1})'(g(a_1))\big|^q h_1(t) + \big|(f\circ g^{-1})'(g(a_2))\big|^q h_2(t)\Big]dt\right)^{\frac{1}{q}} \\
&\quad + \sqrt[p]{B}\left(\int_{\varpi_x}^1\Big[\big|(f\circ g^{-1})'(g(a_1))\big|^q h_1(t) + \big|(f\circ g^{-1})'(g(a_2))\big|^q h_2(t)\Big]dt\right)^{\frac{1}{q}}\Bigg] \\
&= \varsigma(g(a_2),g(a_1))\Bigg\{\sqrt[p]{A}\sqrt[q]{\big|(f\circ g^{-1})'(g(a_1))\big|^q J_1 + \big|(f\circ g^{-1})'(g(a_2))\big|^q J_2} \\
&\quad + \sqrt[p]{B}\sqrt[q]{\big|(f\circ g^{-1})'(g(a_1))\big|^q G_1 + \big|(f\circ g^{-1})'(g(a_2))\big|^q G_2}\Bigg\}.
\end{aligned}$$

The proof of Theorem 2 is completed. ■

Corollary 1. *In Theorem 2 if* $\left|(f\circ g^{-1})'\right|\le K$, *we get*

$$\left|I_1\right| \le \frac{K}{\varsigma^{\frac{p(\alpha-1)+1}{p}}(g(a_2),g(a_1))}\sqrt[p]{\frac{1}{p\alpha+1}} \times\Big\{\varsigma^{\frac{p\alpha+1}{p}}(g(a_2),g(x))\sqrt[q]{J_1+J_2} + \left(\varsigma(g(a_2),g(a_1))-\varsigma(g(a_2),g(x))\right)^{\frac{p\alpha+1}{p}}\sqrt[q]{G_1+G_2}\Big\}. \tag{17}$$

If one takes $\left|(f\circ g^{-1})'\right|\le K$ *and* $\varsigma(g(a_2),g(x))=g(a_2)-g(x)$ *for all* x, *we have*

$$\begin{aligned}\left|\overline{I_1}\right| = &\Bigg|\left(\frac{(g(a_2)-g(x))^\alpha-(g(x)-g(a_1))^\alpha}{(g(a_2)-g(a_1))^\alpha}\right)\\ &\times\left(f\circ g^{-1}\right)(g(a_1)+g(a_2)-g(x))-\frac{\Gamma(\alpha+1)}{(g(a_2)-g(a_1))^\alpha}\\ &\times\left[J^\alpha_{(g(a_1)+g(a_2)-g(x))^-}f(g(a_1))-J^\alpha_{(g(a_1)+g(a_2)-g(x))^+}f(g(a_2))\right]\Bigg|\\ \le &\frac{K}{(g(a_2)-g(a_1))^{\frac{p(\alpha-1)+1}{p}}}\sqrt[p]{\frac{1}{p\alpha+1}}\\ &\times\left\{(g(a_2)-g(x))^{\frac{p\alpha+1}{p}}\sqrt[q]{J_1^*+J_2^*}+(g(x)-g(a_1))^{\frac{p\alpha+1}{p}}\sqrt[q]{G_1^*+G_2^*}\right\},\end{aligned} \tag{18}$$

where

$$J_i^*:=\int_0^{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}h_i(t)dt,\quad G_i^*:=\int_{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}^1 h_i(t)dt,\quad \forall\, i=1,2. \tag{19}$$

Corollary 2. *In Theorem 2 for* $h_1(t)=h(1-t)$ *and* $h_2(t)=h(t)$, *we get*

$$\begin{aligned}\left|I_1\right| \le\ &\varsigma(g(a_2),g(a_1))\\ &\times\Big\{\sqrt[p]{A}\sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q\overline{J_1}+\left|(f\circ g^{-1})'(g(a_2))\right|^q\overline{J_2}}\\ &+\sqrt[p]{B}\sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q\overline{G_1}+\left|(f\circ g^{-1})'(g(a_2))\right|^q\overline{G_2}}\Big\},\end{aligned} \tag{20}$$

where

$$\overline{J_1} := \int_{1-\varpi_x}^{1} h(t)dt, \quad \overline{J_2} := \int_{0}^{\varpi_x} h(t)dt \tag{21}$$

and

$$\overline{G_1} := \int_{0}^{1-\varpi_x} h(t)dt, \quad \overline{G_2} := \int_{\varpi_x}^{1} h(t)dt. \tag{22}$$

Corollary 3. *In Theorem 2 for $h_1(t) = h_2(t) = t(1-t)$, we get*

$$\begin{aligned} \left|I_1\right| \le\; & \varsigma(g(a_2), g(a_1))\left[\sqrt[p]{A}\sqrt[q]{J^*} + \sqrt[p]{B}\sqrt[q]{G^*}\right] \\ & \times \sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q + \left|(f\circ g^{-1})'(g(a_2))\right|^q}, \end{aligned} \tag{23}$$

where

$$J^* := \int_{0}^{\varpi_x} t(1-t)dt = \frac{\varpi_x^2}{2} - \frac{\varpi_x^3}{3} \tag{24}$$

and

$$G^* := \int_{\varpi_x}^{1} t(1-t)dt = \frac{1-\varpi_x^2}{2} - \frac{1-\varpi_x^3}{3}. \tag{25}$$

