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SOME NEW GENERALIZED RIESZ SPACES OVER MODULUS FUNCTION

Abstract. The structure of this paper is to introduce the new sequence space of Riesz type of the form $r^q_{\mathcal{F}}(\triangle^p_s)$ by using modulus function. We will prove that it is complete linear paranormed space. It will be shown to be linearly isomorphism with $\ell(p)$. Further, some inclusion relation will be computed.

Key words: modulus function, paranormed sequence, infinite matrices.

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1. Introduction

We denote the set of all sequences with complex terms by Ω. It is a routine verification that Ω is a linear space with respect to the coordinate wise addition and scalar multiplication of sequences which are defined, as usual, by

$$\zeta + \eta = (\zeta_k) + (\eta_k) = (\zeta_k + \eta_k)$$

and

$$\beta\zeta = \beta(\zeta_k) = (\beta\zeta_k),$$

respectively; with $\zeta = (\zeta_k)$, $\eta = (\eta_k) \in \Omega$ and $\beta \in \mathbf{C}$. By a *sequence space* we define a linear subspace of Ω i.e., the sequence space is the set of scalar sequences(real or complex) which is closed under co-ordinate wise addition and scalar multiplication. Throughout the paper N and C denotes the set of non-negative integers and the set of complex numbers, respectively. As in [8, 11, 12], we denote by ℓ_∞, c and c_0, respectively, the space of all bounded sequences, the space of convergent sequences and the sequences converging to zero. Also, by ℓ_1, $\ell(p)$, cs and bs we denote the spaces of all absolutely, p-absolutely convergent, convergent and bounded series, respectively, as can be seen in [9, 14, 37].

As in [10], [15]-[18], for an infinite matrix $T = (t_{i,j})$ and $\nu = (\nu_k) \in \Omega$, the T-transform of ν is $T\nu = \{(T\nu)_i\}$ provided it exists $\forall\ i \in \mathbb{N}$, where $(T\nu)_i = \sum_{j=0}^{\infty} t_{i,j}\nu_j$.

For an infinite matrix $T = (t_{i,j})$, the set G_T, where

$$G_T = \{\nu = (\nu_j) \in \Omega \ : \ T\nu \in G\}, \tag{1}$$

is known as the matrix domain of T in G [23, 24, 33].

A infinite matrix $G = (\varrho_{nk})$ is said to be regular if and only if the following conditions hold:

(i) $\lim\limits_{n\to\infty} \sum\limits_{k=0}^{\infty} \varrho_{nk} = 1$,

(ii) $\lim\limits_{n\to\infty} \varrho_{nk} = 0$, $(k = 0, 1, 2, ...)$,

(iii) $\sum\limits_{k=0}^{\infty} |\varrho_{nk}| < M$, $(M > 0,\ n = 0, 1, 2, ...)$.

Let (q_k) be a sequence of positive numbers and let us write, $Q_n = \sum\limits_{k=0}^{n} q_k$ for $n \in N$. Then the matrix $R^q = (r^q_{nk})$ of the Riesz mean (R, q_n) is given by

$$r^q_{nk} = \begin{cases} \frac{q_k}{Q_n}, & \text{if} \quad 0 \le k \le n, \\ 0 & \text{if} \quad k > n. \end{cases}$$

The Riesz mean (R, q_n) is regular if and only if $Q_n \to \infty$ as $n \to \infty$ as can be seen in [29, 35].

Quite recently, in [32], the author has introduced the following:

$$r^q(u,p) = \left\{ \zeta = (\zeta_k) \in \Omega : \sum_k \left| \frac{1}{Q_k} \sum_{j=0}^{k} u_j q_j \zeta_j \right|^{p_k} < \infty \right\}$$

where, $0 < p_k \le H < \infty$.

In [20], the author introduced the following difference sequence spaces $W(\triangle)$:

$$W(\triangle) = \{\zeta = (\zeta_k) \in \Omega : (\triangle\zeta_k) \in W\},$$

where, $W \in \{\ell_\infty, c, c_0\}$ and $\triangle\zeta_k = \zeta_k - \zeta_{k+1}$.

