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MINIMAL SEPARATION AXIOMS IN NEUTROSOPHIC TOPOLOGICAL SPACES

Abstract. The main purpose of this paper is to introduce the notion of minimal separation axioms in neutrosophic topological spaces. We have defined some separation axioms in neutrosophic topological spaces in minimal sructure. We have investigated some basic properties of the new class of minimal separation axioms in neutrosophic topological spaces.

Key words: neutrosophic set, neutrosophic topology, minimal structure, separation axioms.

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1. Introduction

Zadeh [24] introduced the notion of fuzzy set. But it was not sufficient to control uncertainty. Thereafter, Atanaosv [5] introduced the notion of intuitionistic fuzzy set with membership and non - membership values. Smarandache [20, 21, 22] introduced the notions of neutrosophic theory and introduced the neutrosophic components, (T, I, F) which represent, the membership, indeterminacy and non - membership values respectively, where $]-0, 1+[$ is a non standard unit interval. Considering the elements with membership, non - membership and indeterministic values, Smarandache [19] introduced the notion of neutrosophic set in order to overcome all sorts of difficulty to handle all types of problems under uncertainty. The notion of neutrosophic topological space was first introduced by Salama and Alblowi [16], followed by Salama and Alblowi [17]. The notion of minimal structure in topological space was introduced by Makai et al. [9]. It is found to have useful applications and the notion was investigated by Madok [10]. The notion of minimal structure in a fuzzy topological space was introduced by Alimohammady and Roohi [2] and further investigated by Tripathy and Debnath [23] and others. Pal et al [11, 12] introduced the notion of grill and minimal continuity in neutrosophic topological spaces. Alimohammady et al. [3] introduced the notion of fuzzy minimal separation axioms. Besides them, so many researchers [1, 4, 6, 7, 8, 13, 18] have contributed to the study of neutrosophic spaces, minimal spaces and separation axioms. Following their works we have introduced and studied minimal separation axioms in neutrosophic topological spaces.

2. Preliminaries

In this section, we procure some definitions and preliminary results, those motivated us for the introduction of the notion of minimal separation axioms and for investigation of their properties in neutrosophic topological spaces.

Definition 1 ([16]). *Let X be an universal set. A neutrosophic set A in X is a set contains triplet having truthness, falseness and indeterminacy membership values that can be characterized independently, denoted by T_A, F_A, I_A in $[0,1]$. The neutrosophic set is denoted as follows:*
$A = \{(x, T_A(x), F_A(x), I_A(x)) : x \in X$, *and* $T_A(x), F_A(x), I_A(x) \in [0,1]\}$.
There is no restriction on the sum of $T_A(x), F_A(x)$ and $I_A(x)$, so

$$0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3.$$

Throughout the article, we denote a neutrosophic set A by $A = \{(x, T_A(x), F_A(x), I_A(x)) : x \in X$ and $T_A(x), F_A(x), I_A(x) \in [0,1]\}$.

The null and full NSs on a non - empty set X are denoted by 0_N and 1_N, defined as follows:

Definition 2 ([16]). *The neutrosophic sets 0_N and 1_N in X are represented as follows:*
(i) $0_N = \{< x, 0, 0, 1 >: x \in X\}$.
(ii) $0_N = \{< x, 0, 1, 1 >: x \in X\}$.
(iii) $0_N = \{< x, 0, 1, 0 >: x \in X\}$.
(iv) $0_N = \{< x, 0, 0, 0 >: x \in X\}$.
(v) $1_N = \{< x, 1, 0, 0 >: x \in X\}$
(vi) $1_N = \{< x, 1, 0, 1 >: x \in X\}$.
(vii) $1_N = \{< x, 1, 1, 0 >: x \in X\}$.
(viii) $1_N = \{< x, 1, 1, 1 >: \in X\}$.
Clearly, $0_N \subseteq 1_N$. We have, for any neutrosophic set $A, 0_N \subseteq A \subseteq 1_N$.