Theorem 3. *Suppose $P \subseteq \mathbb{R}$ be an open invex subset and $\varsigma : P \times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1, a_2) and $h_1, h_2 : [0,1] \longrightarrow [0, +\infty)$ are continuous. Let $f : P \longrightarrow \mathbb{R}$ be a differentiable on P° and $(f\circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, where $\varsigma(g(a_2), g(a_1)) > 0$. If $|f'|^q$ is composite-g^{-1}-(h_1, h_2)-preinvex function, $q \ge 1$, then for $\alpha > 0$*

$$\begin{aligned} \left|I_1\right| \le\; & \varsigma(g(a_2), g(a_1)) \\ & \times \Bigg\{ C^{1-\frac{1}{q}} \sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q H_1 + \left|(f\circ g^{-1})'(g(a_2))\right|^q H_2} \\ & + D^{1-\frac{1}{q}} \sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q M_1 + \left|(f\circ g^{-1})'(g(a_2))\right|^q M_2} \Bigg\}, \end{aligned} \tag{26}$$

where

$$C := \int_{0}^{\varpi_x} t^\alpha dt = \frac{\varpi_x^{\alpha+1}}{\alpha+1}, \quad D := \int_{\varpi_x}^{1} (1-t)^\alpha dt = \frac{(1-\varpi_x)^{\alpha+1}}{\alpha+1} \tag{27}$$

and

$$H_i := \int_{0}^{\varpi_x} t^\alpha h_i(t)dt, \quad M_i := \int_{\varpi_x}^{1} (1-t)^\alpha h_i(t)dt, \quad \forall\, i = 1, 2. \tag{28}$$

Proof. From Lemma 1, composite-g^{-1}-(h_1,h_2)-preinvexity of $|f'|^q$, power mean inequality and properties of the modulus, we get

$$\begin{aligned}
\left|I_1\right| &\le |\varsigma(g(a_2),g(a_1))|\Bigg[\int_0^{\varpi_x} t^{\alpha}\left|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\right|dt \\
&+\int_{\varpi_x}^1 (1-t)^{\alpha}\left|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\right|dt\Bigg] \le \varsigma(g(a_2),g(a_1)) \\
&\times\Bigg[\left(\int_0^{\varpi_x} t^{\alpha}dt\right)^{1-\frac{1}{q}}\left(\int_0^{\varpi_x} t^{\alpha}\left|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\right|^q dt\right)^{\frac{1}{q}} \\
&+\left(\int_{\varpi_x}^1 (1-t)^{\alpha}dt\right)^{1-\frac{1}{q}} \\
&\times\left(\int_{\varpi_x}^1 (1-t)^{\alpha}\left|(f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\right|^q dt\right)^{\frac{1}{q}}\Bigg] \\
&\le \varsigma(g(a_2),g(a_1)) \\
&\times\Bigg[C^{1-\frac{1}{q}}\left(\int_0^{\varpi_x} t^{\alpha}\left[\left|(f\circ g^{-1})'(g(a_1))\right|^q h_1(t)+\left|(f\circ g^{-1})'(g(a_2))\right|^q h_2(t)\right]dt\right)^{\frac{1}{q}} \\
&+D^{1-\frac{1}{q}} \\
&\times\left(\int_{\varpi_x}^1 (1-t)^{\alpha}\left[\left|(f\circ g^{-1})'(g(a_1))\right|^q h_1(t)+\left|(f\circ g^{-1})'(g(a_2))\right|^q h_2(t)\right]dt\right)^{\frac{1}{q}}\Bigg] \\
&=\varsigma(g(a_2),g(a_1))\Bigg\{C^{1-\frac{1}{q}}\sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q H_1+\left|(f\circ g^{-1})'(g(a_2))\right|^q H_2} \\
&+D^{1-\frac{1}{q}}\sqrt[q]{\left|(f\circ g^{-1})'(g(a_1))\right|^q M_1+\left|(f\circ g^{-1})'(g(a_2))\right|^q M_2}\Bigg\}.
\end{aligned}$$

The proof of Theorem 3 is completed. ■

Remark 5. Using Theorems 2 and 3, for different choices of function g, for example $g(t)=e^t,\ln t,-\frac{1}{t}$ and t^p where $p>0$, we can get some Ostrowski inequalities for composite preinvex functions using fractional integrals. For $\alpha=1$, we derive some Ostrowskiinequalities for composite preinvex functions using classical integrals.