In [2], the author has studied the sequence space as

$$bv_p = \left\{ \zeta = (\zeta_k) \in \Omega : \sum_k |\Delta x_k|^p < \infty \right\},$$

where $1 \le p < \infty$. With the notation of (1), the space bv_p can be redefined as

$$bv_p = (l_p)_\triangle, 1 \le p < \infty$$

where, $\triangle$ denotes the matrix $\triangle = (\triangle_{nk})$ defined as

$$\triangle_{nk} = \begin{cases} (-1)^{n-k}, & \text{if } n-1 \leq k \leq n, \\ \\ 0, & \text{if } k < n-1 \text{ or } k > n. \end{cases}$$

In [25], the author introduced the concept of modulus function. We call a function $\mathcal{F} : [0, \infty) \to [0, \infty)$ to be modulus function if

(i) $\mathcal{F}(\zeta) = 0$ if and only if $zeta = 0$,
(ii) $\mathcal{F}(\zeta + \eta) \leq \{(\zeta) + \{(\eta) \ \forall \ \zeta \geq 0, \eta \geq 0$,
(iii) $\mathcal{F}$ is increasing, and
(iv) $\mathcal{F}$ is continuous from the right at 0.

One can easily see that if $\mathcal{F}_1$ and $\mathcal{F}_2$ are modulus functions then so is $\mathcal{F}_1+\mathcal{F}_2$; and the function $\mathcal{F}^j$ ($j \in \mathbf{N}$), the composition of a modulus function $\mathcal{F}$ with itself j times is also modulus function.

Recently, in [30] the new space was introduced by using notion of modulus function as follows:

$$L(\mathcal{F}) = \left\{ \zeta = (\zeta_r) : \ \sum_r |\mathcal{F}(\zeta_r|)| < \infty \right\}.$$

The approach of constructing a new sequence space by means of matrix domain of a particular limitation method has been studied by several authors. $(\ell_\infty)_{N_q}$ and c_{N_q}(see, [36]), $(\ell_p)_{C_1} = X_p$ and $(\ell_\infty)_{C_1} = X_\infty$(see, [28]), $(\ell_\infty)_{R^t} = r^t_\infty$, $(c)_{R^t} = r^t_c$ and $(c_o)_{R^t} = r^t_0$(see, [19]), $(\ell_p)_{R^t} = r^t_p$ (see, [1]), $(\ell_p)_{E^r} = e^r_p$ and $(l_\infty)_{E^r} = e^r_\infty$ (see, [3]), $(c_0)_{A^r} = a^r_0$ and $c_{A^r} = a^r_c$(see, [4]), $[c_0(u,p)]_{A^r} = a^r_0(u,p)$ and $[c(u,p)]_{A^r} = a^r_c(u,p)$ (see, [5], $r^q(u,p) = \{l(p)\}_{R^q_u}$ (see, [32])and etc.

2. The sequence space $r^q_{\mathcal{F}}(\triangle^p_s)$ of non-absolute type

In this section, we define the Riesz sequence space $r^q_{\mathcal{F}}(\triangle^p_s)$, and prove that the space $r^q_{\mathcal{F}}(\triangle^p_s)$ is a complete paranormed linear space and show it is linearly isomorphic to the space $l(p)$.

Let Λ be a real or complex linear space, define the function $\tau : \Lambda \to \mathbb{R}$ with $\mathbb{R}$ as set of real numbers. Then, the paranormed space is a pair $(\Lambda; \tau)$ and τ is a paranorm for Λ, if the following axioms are satisfied for all ζ, $\eta \in \Lambda$ and for all scalars β:

(i) $\tau(\theta) = 0$,
(ii) $\tau(-\zeta) = \tau(\zeta)$,
(iii) $\tau(\zeta + \eta) \leq \tau(\zeta) + \tau(\eta)$, and
(iv) scalar multiplication is continuous, that is,

$|\beta_n - \beta| \to 0$ and $h(\zeta_n - \zeta) \to 0$ imply $\tau(\beta_n\zeta_n - \beta\zeta) \to 0$ for all $\beta's$ in $\mathbb{R}$ and $\zeta's$ in Λ , where θ is a zero vector in the linear space Λ. Assume here and after that (p_k) be a bounded sequence of strictly positive real numbers with $\sup_k p_k = H$ and $M = \max\{1, H\}$. Then, the linear space $\ell_\infty(p)$ was defined by Maddox [23] as follows :

$$\ell_\infty(p) = \{\zeta = (\zeta_k) : \sup_k |\zeta_k|^{p_k} < \infty\}$$

which is complete space paranormed by

$$\tau_1(\zeta) = \left[\sup_k |\zeta_k|^{p_k}\right]^{1/M}.$$

We shall assume throughout that $p_k^{-1} + \{p_k^{'}\}^{-1}$ provided $1 < inf p_k \leq H < \infty$, and we denote the collection of all finite subsets of N by F, where N={0, 1, 2, ...