Definition 3 ([16]). *Let $A = (x, T_A, F_A, I_A)$ be a NS over X. Then the complement of A is defined by $A^c = \{(x, F_A(x), 1 - I_A(x), T_A(x)) : x \in X\}$.*

Definition 4 ([16]). *A neutrosophic set $A = (x, T_A, F_A, I_A)$ is contained in the other neutrosophic set $B = (x, T_B, F_B, I_B)$ (i.e. $A \subseteq B$) if and only if $T_A(x) \leq T_B(x), F_A(x) \geq F_B(x), I_A(x) \geq I_B(x)$, for each $x \in X$*

Definition 5 ([16]). *If $A = (x, T_A, F_A, I_A)$ and $B = (x, T_B, F_B, I_B)$ are any two NSs over X, then $A \cup B$ and $A \cap B$ is defined by*
$A \cup B = \{(x, T_A(x) \vee T_B(x), F_A(x) \wedge F_B(x), I_A(x) \wedge I_B(x)) : x \in X\}$
$A \cap B = \{(x, T_A(x) \wedge T_B(x), F_A(x) \vee F_B(x), I_A(x) \vee I_B(x)) : x \in X\}$.

Definition 6 ([16]). *Let X be a non-empty set and T be the collection of neutrosophic subsets of X. Then T is said to be a neutrosophic topology (in short NT) on X if the following properties hold:*
(i) $0_N, 1_N \in T$
(ii) $U_1, U_2 \in T \Rightarrow U_1 \cap U_2 \in T$.
(iii) $\cup_{i \in \Delta} u_i \in T$, *for every* $\{u_i : i \in \Delta\} \subseteq \mathrm{T}$.
Then (X, T) is called a neutrosophic topological space (in short NTS) over X. The members of T are called neutrosophic open sets (in short NOS). A neutrosophic set D is called neutrosophic closed set (in short NCS) if and only if D^c is a neutrosophic open set.

Definition 7 ([16]). *Let (X, T) be a NTS and U be a NS in X. Then the neutrosophic interior (in short N_{int}) and neutrosophic closure (in short N_{cl}) of U are defined by*
$N_{int}(U) = \cup\{E : E \text{ is a } NOS \text{ in } X \text{ and } E \subseteq U\}$.
$N_{cl}(U) = \cap\{F : F \text{ is a } NCS \text{ in } X \text{ and } U \subseteq F\}$.

Remark 1 ([16]). Clearly $N_{\mathrm{int}}(U)$ is the largest neutrosophic open set over X which is contained in U and $N_{cl}(U)$ is the smallest neutrosophic closed set over X which contains U.

Proposition 1 ([16]). *For any NS B in (X, T), we have*
(i) $N_{int}(B^c) = (N_{cl}(B))^c$.
(ii) $N_{cl}(B^c) = (N_{int}(B))^c$.

Definition 8 ([16]). *Let X be a set and $P(X)$ denotes the power set of X. A family M of neutrosophic subsets of X where $M \subset P(X)$, is said to be a minimal structure on X if 0_N and 1_N belong to M. By (X, M), we denote the nutrosophic minimal space.*
We consider the elements of M as neutrosophic m-open subset of X.

Definition 9 ([11]). *The complement of neutrosophic m - open set A is called a neutrosophic m - closed set.*

Definition 10 ([15]). *Let $N(X)$ be the set of all neutrosophic sets over X. A NS $P = \{(x,\ T_P(x), I_P(x), F_P(x)) : x \in X\}$ is called a neutrosophic point (NP for short) if and only if for any element $y \in X, T_P(y) = \alpha, I_P(y) = \beta, F_P(y) = \gamma$ for $y = x$ and $T_P(y) = 0, I_P(y) = 1, F_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$.*

A neutrosophic point $P = \{(x, T_P(x), I_P(x), F_P(x)) : x \in X\}$ *will be denoted by* $P^x_{\alpha,\beta,\gamma}$ *or simply by* $x_{\alpha,\beta,\gamma}$. *For the* $NP, x_{\alpha,\beta,\gamma}, x$ *will be called its support.*

The complement of the NP $x_{\alpha,\beta,\gamma}$ *will be denoted by* $x^c_{\alpha,\beta,\gamma}$. *A NS* $P = \{(x, T_P(x), I_P(x), F_P(x)) : x \in X\}$ *is called a neutrosophic crisp point (NCP for short) if and only if for any element* $y \in X$, $T_P(y) = 1, I_P(y) = 0, F_P(y) = 0$ *for* $y = x$ *and* $T_P(y) = 0, I_P(y) = 1, F_P(y) = 1$ *for* $y \neq x$.

Definition 11 ([15]). *Let* (X, T) *be a neutrosophic topological space. A NS* $A \in N(X)$ *is called a neutrosophic neighbourhood or simply neighbourhood (nhbd for short) of a NP* $x_{\alpha,\beta,\gamma}$ *if and only if there exists a NS* $B \in T$ *such that* $x_{\alpha,\beta,\gamma} \in B \subseteq A$.