Corollary 4. *In Theorem 3 if* $\left|(f\circ g^{-1})'\right|\le K$, *we get*

$$\left|I_1\right| \le \frac{K\varsigma(g(a_2),g(a_1))}{\left((\alpha+1)\varsigma^{\alpha+1}(g(a_2),g(a_1))\right)^{1-\frac{1}{q}}} \times\Bigg\{\varsigma^{(\alpha+1)\left(1-\frac{1}{q}\right)}(g(a_2),g(x))\sqrt[q]{H_1+H_2} \tag{29}$$

$$+ \left(\varsigma(g(a_2), g(a_1)) - \varsigma(g(a_2), g(x))\right)^{(\alpha+1)\left(1-\frac{1}{q}\right)} \sqrt[q]{M_1 + M_2}\Bigg\}.$$

If one takes $\left|(f \circ g^{-1})'\right| \leq K$ *and* $\varsigma(g(a_2), g(x)) = g(a_2) - g(x)$ *for all* x, *we have*

$$\left|\overline{I_1}\right| \leq \frac{K(g(a_2) - g(a_1))}{\left((\alpha+1)(g(a_2) - g(a_1))^{\alpha+1}\right)^{1-\frac{1}{q}}} \times \Bigg\{(g(a_2) - g(x))^{(\alpha+1)\left(1-\frac{1}{q}\right)} \sqrt[q]{H_1^* + H_2^*} + (g(x) - g(a_1))^{(\alpha+1)\left(1-\frac{1}{q}\right)} \sqrt[q]{M_1^* + M_2^*}\Bigg\}, \tag{30}$$

where

$$H_i^* := \int_0^{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}} t^\alpha h_i(t)dt, \quad M_i^* := \int_{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}^1 (1-t)^\alpha h_i(t)dt, \quad \forall\, i = 1, 2. \tag{31}$$

Corollary 5. *In Theorem 3 for* $h_1(t) = h(1-t)$ *and* $h_2(t) = h(t)$, *we get*

$$\left|I_1\right| \leq \varsigma(g(a_2), g(a_1)) \times \Bigg\{C^{1-\frac{1}{q}} \sqrt[q]{\left|(f \circ g^{-1})'(g(a_1))\right|^q \overline{H_1} + \left|(f \circ g^{-1})'(g(a_2))\right|^q \overline{H_2}} + D^{1-\frac{1}{q}} \sqrt[q]{\left|(f \circ g^{-1})'(g(a_1))\right|^q \overline{M_1} + \left|(f \circ g^{-1})'(g(a_2))\right|^q \overline{M_2}}\Bigg\}, \tag{32}$$

where

$$\overline{H_1} := \int_0^{\varpi_x} t^\alpha h(1-t)dt, \quad \overline{H_2} := \int_0^{\varpi_x} t^\alpha h(t)dt \tag{33}$$

and

$$\overline{M_1} := \int_{\varpi_x}^1 (1-t)^\alpha h(1-t)dt, \quad \overline{M_2} := \int_{\varpi_x}^1 (1-t)^\alpha h(t)dt. \tag{34}$$

Corollary 6. *In Theorem 3 for* $h_1(t) = h_2(t) = t(1-t)$, *we get*

$$\left|I_1\right| \leq \varsigma(g(a_2), g(a_1))\Big[C^{1-\frac{1}{q}} \sqrt[q]{H^*} + D^{1-\frac{1}{q}} \sqrt[q]{M^*}\Big] \times \sqrt[q]{\left|(f \circ g^{-1})'(g(a_1))\right|^q + \left|(f \circ g^{-1})'(g(a_2))\right|^q}, \tag{35}$$

where

$$H^* := \int_0^{\varpi_x} t^\alpha t(1-t)dt = \frac{\varpi_x^{\alpha+2}}{\alpha+2} - \frac{\varpi_x^{\alpha+3}}{\alpha+3} \tag{36}$$

and

$$M^* := \int_{\varpi_x}^1 (1-t)^\alpha t(1-t)dt = \frac{(1-\varpi_x)^{\alpha+2}}{\alpha+2} - \frac{(1-\varpi_x)^{\alpha+3}}{\alpha+3}. \tag{37}$$

Theorem 4. *Suppose $P \subseteq \mathbb{R}$ be an open invex subset and $\varsigma : P \times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1, a_2) and $h_1, h_2 : [0,1] \longrightarrow [0, +\infty)$ are continuous. Let $f : P \longrightarrow \mathbb{R}$ be a differentiable on P° and $(\boldsymbol{k} \circ f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, where $\varsigma(g(a_2), g(a_1)) > 0$ and $\boldsymbol{k} : I \longrightarrow \mathbb{R}$ a continuous function on I that is strictly increasing on I. If $|f'|^q$ is $\boldsymbol{k}$-composite-g^{-1}-(h_1, h_2)-preinvex function, $q > 1$ and $p^{-1} + q^{-1} = 1$, then for $\alpha > 0$*

$$\begin{aligned} \big|I_2\big| &\le \varsigma(g(a_2), g(a_1)) \\ &\times \Bigg\{ \sqrt[p]{A}\sqrt[q]{\big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_1))\big|^q J_1 + \big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_2))\big|^q J_2} \\ &+ \sqrt[p]{B}\sqrt[q]{\big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_1))\big|^q G_1 + \big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_2))\big|^q G_2}\Bigg\}, \end{aligned} \tag{38}$$

where A, B, J_1, J_2, G_1 and G_2 are defined as in Theorem 2.