Following Altay [1]-[3], Başarir and Öztürk [6], Choudhary and Mishra [7], Ganie and Dowlath [10, 14], Mursaleen [26], Sheikh and Ganie [32]-[34], Ruckle [30], Sengönül [31], we define the difference sequence space $r^q_{\mathcal{F}}(\triangle^p_s)$ as follows:

$$r^q_{\mathcal{F}}(\triangle^p_s) = \left\{\zeta = (\zeta_k) \in \Omega : \sum_k \left|\mathcal{F}\left(\frac{1}{Q^s_k}\sum_{j=0}^{k} q_j \triangle\zeta_j\right)\right|^{p_k} < \infty\right\}$$

where, $0 < p_k \leq H < \infty$ and $s \geq 0$.

By (1), it can be redefined as

$$r^q_{\mathcal{F}}(\triangle^p_s) = \{l(p)\}_{R^q_{\mathcal{F}}(\triangle_g)}.$$

Define the sequence $\xi = (\xi_k)$, which will be used, by the $R^q_{\mathcal{F}}\triangle_g$-transform of a sequence $\zeta = (\zeta_k)$, i.e.,

$$\xi_k = f\frac{1}{Q^s_k}\sum_{j=0}^{k} q_j \triangle\zeta_j. \tag{2}$$

Now, we begin with the following theorem which is essential in the text.

Theorem 1. *$r^q_{\mathcal{F}}(\triangle^p_s)$ is a complete linear metric space paranormed by $h_\triangle$, defined as*

$$h_\triangle(\zeta) = \left[\sum_k \left|\mathcal{F}\left[\frac{1}{Q^s_k}\sum_{j=0}^{k-1}(q_j - q_{j+1})\zeta_j + \frac{q_k}{Q^s_k}\zeta_k\right]\right|^{p_k}\right]^{\frac{1}{M}}$$

with $0 < p_k \leq H < \infty$.

Proof. The linearity of $r^q_{\mathcal{F}}(\triangle^p_s)$ with respect to the co-ordinatewise addition and scalar multiplication follows from from the inequalities which are satisfied for $\zeta, \xi \in r^q_{\mathcal{F}}(\triangle^p_s)$(see [14], p.30])

$$\left[\sum_k \left|\mathcal{F}\left[\frac{1}{Q^s_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})(\zeta_j+\eta_j)+\frac{q_k}{Q^s_k}(\zeta_k+\eta_k)\right]\right|^{p_k}\right]^{\frac{1}{M}} \tag{3}$$
$$\leq \left[\sum_k \left|\mathcal{F}\left(\frac{1}{Q^s_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q^s_k}\zeta_k\right)\right|^{p_k}\right]^{\frac{1}{M}}$$
$$+\left[\sum_k \left|\mathcal{F}\left(\frac{1}{Q^s_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\eta_j+\frac{q_k}{Q^s_k}\eta_k\right)\right|^{p_k}\right]^{\frac{1}{M}}$$

and for any $\alpha \in \mathbf{R}$ (see, [13])

$$|\alpha|^{p_k} \leq \max(1, |\alpha|^M). \tag{4}$$

It is clear that, $h_\triangle(\theta)$=0 and $h_\triangle(\zeta) = h_\triangle(-\zeta)$ for all $\zeta \in r^q_{\mathcal{F}}(\triangle^p_s)$. Again the inequality (3) and (4), yield the subadditivity of $h_\triangle$ and

$$h_\triangle(\alpha\zeta) \leq \max(1, |\alpha|)h_\triangle(\zeta).$$

Let $\{\zeta^n\}$ be any sequence of points of the space $r^q_{\mathcal{F}}(\triangle^p_s)$ such that $h_\triangle(\zeta^n - \zeta) \to 0$ and (α_n) is a sequence of scalars such that $\alpha_n \to \alpha$. Then, since the inequality,

$$h_\triangle(x^n) \leq h_\triangle(x) + h_\triangle(x^n - x)$$

holds by subadditivity of $h_\triangle$, $\{h_\triangle(\zeta^n)\}$ is bounded and we thus have

$$h_\triangle(\alpha_n\zeta^n - \alpha\zeta) = \left[\sum_k \left|\mathcal{F}\left(\frac{1}{Q^s_k}\sum_{j=0}^{k}(q_j-q_{j+1})(\alpha_n\zeta^n_j-\alpha\zeta_j)\right)\right|^{p_k}\right]^{\frac{1}{M}}$$
$$\leq |\alpha_n - \alpha|^{\frac{1}{M}}\, h_\triangle(\zeta^n) + |\alpha|^{\frac{1}{M}}\, h_\triangle(\zeta^n - \zeta)$$

which tends to zero as $n \to \infty$, which shows that the scalar multiplication is continuous. Hence, $h_\triangle$ is paranorm on the space $r^q_{\mathcal{F}}(\triangle^p_u)$.