A neighbourhood A *of the NP* $x_{\alpha,\beta,\gamma}$ *is said to be a neutrosophic open neighbourhood of* $x_{\alpha,\beta,\gamma}$ *if* A *is a neutrosophic open set. The family consisting of all the neighbourhoods of the NP* $x_{\alpha,\beta,\gamma}$ *is called the system of neighbourhoods (or neighbourhood system) of* $x_{\alpha,\beta,\gamma}$. *This family is denoted by* $N(x_{\alpha,\beta,\gamma})$.

Definition 12 ([14]). *A fuzzy point* x_p *is said to be quasi - coincident with a fuzzy subset A, denoted by* $x_p q A$ *if and only if* $p > A^c(x)$ *or,* $p + A(x) > 1$.

A fuzzy subset A *is said to be quasi - coincident with a fuzzy subset* B*, denoted by* AqB*, if and only if there exists* $x \in X$ *such that* $A(x) > B^c(x)$ *or,* $A(x) + B(x) > 1$. *If this is true, we also say that* A *and* B *are quasi - coincident (with each other) at* x.

A fuzzy subset A *is said to be non - quasi - coincident with a fuzzy subset* B*, denoted by* $A\not{q}B$*, if and only if there exists* $x \in X$ *such that* $A(x) + B(x) \leq 1$.

3. Neutrosophic minimal separation axioms

In this section, our main aim is to propose the concept of minimal separation axioms in neutrosophic topological spaces. We would investigate some basic properties and characterization theorems in neutrosophic topological spaces.

Definition 13. *A neutrosophic set* N *in a neutrosophic minimal space* (X, M) *is said to be a neutrosophic minimal neighborhood of a neutrosophic point* x_α *if there is a neutrosophic* m *- open set* μ *in* X *with* $x_\alpha \in \mu$ *and* $\mu \leq N$.

Definition 14. *Suppose* (X, M) *is a neutrosophic minimal space. A neutrosophic set* N *in* X *is said to be a neutrosophic minimal* q *- neighborhood of a neutrsophic point* x_α *if there is a neutrosophic* m *- open set* μ *in* X *with* $x_\alpha q \mu$ *and* $\mu \leq A$.

Definition 15. *Suppose (X, M) is a neutrosophic minimal space. A neutrosophic point x_α in X is said to be neutrosophic minimal cluster point of a neutrosophic set A if every neutrosophic minimal q - neighborhood of x_α is q - coincident with A.*

Theorem 1. *Suppose (X, M) is a neutrosophic minimal space. A neutrosophic point x_α is a neutrsophic minimal cluster point of a neutrosophic set A if and only if $x_\alpha \in m$ - $Cl(A)$.*

Proof. Suppose $x_\alpha \notin m$ - Cl(A). Then, one can easily see that there exists neutrosophic m - closed set F in X with $A \leq F$ and $F(x) < \alpha$. Therefore, $x_\alpha q F^c$ and A is not q - neighborhood with F^c, i.e., x_α is not a neutrosophi minimal cluster point of A. Conversely, suppose x_α is not a neutrosophic minimal cluster point of A. There exists a neutrosophic minimal q - neighborhood N of x_α for which N is not q - coincident with A. Then there exists a neutrosophic m - open set μ in X with $x_\alpha q \mu$ and $\mu \leq N$. Therefore, μ is not q - coincident A which implies that $A \leq \mu^c$. Since μ^c is m - closed, so m - Cl$(A) \leq \mu^c$. That x$_\alpha \notin m$ - Cl(A) follows from the fact that $x_\alpha \notin \mu^c$. ■

Definition 16. *A neutrosophic minimal space (X, M) is said to be neutrosophic minimal T_0 - space if for every pair of distinct neutrosophic points x_α and y_β,*

(a) when $x \neq y$, either x_α has a neutrosophic minimal neighborhood which is not q - coincident with y_β or y_β has a neutrosophic minimal neighborhood which is not q - coincident with x_α,

(b) when $x = y$ and $\alpha < \beta$ (say), there is a neutrosohic minimal q - neighborhood of y_β which is not q - coincident with x_α.