Proof. See Theorem 2 using Lemma 2. ■

Theorem 5. *Suppose $P \subseteq \mathbb{R}$ be an open invex subset and $\varsigma : P \times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1, a_2) and $h_1, h_2 : [0,1] \longrightarrow [0, +\infty)$ are continuous. Let $f : P \longrightarrow \mathbb{R}$ be a differentiable on P° and $(\boldsymbol{k} \circ f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, where $\varsigma(g(a_2), g(a_1)) > 0$ and $\boldsymbol{k} : I \longrightarrow \mathbb{R}$ a continuous function on I that is strictly increasing on I. If $|f'|^q$ is $\boldsymbol{k}$-composite-g^{-1}-(h_1, h_2)-preinvex function, $q \ge 1$, then for $\alpha > 0$*

$$\begin{aligned} \big|I_2\big| &\le \varsigma(g(a_2), g(a_1)) \\ &\times \Bigg\{ C^{1-\frac{1}{q}}\sqrt[q]{\big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_1))\big|^q H_1 + \big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_2))\big|^q H_2} \\ &+ D^{1-\frac{1}{q}}\sqrt[q]{\big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_1))\big|^q M_1 + \big|(\boldsymbol{k}\circ f\circ g^{-1})'(g(a_2))\big|^q M_2}\Bigg\}, \end{aligned} \tag{39}$$

where C, D, H_1, H_2, M_1 and M_2 are defined as in Theorem 3.

Proof. See Theorem 3 using Lemma 2. ■

Remark 6. Using Theorems 4 and 5, for different choices of function g, for example $g(t) = e^t, \ln t, -\frac{1}{t}$ and t^p where $p > 0$ and for different choices of function $\mathbf{k}$, for example $\mathbf{k}(t) = \ln t$ and $\frac{1}{t}$, we can get some Ostrowski inequalities for $\mathbf{k}$-composite preinvex functions using fractional integrals. Also, for functions h_1, h_2 as we taked above, we can get some Ostrowski inequalities for $\mathbf{k}$-composite preinvex functions using fractional integrals. For $\alpha = 1$, we get some Ostrowski inequalities for $\mathbf{k}$-composite preinvex functions using classical integrals.

Theorem 6. *Suppose $P \subseteq \mathbb{R}$ be an open invex subset and $\varsigma : P \times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1, a_2) and $h_1, h_2 : [0, 1] \longrightarrow [0, +\infty)$ are continuous. Let $f : P \longrightarrow (0, +\infty)$ be a differentiable on P° and $(f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, where $\varsigma(g(a_2), g(a_1)) > 0$. If f'^q is composite-g^{-1}-$(r; h_1, h_2)$-preinvex function, $0 < r \le 1$, $q > 1$ and $p^{-1} + q^{-1} = 1$, then for $\alpha > 0$*

$$\begin{aligned} |I_1| &\le \varsigma(g(a_2), g(a_1)) \\ &\times \Bigg\{ \sqrt[p]{A} \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq} J_{1,r}^r + ((f \circ g^{-1})'(g(a_2)))^{rq} J_{2,r}^r} \\ &+ \sqrt[p]{B} \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq} G_{1,r}^r + ((f \circ g^{-1})'(g(a_2)))^{rq} G_{2,r}^r} \Bigg\}, \end{aligned} \tag{40}$$

where

$$J_{i,r} := \int_0^{\varpi_x} h_i^{\frac{1}{r}}(t)dt, \quad G_{i,r} := \int_{\varpi_x}^1 h_i^{\frac{1}{r}}(t)dt, \quad \forall\, i = 1, 2 \tag{41}$$

and A, B are defined as in Theorem 2.

Proof. From Lemma 1, composite-g^{-1}-$(r; h_1, h_2)$-preinvexity of f'^q, Hölder inequality, Minkowski inequality and properties of the modulus, we get

$$\begin{aligned} |I_1| &\le |\varsigma(g(a_2), g(a_1))| \Bigg[\int_0^{\varpi_x} t^\alpha \big|(f \circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1)))\big| dt \\ &+ \int_{\varpi_x}^1 (1-t)^\alpha \big|(f \circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1)))\big| dt \Bigg] \\ &\le \varsigma(g(a_2), g(a_1)) \\ &\times \Bigg[\left(\int_0^{\varpi_x} t^{p\alpha} dt \right)^{\frac{1}{p}} \left(\int_0^{\varpi_x} ((f \circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1))))^q \, dt \right)^{\frac{1}{q}} \end{aligned}$$

$$+\left(\int_{\varpi_x}^{1}(1-t)^{p\alpha}dt\right)^{\frac{1}{p}}\left(\int_{\varpi_x}^{1}\left((f\circ g^{-1})'(g(a_1)+t\varsigma(g(a_2),g(a_1)))\right)^{q}dt\right)^{\frac{1}{q}}\Bigg]$$

$$\leq \varsigma(g(a_2),g(a_1))$$

$$\times\left[\sqrt[p]{A}\left(\int_{0}^{\varpi_x}\left[\left((f\circ g^{-1})'(g(a_1))\right)^{rq}h_1(t)+\left((f\circ g^{-1})'(g(a_2))\right)^{rq}h_2(t)\right]^{\frac{1}{r}}dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\sqrt[p]{B}\left(\int_{\varpi_x}^{1}\left[\left((f\circ g^{-1})'(g(a_1))\right)^{rq}h_1(t)+\left((f\circ g^{-1})'(g(a_2))\right)^{rq}h_2(t)\right]^{\frac{1}{r}}dt\right)^{\frac{1}{q}}\right]$$