It remains to prove the completeness of the space $r^q_{\mathcal{F}}(\triangle^p_s)$. Let $\{\zeta^j\}$ be any Cauchy sequence in the space $r^q_{\mathcal{F}}(\triangle^p_s)$, where $\zeta^i = \{\zeta^i_0, \zeta^i_1, ...\}$. Then, for a given $\epsilon > 0$, there exists a positive integer $n_0(\epsilon)$ such that

$$h_\triangle(\zeta^i - \zeta^j) < \epsilon \tag{5}$$

for all $i, j \geq n_0(\epsilon)$. Using definition of $h_\triangle$ and for each fixed $k \in \mathbf{N}$, we have

$$\left|(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k - (R^q_s\triangle\zeta^j)_k\right| \leq \left[\sum_k \left|(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k - (R^q_{\mathcal{F}}\triangle_s\zeta^j)_k\right|^{p_k}\right]^{\frac{1}{M}} < \epsilon$$

for $i, j \geq n_0(\epsilon)$, which leads us to the fact that $\{(R^q_{\mathcal{F}}\triangle_s\zeta^0)_k, (R^q_{\mathcal{F}}\triangle_s\zeta^1)_k, \dots\}$ is a Cauchy sequence of real numbers for every fixed $k \in \mathbf{N}$. Since R is complete, it converges, say, $(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k \to ((R^q_{\mathcal{F}}\triangle_s\zeta)_k$ as $i \to \infty$. Using these infinitely many limits $(R^q_{\mathcal{F}}\triangle_s\zeta)_0, (R^q_s\triangle\zeta)_1, \dots$, we define the sequence $\{(R^q_{\mathcal{F}}\triangle_s\zeta)_0, (R^q_{\mathcal{F}}\triangle_s\zeta)_1, \dots\}$. From (5) for each $m \in \mathbf{N}$ and $i, j \geq n_0(\epsilon)$,

$$\sum_{k=0}^{m} \left|(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k - (R^q_{\mathcal{F}}\triangle_s\zeta^j)_k\right|^{p_k} \leq h_\triangle(\zeta^i - \zeta^j)^M < \epsilon^M. \tag{6}$$

Take any $i, j \geq n_0(\epsilon)$. First, let $j \to \infty$ in (6) and then $m \to \infty$, we obtain

$$h_\triangle(\zeta^i - \zeta) \leq \epsilon.$$

Finally, taking $\epsilon = 1$ in (6) and by letting $i \geq n_0(1)$, we have by Minkowski's inequality for each $m \in N$ that

$$\left[\sum_{k=0}^{m} \left|(R^q_{\mathcal{F}}\triangle_g\zeta)_k\right|^{p_k}\right]^{\frac{1}{M}} \leq h_\triangle(\zeta^i - \zeta) + h_\triangle(\zeta^i) \leq 1 + h_\triangle(\zeta^i)$$

which implies that $\zeta \in r^q_{\mathcal{F}}(\triangle^p_s)$. Since $h_\triangle(\zeta - \zeta^i) \leq \epsilon$ for all $i \geq n_0(\epsilon)$, it follows that $\zeta^i \to \zeta$ as $i \to \infty$, hence we have shown that $r^q_{\mathcal{F}}(\triangle^p_s)$ is complete, hence the proof of the theorem follows. ■

Note that one can easily see the absolute property does not hold on the spaces $r^q_{\mathcal{F}}(\triangle^p_s)$, that is, $h_\triangle(\zeta) \neq h_\triangle(|\zeta|)$ for atleast one sequence in the space $r^q_{\mathcal{F}}(\triangle^p_s)$ and this says that $r^q_{\mathcal{F}}(\triangle^p_s)$ is a sequence space of non-absolute type.

3. Inclusion relations

In this section, we investigate some of its inclusions properties .