Definition 17. *A neutrosophic minimal space (X, M) is said to be neutrosophic minimal T_1 - space if for every pair of distinct neutrosophic points x_α and x_β,*

(a) when $x \neq y$, there is a neutrosophic minimal neighborhood μ of x_α and a neutrosophic minimal neighborhood v of y_β with $\mu \not{q} y_\beta$ and $x_\alpha \not{q} v$,

(b) when $x = y$ and $\alpha < \beta$ (say), y_β has a neutrosophic minimal q - neighborhood which is not q - coincident with x_α.

Definition 18. *A neutrosophic minimal space (X, M) is said to be neutrosophic minimal T_2 - space if for every pair of distinct neutrosophic points x_α and y_β,*

(a) when $x \neq y, x_\alpha$ and y_β have neutrosophic minimal q - neighborhoods which are not q - coincident,

(b) when $x = y$ and $\alpha < \beta$ (say), x_α has a neutrosophic minimal neighborhood μ and y_β has a neutrosophic minimal q - neighborhood v in which $\mu \not q v$.

In short, neutrosophic m - $T_i(i = 0, 1, 2)$ spaces are used for neutrosophic minimal T_i - spaces.

Theorem 2. *Every neutrosophic m - T_2 - space is a neutrosophic m - T_1 - space and also every neutrosophic m - T_1 - space is a neutrosophic m - T_0 - space.*

Proof. The proof is straightforward from the definitions. ■

Theorem 3. *A neutrosophic minimal space (X, M) is neutrosophic m - T_1 - space if every neutrosophic point x_α is neutrosophic m - closed in X.*

Proof. Suppose x_α and y_β are distinct neutrosophic points in X. Then there are two cases:
(*i*) $x \neq y$
(*ii*) $x = y$ and $\alpha < \beta$ (say). Assume that $x \neq y$. By hypothesis x_α^c and y_β^c are neutrosophic m - open sets. It is easy to see that $x_\alpha \in y_\beta^c, y_\beta \in x_\alpha^c$, $\text{x}_\alpha \not q x_\alpha^c$ and $\text{y}_\beta \not q y_\beta^c$. In case that $x = y$ and $\alpha < \beta$, one can deduce that x_α^c is a neutrosophic m - open set with $y_\beta q x_\alpha^c$ and $\text{x}_\alpha \not q x_\alpha^c$ which implies that (X, M) is neutrosophic m - T_1 - space. ■

Theorem 4. *Let (X, M) be a neutrosophic minimal space. Then (X, M) is neutrosophic minimal T_1 - space if for each $x \in X$ and each $\alpha \in [0, 1]$ there exists a neutrosophic minimal open set μ such that $\mu(x) = 1 - \alpha$ and $\mu(y) = 1$ for $y \neq x$.*

Proof. Let x_α be an arbitrary neutrosophic point of X. We shall show that the neutrosophic point x_α is neutrosophic minimal closed. By hypothesis, there exists a neutrosophic minimal open set μ such that $\mu(x) = 1$ - α and $\mu(y) = 1$ for $y \neq x$. We have $\mu^c = x_\alpha$. Thus, the neutrosophic point x_α is neutrosophic minimal closed and hence by Theorem 3, the neutrosophic minimal space X is neutrosophic minimal T_1 - space. ■

Theorem 5. *Suppose $i = 0, 1$, 2. A neutrosophic minimal space (X, M) is neutrosophic m - T_i - space if and only if for any pair of distinct neutrosophic points x_α and y_β with distinct supports, there exists a neutrosophic m - continuous mapping f from X into a neutrosophic m - T_i - space (Y, N) such that $f(x) \neq f(y)$.*

Proof. We only prove the case that $i = 2$ and others are similar. Suppose (X, M) is neutrosophic m - T_2 - space. Let $f : (X, M) \to (Y, N)$ be a mapping. Clearly, (Y, N) and f have the required properties. Conversely, suppose x_α and y_β are distinct neutrosophic points in X. There are two cases:

(i) $x \neq y$

(ii) $x = y$ and $\alpha < \beta$ (say).