$$\leq \varsigma(g(a_2),g(a_1))\Bigg\{\sqrt[p]{A}\Bigg[\left(\int_{0}^{\varpi_x}\left((f\circ g^{-1})'(g(a_1))\right)^{q}h_1^{\frac{1}{r}}(t)dt\right)^{r}$$

$$+\left(\int_{0}^{\varpi_x}\left((f\circ g^{-1})'(g(a_2))\right)^{q}h_2^{\frac{1}{r}}(t)dt\right)^{r}\Bigg]^{\frac{1}{rq}}$$

$$+\sqrt[p]{B}\Bigg[\left(\int_{\varpi_x}^{1}\left((f\circ g^{-1})'(g(a_1))\right)^{q}h_1^{\frac{1}{r}}(t)dt\right)^{r}$$

$$+\left(\int_{\varpi_x}^{1}\left((f\circ g^{-1})'(g(a_2))\right)^{q}h_2^{\frac{1}{r}}(t)dt\right)^{r}\Bigg]^{\frac{1}{rq}}\Bigg\}$$

$$=\varsigma(g(a_2),g(a_1))\Bigg\{\sqrt[p]{A}\sqrt[rq]{\left((f\circ g^{-1})'(g(a_1))\right)^{rq}J_{1,r}^{r}+\left((f\circ g^{-1})'(g(a_2))\right)^{rq}J_{2,r}^{r}}$$

$$+\sqrt[p]{B}\sqrt[rq]{\left((f\circ g^{-1})'(g(a_1))\right)^{rq}G_{1,r}^{r}+\left((f\circ g^{-1})'(g(a_2))\right)^{rq}G_{2,r}^{r}}\Bigg\}.$$

The proof of Theorem 6 is completed. ■

Remark 7. For $r=1$ in Theorem 6, we get Theorem 2.

Corollary 7. *In Theorem 6 if $(f\circ g^{-1})'\leq K$, we get*

$$|I_1|\leq K\varsigma(g(a_2),g(a_1))\left\{\sqrt[p]{A}\sqrt[rq]{J_{1,r}^{r}+J_{2,r}^{r}}+\sqrt[p]{B}\sqrt[rq]{G_{1,r}^{r}+G_{2,r}^{r}}\right\}. \tag{42}$$

If one takes $(f\circ g^{-1})'\leq K$ and $\varsigma(g(a_2),g(x))=g(a_2)-g(x)$ for all x, we have

$$|\overline{I_1}|\leq K(g(a_2)-g(a_1))\times\left\{\sqrt[p]{A^*}\sqrt[rq]{J_{1,r}^{*r}+J_{2,r}^{*r}}+\sqrt[p]{B^*}\sqrt[rq]{G_{1,r}^{*r}+G_{2,r}^{*r}}\right\}, \tag{43}$$

where

$$A^*:=\int_{0}^{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}t^{p\alpha}dt=\frac{\left(\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}\right)^{p\alpha+1}}{p\alpha+1}, \tag{44}$$

$$B^* := \int_{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}^{1} (1-t)^{p\alpha} dt = \frac{\left(1 - \frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}\right)^{p\alpha+1}}{p\alpha+1} \tag{45}$$

and

$$J^*_{i,r} := \int_0^{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}} h_i^{\frac{1}{r}}(t)dt, \;\; G^*_{i,r} := \int_{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}^{1} h_i^{\frac{1}{r}}(t)dt, \;\; \forall\, i = 1,2. \tag{46}$$

Corollary 8. *In Theorem 6 for $h_1(t) = h(1-t)$ and $h_2(t) = h(t)$, we get*

$$\begin{aligned} \big|I_1\big| &\le \varsigma(g(a_2), g(a_1)) \\ &\times \left\{ \sqrt[p]{A}\, \sqrt[rq]{((f\circ g^{-1})'(g(a_1)))^{rq}\, \overline{J_{1,r}}^{\,r} + ((f\circ g^{-1})'(g(a_2)))^{rq}\, \overline{J_{2,r}}^{\,r}} \right. \\ &\left. + \sqrt[p]{B}\, \sqrt[rq]{((f\circ g^{-1})'(g(a_1)))^{rq}\, \overline{G_{1,r}}^{\,r} + ((f\circ g^{-1})'(g(a_2)))^{rq}\, \overline{G_{2,r}}^{\,r}} \right\}, \end{aligned} \tag{47}$$

where

$$\overline{J_{1,r}} := \int_{1-\varpi_x}^{1} h^{\frac{1}{r}}(t)dt, \quad \overline{J_{2,r}} := \int_0^{\varpi_x} h^{\frac{1}{r}}(t)dt \tag{48}$$

and

$$\overline{G_{1,r}} := \int_0^{1-\varpi_x} h^{\frac{1}{r}}(t)dt, \quad \overline{G_{2,r}} := \int_{\varpi_x}^{1} h^{\frac{1}{r}}(t)dt. \tag{49}$$