Theorem 2. *If p_k and t_k are bounded sequences of positive real numbers with $0 < p_k \leq t_k < \infty$ for each $k \in \mathbf{N}$, then for any modulus function $\mathcal{F}$, $r^q_{\mathcal{F}}(\triangle^p_s) \subseteq r^q_{\mathcal{F}}(\triangle^t_s)$.*

Proof. For $\zeta \in r^q_{\mathcal{F}}(\triangle^p_s)$ it is obvious that

$$\sum_k \left|\mathcal{F}\left(\frac{1}{Q^s_k}\sum_{j=0}^{k-1}(q_j - q_{j+1})\zeta_j + \frac{q_k}{Q^s_k}\zeta_k\right)\right|^{p_k} < \infty.$$

Consequently, for sufficiently large values of k say $k \geq k_0$ for some fixed $k_0 \in \mathbf{N}$.

$$\left| \mathcal{F}\left(\frac{1}{Q_k^s} \sum_{j=0}^{k-1} (q_j - q_{j+1})\zeta_j + \frac{q_k}{Q_k^s}\zeta_k \right) \right| < \infty.$$

But $\mathcal{F}$ being increasing and $p_k \leq t_k$, we have

$$\sum_{k \geq k_0} \left| \mathcal{F}\left(\frac{1}{Q_k^s} \sum_{j=0}^{k-1} (q_j - q_{j+1})\zeta_j + \frac{q_k}{Q_k^s}\zeta_k \right) \right|^{t_k} \leq \sum_{k \geq k_0} \left| \mathcal{F}\left(\frac{1}{Q_k^s} \sum_{j=0}^{k-1} (q_j - q_{j+1})\zeta_j + \frac{q_k}{Q_k^s}\zeta_k \right) \right|^{p_k} < \infty.$$

From this, it is clear that $\zeta \in r_{\mathcal{F}}^q(\triangle_s^t)$ and the result follows. ■

Conclusion: In this paper, we have introduced the various generalized spaces using modulus function. The sequence space $r_{\mathcal{F}}^q(\triangle_s^p)$ of non-absolute type have been studied and various topological properties have been computed corresponding to it. Also, we have given the structure of some inclusion relation concerning to this new space. The results are more general than the pre-existing results.

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References

[1] Altay B. Başar F., On the paranormed Riesz sequence space of nonabsolute type, *Southeast Asian Bull. Math.,* 26(2002), 701-715.

[2] Altay B. Başar F., On the space of sequences of p-bounded variation and related matrix mappings, *Ukarnian Math. J.,* 1(1)(2003), 136-147, (DOI:10. 1023/A:1025080820961).

[3] Altay B. Başar F., Mursaleen M., On the Euler sequence spaces which include the spaces l_p and l_∞-II, *Nonlinear Anal.,* 176(2006), 1465-1462.

[4] Aydin C., Başar F., On the new sequence spaces which include the spaces c_o and c, *Hokkaido Math. J.,* 33(2002), 383-398.

[5] Aydin C., Başar F., Some new paranormed sequence spaces, *Inf. Sci.,* 160 (2004), 27-40.

[6] Başarir M., Öztürk M., On the Riesz difference sequence space, *Rendiconti del Cirocolo di Palermo,* 57(2008), 377-389.

[7] Choudhary B., Mishra S.K., On Köthe Toeplitz Duals of certain sequence spaces and matrix Transformations, *Indian J. Pure Appl. Math.,* 24(4)(1993), 291-301.

[8] Fathima D., Ganie A.H., On some new scenario of Δ- spaces, *J. Nonlinear Sci. Appl.,* 14(2021), 163-167, (http://dx.doi.org/10.22436/jnsa.014.03.05).

[9] FATHIMA D., ALBAIDANI M.M., GANIE A.H., AKHTER A., New structure of Fibonacci numbers using concept of Δ-operator, *J. Math. Comput. Sci.,* 26(2)(2022), 101-112, (http://dx.doi.org/10.22436/jmcs.026.02.01).

[10] GANIE A.H., *New spaces over modulus function,* Boletim Sociedade Paranaense de Matematica (in press), 1-6: http://www.spm.uem.br/bspm/pdf/next/313.pdf.

[11] GANIE A.H., AHMAD M., SHEIKH N.A., Generalized difference paranormed sequence space with respect to modulus function and almost convergence, *Journal of Global Research in Computer Science,* 6(11)(2015), 23-26.

[12] GANIE A.H., D. FATHIMA D., Almost convergence property of generalized Riesz spaces, *Journal of Applied Mathematics and Computation,* 4(4)(2020), 249-253, (DOI:http://dx.doi.org/10.26855/jamc.2020.12.016).