When x $\neq y$, by assumption there is neutrosophic m - continuous mapping f from (X, M) into a neutrosophic m - T_2 - space (Y, N) with $f(x) \neq f(y)$. Since (Y, N) is neutrosophic m - T_2 - space and $(f(x))_\alpha$ and $(f(y))_\beta$ are distinct neutrosophic points in Y, so there are neutrosophic minimal neighborhoods μ and v of $(f(x))_\alpha$ and $(f(y))_\beta$ respectively for which $\mu \not{q} v$. It follows from m - continuity of f that $f^{-1}(\mu)$ and $f^{-1}(v)$ are neutrosophic minimal neighborhood of x_α and y_β respectively. Since $\mu \not{q} v$, so $f^{-1}(\mu) \not{q} f^{-1}(v)$. In case that $x = y$ and $\alpha < \beta$ (say), $(f(x))_\alpha$ and $(f(y))_\beta$ are neutrosophic points in Y with $f(x) = f(y)$. Since (Y, N) is neutrosophic m - T_2 - space, so $(f(x))_\alpha$ has a neutrosophic minimal neighborhood μ and $(f(y))_\beta$ has a neutrosophic minimal q - neighborhood v for which $\mu \not{q} v$. Then $f^{-1}(\mu)$ is a neutrosoophic minimal q - neighborhood of x_α and $f^{-1}(v)$ is a neutrosophic minimal q - neighborhood of y_β with $f^{-1}(\mu) \not{q} f^{-1}(v)$. Therefore, $(X,\ M\ )$ is neutrosophic m - T_2 - space. The following result is an immediate consequence of Theorem 5. ■

Corollary 1. *Suppose (X, M) and (Y, N) are neutrosophic minimal spaces and $f : X \to Y$ is injective and neutrosophic m - continuous. (X, M) is neutrosophic m - T_i - space if (Y, N) is neutrosophic m - T_i - space.*

Theorem 6. *Let (X, M) be a neutrosophic minimal space. If (X, M) is neutrosophic minimal T_2 - space, then for any two distinct neutrosophic points x_α and y_β, the following properties hold:*

(i) If $x \neq y$, then there exist neutrosophic open neighborhoods μ and v of x_α and y_β, respectively, such that m - $cl(v) \leq \mu^c$ and m - $cl(\mu) \leq v^c$.

(ii) If $x = y$ and $\alpha < \beta$ (say), then there exists a neutrosophc open neighborhood μ of x_α such that $y_\beta \notin m$ - $cl(\mu)$

Proof. (i): Let $x \neq y$. Then there exist neutrosophic m - open neighborhoods μ and v of x_α and y_β, respectively, such that $\mu \not{q} v$. Since $\mu \not{q} v$, then $\mu(z) \leq 1 - v(z)$ and $v(z) \leq 1 - \mu(z)$ for all $z \in X$. Since μ^{c} and v^{c} are neutrosophic m - closed, then $m - \mathrm{cl}(v) \leq \mu^{\mathrm{c}}$ and $m - \mathrm{cl}(\mu) \leq v^{\mathrm{c}}$.

(ii): Let $x = y$. Then there exists a neutrosophic minimal q - neighborhood λ of y_β and a neutrosophic open neighborhood μ of x_α such that $\lambda \not{q} \mu$. Now, let v be a neutrosophic m - open set in X such that $y_\beta q v$ and $v \leq \lambda$. Since $\beta > 1 - v(y) = (m\text{ - cl}\,(v^c))(y), v \leq \lambda$ and $\mu \leq \lambda^c$, then $\beta > m$ - $\mathrm{cl}(\mu)(y)$ for all $y \in X$. Thus, $y_\beta \notin m$ - $\mathrm{cl}(\mu)$. ■

Theorem 7. *Let (X, M) be a neutrosophic minimal space. Suppose that (X, M) enjoys the property U. Then (X, M) is neutrosophic minimal T_2 - space if and only if for any two distinct neutrosophic points x_α and y_β, the following properties hold:*

(i) If $x \neq y$, then there exist neutrosophic m - open neighborhoods μ and v of x_α and y_β, respectively, such that m - $cl(v) \leq \mu^c$ and m - $cl(\mu) \leq v^c$.

(ii) If $x = y$ and $\alpha < \beta$ (say), then there exists a neutrosophic m - open neighborhood μ of x_α such that $y_\beta \notin m$ - $cl(\mu)$.

Proof. $(\Rightarrow)$: It follows from Theorem 6.