Corollary 9. *In Theorem 6 for $h_1(t) = h_2(t) = t(1-t)$, we get*

$$\begin{aligned} \big|I_1\big| \;&\le\; \varsigma(g(a_2), g(a_1)) \Big[\sqrt[p]{A}\, \sqrt[q]{J^*_r} + \sqrt[p]{B}\, \sqrt[q]{G^*_r} \Big] \\ &\times\; \sqrt[rq]{((f\circ g^{-1})'(g(a_1)))^{rq} + ((f\circ g^{-1})'(g(a_2)))^{rq}}, \end{aligned} \tag{50}$$

where

$$J^*_r := \int_0^{\varpi_x} \sqrt[r]{t(1-t)}dt, \quad G^*_r := \int_{\varpi_x}^{1} \sqrt[r]{t(1-t)}dt. \tag{51}$$

Theorem 7. *Suppose $P \subseteq \mathbb{R}$ be an open invex subset and $\varsigma : P\times P \longrightarrow \mathbb{R}$. Also, let $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ be a continuous strictly increasing function that is differentiable on (a_1, a_2) and $h_1, h_2 : [0,1] \longrightarrow [0,+\infty)$ are continuous. Let $f : P \longrightarrow (0,+\infty)$ be a differentiable on P° and $(f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, where $\varsigma(g(a_2), g(a_1)) > 0$. If f'^q is* *composite-g^{-1}-$(r; h_1, h_2)$-preinvex function, $0 < r \leq 1$ and $q \geq 1$, then for $\alpha > 0$*

$$(52)\quad \left|I_1\right| \leq \varsigma(g(a_2), g(a_1)) \times \left\{ C^{1-\frac{1}{q}} \sqrt[rq]{((f\circ g^{-1})'(g(a_1)))^{rq} H_{1,r}^r + ((f\circ g^{-1})'(g(a_2)))^{rq} H_{2,r}^r} + D^{1-\frac{1}{q}} \sqrt[rq]{((f\circ g^{-1})'(g(a_1)))^{rq} M_{1,r}^r + ((f\circ g^{-1})'(g(a_2)))^{rq} M_{2,r}^r} \right\},$$

where

$$(53)\qquad H_{i,r} := \int_0^{\varpi_x} t^\alpha h_i^{\frac{1}{r}}(t)dt, \quad M_{i,r} := \int_{\varpi_x}^1 (1-t)^\alpha h_i^{\frac{1}{r}}(t)dt, \quad \forall\, i = 1, 2$$

and C, D are defined as in Theorem 3.

Proof. From Lemma 1, composite-g^{-1}-$(r; h_1, h_2)$-preinvexity of f'^q, power mean inequality, Minkowski inequality and properties of the modulus, we have

$$\begin{aligned}
\left|I_1\right| &\leq |\varsigma(g(a_2), g(a_1))| \left[\int_0^{\varpi_x} t^\alpha \left|(f\circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1)))\right| dt \right.\\
&\quad \left. + \int_{\varpi_x}^1 (1-t)^\alpha \left|(f\circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1)))\right| dt \right]\\
&\leq \varsigma(g(a_2), g(a_1)) \left[\left(\int_0^{\varpi_x} t^\alpha dt \right)^{1-\frac{1}{q}} \right.\\
&\quad \times \left(\int_0^{\varpi_x} t^\alpha \left((f\circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1)))\right)^q dt \right)^{\frac{1}{q}}\\
&\quad + \left(\int_{\varpi_x}^1 (1-t)^\alpha dt \right)^{1-\frac{1}{q}}\\
&\quad \left. \times \left(\int_{\varpi_x}^1 (1-t)^\alpha \left((f\circ g^{-1})'(g(a_1) + t\varsigma(g(a_2), g(a_1)))\right)^q dt \right)^{\frac{1}{q}} \right]\\
&\leq \varsigma(g(a_2), g(a_1)) \left[C^{1-\frac{1}{q}} \left(\int_0^{\varpi_x} t^\alpha \left[\left((f\circ g^{-1})'(g(a_1))\right)^{rq} h_1(t) \right.\right.\right.\\
&\quad \left.\left. + \left((f\circ g^{-1})'(g(a_2))\right)^{rq} h_2(t) \right]^{\frac{1}{r}} dt \right)^{\frac{1}{q}}\\
&\quad + D^{1-\frac{1}{q}} \left(\int_{\varpi_x}^1 (1-t)^\alpha \left[\left((f\circ g^{-1})'(g(a_1))\right)^{rq} h_1(t) \right.\right.\\
&\quad \left.\left.\left. + \left((f\circ g^{-1})'(g(a_2))\right)^{rq} h_2(t) \right]^{\frac{1}{r}} dt \right)^{\frac{1}{q}} \right]
\end{aligned}$$

$$
\begin{aligned}
&\le \varsigma(g(a_2), g(a_1)) \times \Bigg\{ C^{1-\frac{1}{q}} \Bigg[\left(\int_0^{\varpi_x} t^{\alpha} \left((f \circ g^{-1})'(g(a_1))\right)^q h_1^{\frac{1}{r}}(t)dt \right)^r \\
&+ \left(\int_0^{\varpi_x} t^{\alpha} \left((f \circ g^{-1})'(g(a_2))\right)^q h_2^{\frac{1}{r}}(t)dt \right)^r \Bigg]^{\frac{1}{rq}} \\
&+ D^{1-\frac{1}{q}} \Bigg[\left(\int_{\varpi_x}^1 (1-t)^{\alpha} \left((f \circ g^{-1})'(g(a_1))\right)^q h_1^{\frac{1}{r}}(t)dt \right)^r \\
&+ \left(\int_{\varpi_x}^1 (1-t)^{\alpha} \left((f \circ g^{-1})'(g(a_2))\right)^q h_2^{\frac{1}{r}}(t)dt \right)^r \Bigg]^{\frac{1}{rq}} \Bigg\} \\
&= \varsigma(g(a_2), g(a_1)) \\
&\quad \times \Bigg\{ C^{1-\frac{1}{q}} \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq} H_{1,r}^r + ((f \circ g^{-1})'(g(a_2)))^{rq} H_{2,r}^r} \\
&+ D^{1-\frac{1}{q}} \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq} M_{1,r}^r + ((f \circ g^{-1})'(g(a_2)))^{rq} M_{2,r}^r} \Bigg\}.
\end{aligned}
$$