[13] GANIE A.H., ALBAIDAN M.M., Matrix Structure of Jacobsthal numbers, *J. Funct. Spaces.,* 2021(2021) Article ID 2888840, (https://doi.org/10.1155/2021/2888840).

[14] GANIE A.H., SHEIKH N.A., On some new sequence spaces of non-absolute type and matrix transformations, *J. Egyp. Math. Soc.,* 21(2) (2013), 34-40, (https://doi.org/10.1016/j.joems.2013.01.006).

[15] GANIE A.H., AHMAD M., SHEIKH N.A., JALAL T., GUPKARI S.A., Some new type of difference sequence space of non-absolute type, *Int. J. Modern Math.Sci.,* 14(1)(2016), 116-122.

[16] GANIE A.H., GUPKARIS.A., AKHTER A., Invariant means of sequences with statistical behaviour, *Int. J. Creative Resh. Thoughts (IJCRT),* 8(8)(2020), 2702-2705.

[17] JALAL T., GUPKARI S.A., GANIE A.H., Infinite matrices and sigma convergent sequences, *Southeast Asian Bull. Math.,* 36(2012), 825-830.

[18] T. JALAL T., GANIE A.H., Almost convergence and some matrix transformation, *International Jour. Math.(Shekhar New Series),* 1(1)(2009), 133-138.

[19] GROSS ERDMANN K.G., Matrix transformations between the sequence spaces of Maddox, *J. Math. Anal. Appl.,* 180(1993), 223-238, (https://doi.org/10.1006/jmaa.1993.1398).

[20] KIZMAZ H., On certain sequence, *Canad. Math. Bull.,* 24(2) (1981), 169-176, (https://doi.org/10.4153/CMB-1981-027-5).

[21] LASCARIDES C.G., MADDOX I.J., Matrix transformations between some classes of sequences, *Proc. Camb. Phil. Soc.,* 68 (1970), 99-104.

[22] MADDOX I.J., Paranormed sequence spaces generated by infinite matrices, *Proc. Camb. Phil. Soc.,* 64(1968), 335-340, (https://doi.org/10.1017/S0305004100042894).

[23] MADDOX I.J., *Elements of Functional Analysis* 2^{nd}*ed,* The University Press, Cambridge, (1988).

[24] MURSALEEN M., GANIE A.H., SHEIKH N.A., New type of difference sequence space and matrix transformation, *FILOMAT,* 28(7)(2014), 1381-1392.

[25] NAKANO N., Cancave modulars, *J. Math. Soc. Japan,* 5(1953), 29-49.

[26] MURSALEE M., BASÀR F., ALTAY B., On the Euler sequence spaces which include the spaces l_p and l_∞-II, *Nonlinear Anal.,* 65(2006), 707-717.

[27] MURSALEEN M, NOMAN A.K., On some new difference sequence spaces of non-absolute type, *Math. Comput. Mod.,* 52(2010), 603-617.

[28] Ng P.-N., Lee P.-Y., Cesáro sequences spaces of non-absolute type, *Comment. Math. Prace Mat.,* 20(2)(1978), 429-433.

[29] Petersen G.M., *Regular matrix transformations,* Mc Graw-Hill, London, (1966).

[30] Ruckle W.H., FK spaces in which the sequence of coordinate vectors is bounded, *Canad. J. Math.,* 25(1973), 973-978.

[31] Sengönül M., Basar F., Some new Cesáro sequences spaces of non-absolute type, which include the spaces c_o and c, *Soochow J. Math.,* 1(2005), 107-119.

[32] Sheikh N.A., Ganie A.H., A new paranormed sequence space and some matrix transformations, *Acta Math. Acad. Paedagog. Nyiregyhaziensis,* 28(2012), 47-58.

[33] Sheikh N.A., Ganie A.H., A new type of sequence space of non-absolute type and matrix transformation, *WSEAS Trans. Math.,* 8(12)(2013), 852-859.

[34] Sheikh N.A., Jalal T., Ganie A.H., New type of sequence spaces of non-absolute type and some matrix transformations, *Acta Math. Acad. Paedagog. Nyiregyhaziensis,* 29 (2013), 51-66.

[35] Toeplitz Ö., Über allegemeine Lineare mittelbildungen, *Prace Math. Fiz.,* 22(1991), 113-119.

[36] Wang C.-S., On Nörlund sequence spaces, *Tamkang J. Math.,* 9(1978), 269-274.

[37] Wilansky A., *Summability through Functional Analysis,* North Holland Mathematics Studies, Amsterdam - New York - Oxford, (1984).

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