$(\Leftarrow)$: Let x$_\alpha$ and y$_\beta$ be distinct neutrosophic points in X and let $x \neq y$. Then there exist neutrosophic m - open neighborhoods μ and v of x_α and y_β, respectively, such that m - cl$(v) \leq \mu^c$. This implies that for all $z \in X, \mu(z) + v(z) \leq (m$ - cl$(v))(z) + \mu(z) \leq 1$. Hence, $\mu \not{q} v$. Now, let $x = y$ and $\alpha < \beta$. Then there exists a neutrosophic m - open neighborhood μ of x_α such that $y_\beta \notin m$ - cl(μ). Let $\lambda = (m$ - cl$(\mu))^c$. Since for all $z \in X, \lambda(z) + \mu(z) \leq 1$, then $\lambda \not{q} \mu$. On the other hand, λ is a neutrosophic open set and $\beta + \lambda(y) > \alpha + \lambda(y) \geq 1$. Hence, λ is a neutrosophic minimal q - neighborhood of y_β such that $\lambda \not{q} \mu$. ■

Theorem 8. *Let (X, M) be a neutrosophic minimal space. If (X, M) is neutrsophic minimal T_2 - space, then the following hold:*

(i) for every neutrosophic point x_α in $X, x_\alpha = \wedge\{m$ - $cl(v) : v$ is a neutrosophic minimal neighborhood of $x_\alpha\}$

(ii) for every $x, y \in X$ with $x \neq y$, there exists a neutrosophic minimal neighborhood μ of x such that $y \notin$ supp$(m$ - $cl(\mu))$.

Proof. (i): Let $y_\beta \notin x_\alpha$. We shall show the existence of a neutrosophic minimal neighborhood of x_α such that $y_\beta \notin m$ - cl(v). Let $x \neq y$. Then there exist neutrosophic minimal open sets μ and v containing y_β and x_α, respectively such that $\mu \not{q} v$. Then v is neutrosophic minimal neighborhood of x_α and μ is a neutrosophic minimal q - neighborhood of y_β such that $\mu \not{q} v$. Hence, by using Theorem 1, we get $y_\beta \notin m$ - cl(v).

Let $x = y$. Then $\alpha < \beta$ and there exists a neutrosophic minimal q - neighborhood μ of y_β and neutrosophic minimal neighborhood v of x_α such that $\mu \not{q} v$. Thus, $y_\beta \notin m$ - cl(v).

(ii): For every $x, y \in X$ with $x \neq y$, since (X, M) is neutrosophic minimal T_2 - space, then there exist neutrosophic minimal open sets μ and v such that $x \in \mu, y \in v$ and $\mu \not{q} v$. Then $v^c(y) = 0$ and $\mu \leq v^c$. Since v^c is neutrosophic minimal closed, m - cl$(\mu) \leq v^c$. Thus, m - cl$(\mu)(y) = 0$ and hence, $y \notin$ supp$(m$ - cl$(\mu))$. ■

Theorem 9. *Let (X, M) be a neutrosophic minimal space with property U. Then (X, M) is neutrosophic minimal T_2 - space if and only if:*

(i) for every neutrosophic point x_α in $X, x_\alpha = \wedge\{m$ - $cl(v)$: v is a neutrosophic minimal neighborhood of $x_\alpha\}$,

(ii) for every $x, y \in X$ with $x \neq y$, there exists a neutrosophic minimal neighborhood μ of x such that $y \notin \text{supp}(m - cl(\mu))$.

Proof. $(\Rightarrow)$: It follows from Theorem 8.

$(\Leftarrow)$: Let x_α and y_β be two distinct neutrosophic points in X. Let $x \neq y$. Suppose that $0 < \alpha < 1$. There exists a real number δ such that $0 < \alpha + \delta < 1$. By hypothesis, there exists a neutrosophic minimal neighborhood μ of y_β such that $x_\delta \notin m$ - cl(μ). Then x_δ has a neutrosophic minimal q - neighborhood v such that $\mu \not q v$. On the other hand, $\delta + v(x) > 1$ and $v(x) > 1 - \delta > \alpha$ and hence v is a neutrosophic minimal neighborhood of x_α such that $\mu \not q v$, where μ is a neutrosophic minimal neighborhood of y_β. If $\alpha = \beta = 1$, by hypothesis there exists a neutrosophic minimal neighborhood μ of x such that m - cl$(\mu)(y) = 0$. Thus, $v = (m$ - cl$(\mu))^c$ is a neutrosophic minimal neighborhood of y such that $\mu \not q v$. Let $x = y$ and $\alpha < \beta$. Then there exists a neutrosophic minimal neighborhood of x_α such that $y_\beta \notin m -$ cl(μ). Thus, there exists a neutrosophic minimal q - neighborhood v of y_β such that $\mu \not q v$. Hence, (X, M) is neutrosophic minimal T_2 - space. ■

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