The proof of Theorem 7 is completed. ■

Remark 8. For $r = 1$ in Theorem 7, we get Theorem 3.

Remark 9. Using Theorems 6 and 7, for different choices of function g, for example $g(t) = e^t, \ln t, -\frac{1}{t}$ and t^p where $p > 0$, we can get some Ostrowski type inequalities for composite preinvex functions using fractional integrals. For $\alpha = 1$, we get some Ostrowski inequalities for composite preinvex functions using classical integrals.

Corollary 10. *In Theorem 7 if* $(f \circ g^{-1})' \le K$, *we get*

$$
\begin{aligned}
(54)\quad |I_1| \le\; & \frac{K\varsigma(g(a_2), g(a_1))}{\left((\alpha+1)\varsigma^{\alpha+1}(g(a_2), g(a_1))\right)^{1-\frac{1}{q}}} \\
& \times \Bigg\{ \varsigma^{(\alpha+1)\left(1-\frac{1}{q}\right)}(g(a_2), g(x)) \sqrt[rq]{H_{1,r}^r + H_{2,r}^r} \\
& + \left(\varsigma(g(a_2), g(a_1)) - \varsigma(g(a_2), g(x))\right)^{(\alpha+1)\left(1-\frac{1}{q}\right)} \sqrt[rq]{M_{1,r}^r + M_{2,r}^r} \Bigg\}.
\end{aligned}
$$

If one takes $(f \circ g^{-1})' \le K$ *and* $\varsigma(g(a_2), g(x)) = g(a_2) - g(x)$ *for all* x, *we have*

$$
(55)\qquad |\overline{I_1}| \le \frac{K(g(a_2) - g(a_1))}{\left((\alpha+1)(g(a_2) - g(a_1))^{\alpha+1}\right)^{1-\frac{1}{q}}}
$$

$$\times \left\{ (g(a_2) - g(x))^{(\alpha+1)\left(1-\frac{1}{q}\right)} \sqrt[rq]{H_{1,r}^{*r} + H_{2,r}^{*r}} \right.$$
$$\left. + (g(x) - g(a_1))^{(\alpha+1)\left(1-\frac{1}{q}\right)} \sqrt[rq]{M_{1,r}^{*r} + M_{2,r}^{*r}} \right\},$$

where

$$H_{i,r}^* := \int_0^{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}} t^\alpha h_i^{\frac{1}{r}}(t)dt, \tag{56}$$
$$M_{i,r}^* := \int_{\frac{g(a_2)-g(x)}{g(a_2)-g(a_1)}}^1 (1-t)^\alpha h_i^{\frac{1}{r}}(t)dt, \quad \forall\, i = 1, 2.$$

Corollary 11. *In Theorem 7 for $h_1(t) = h(1-t)$ and $h_2(t) = h(t)$, we get*

$$\left|I_1\right| \leq \varsigma(g(a_2), g(a_1)) \tag{57}$$
$$\times \left\{ C^{1-\frac{1}{q}} \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq}\, \overline{H_{1,r}}^r + ((f \circ g^{-1})'(g(a_2)))^{rq}\, \overline{H_{2,r}}^r} \right.$$
$$\left. + D^{1-\frac{1}{q}} \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq}\, \overline{M_{1,r}}^r + ((f \circ g^{-1})'(g(a_2)))^{rq}\, \overline{M_{2,r}}^r} \right\},$$

where

$$\overline{H_{1,r}} := \int_0^{\varpi_x} t^\alpha h^{\frac{1}{r}}(1-t)dt, \quad \overline{H_{2,r}} := \int_0^{\varpi_x} t^\alpha h^{\frac{1}{r}}(t)dt \tag{58}$$

and

$$\overline{M_{1,r}} := \int_{\varpi_x}^1 (1-t)^\alpha h^{\frac{1}{r}}(1-t)dt, \quad \overline{M_{2,r}} := \int_{\varpi_x}^1 (1-t)^\alpha h^{\frac{1}{r}}(t)dt. \tag{59}$$

Corollary 12. *In Theorem 7 for $h_1(t) = h_2(t) = t(1-t)$, we get*

$$\left|I_1\right| \leq \varsigma(g(a_2), g(a_1)) \left[C^{1-\frac{1}{q}} \sqrt[q]{H_r^*} + D^{1-\frac{1}{q}} \sqrt[q]{M_r^*} \right] \tag{60}$$
$$\times \sqrt[rq]{((f \circ g^{-1})'(g(a_1)))^{rq} + ((f \circ g^{-1})'(g(a_2)))^{rq}},$$

where

$$H_r^* := \int_0^{\varpi_x} t^\alpha \sqrt[r]{t(1-t)}dt, \quad M_r^* := \int_{\varpi_x}^1 (1-t)^\alpha \sqrt[r]{t(1-t)}dt. \tag{61}$$

Theorem 8. *Suppose* $P \subseteq \mathbb{R}$ *be an open invex subset and* $\varsigma : P \times P \longrightarrow \mathbb{R}$. *Also, let* $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ *be a continuous strictly increasing function that is differentiable on* (a_1, a_2) *and* $h_1, h_2 : [0,1] \longrightarrow [0, +\infty)$ *are continuous. Let* $f : P \longrightarrow (0, +\infty)$ *be a differentiable on* P° *and* $(\boldsymbol{k} \circ f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, *where* $\varsigma(g(a_2), g(a_1)) > 0$ *and* $\boldsymbol{k} : (0, +\infty) \longrightarrow (0, +\infty)$ *a continuous function on* $(0, +\infty)$ *that is strictly increasing on* $(0, +\infty)$. *If* f'^q *is* $\boldsymbol{k}$*-composite-*g^{-1}*-*$(r; h_1, h_2)$*-preinvex function,* $0 < r \leq 1$, $q > 1$ *and* $p^{-1} + q^{-1} = 1$, *then for* $\alpha > 0$

$$
\begin{aligned}
(62)\quad \left|I_2\right| &\leq \varsigma(g(a_2), g(a_1)) \\
&\times \Bigg\{ \sqrt[p]{A} \sqrt[rq]{((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_1)))^{rq} J_{1,r}^r + ((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_2)))^{rq} J_{2,r}^r} \\
&+ \sqrt[p]{B} \sqrt[rq]{((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_1)))^{rq} G_{1,r}^r + ((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_2)))^{rq} G_{2,r}^r} \Bigg\},
\end{aligned}
$$

where A, B *are defined as in Theorem 2 and* $J_{1,r}$, $J_{2,r}$, $G_{1,r}$, $G_{2,r}$ *are defined as in Theorem 6.*

Proof. See Theorem 6 using Lemma 2. ■

Remark 10. For $r = 1$ in Theorem 8, we get Theorem 4.

Theorem 9. *Suppose* $P \subseteq \mathbb{R}$ *be an open invex subset and* $\varsigma : P \times P \longrightarrow \mathbb{R}$. *Also, let* $g : [a_1, a_2] \longrightarrow [g(a_1), g(a_2)]$ *be a continuous strictly increasing function that is differentiable on* (a_1, a_2)*and* $h_1, h_2 : [0,1] \longrightarrow [0, +\infty)$ *are continuous. Let* $f : P \longrightarrow (0, +\infty)$ *be a differentiable on* P° *and* $(\boldsymbol{k} \circ f \circ g^{-1})' \in L[g(a_1), g(a_1) + \varsigma(g(a_2), g(a_1))]$, *where* $\varsigma(g(a_2), g(a_1)) > 0$ *and* $\boldsymbol{k} : (0, +\infty) \longrightarrow (0, +\infty)$ *a continuous function on* $(0, +\infty)$ *that is strictly increasing on* $(0, +\infty)$. *If* f'^q *is* $\boldsymbol{k}$*-composite-*g^{-1}*-*$(r; h_1, h_2)$*-preinvex function,* $0 < r \leq 1$ *and* $q \geq 1$, *then for* $\alpha > 0$

$$
\begin{aligned}
(63)\quad \left|I_2\right| &\leq \varsigma(g(a_2), g(a_1)) \\
&\times \Bigg\{ C^{1-\frac{1}{q}} \sqrt[rq]{((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_1)))^{rq} H_{1,r}^r + ((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_2)))^{rq} H_{2,r}^r} \\
&+ D^{1-\frac{1}{q}} \sqrt[rq]{((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_1)))^{rq} M_{1,r}^r + ((\boldsymbol{k} \circ f \circ g^{-1})'(g(a_2)))^{rq} M_{2,r}^r} \Bigg\},
\end{aligned}
$$

where C, D *are defined as in Theorem 3 and* $H_{1,r}$, $H_{2,r}$, $M_{1,r}$, $M_{2,r}$ *are defined as in Theorem 7.*

Proof. See Theorem 7 using Lemma 2. ■

Remark 11. For $r = 1$ in Theorem 9, we get Theorem 5.

Remark 12. Using Theorems 8 and 9, for different choices of function g, for example $g(t) = e^t, \ln t, -\frac{1}{t}$ and t^p where $p > 0$ and for different choices of function $\mathbf{k}$, for example $\mathbf{k}(t) = \ln t$ and $\frac{1}{t}$, we can get some Ostrowski inequalities for $\mathbf{k}$-composite preinvex functions using fractional integrals. Also, for functions h_1, h_2 as we taked above, we can get some Ostrowski inequalities for $\mathbf{k}$-composite preinvex functions using fractional integrals. For $\alpha = 1$, we get some Ostrowski inequalities for $\mathbf{k}$-composite preinvex functions using classical integrals.

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Received on 19.10.2018 and, in revised form, on 14.08.2019.