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SOME GENERALIZATIONS OF NEARLY I-CONTINUOUS MULTIFUNCTIONS IN BITOPOLOGICAL SPACES

Abstract. Let (X, τ_1, τ_2, I) be an ideal bitopological space. Recently, many generalizations of open sets in (X, τ_1, τ_2, I) are introduced and investigated. By using these sets, we introduce a unified form of several generalizations of nearly continuous multifunctions on ideal bitopological spaces.

Key words: m-structure, m-space, ideal topological space, bitopological space, nearly continuous, m_{ij}-I-BTP-continunous, multifunction.

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1. Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets and b-open sets play an important part in the researches of generalizations of continuity for functions and multifunctions in topological spaces. By using these sets, various types of continuous multifunctions are introduced and studied. The notions of minimal structures, m-spaces, m-continuity and M-continuity are introduced and studied by the present authors [25], [27]. By using these notions, the present authors obtained the unified theory of continuity for functions and multifunctions in [19], [20], [21], and [28].

The notion of N-closed sets in topological spaces is investigated in [17] and [18]. The study of upper/lower nearly continuous multifunctions is given in [7] and [8]. A generalization of nearly continuous multifunctions is obtained in [20]. In [11], [16], C-continuous functions are investigated. Some characterizations of C-quasi continuous multifunctions are published in [30]. Some forms of C-continuous multifunctions are published in [20]. Some forms of S-continuous multifunctions are studied in [10], [23] and [24]. A unified theory of S-continuity for multifunctions is obtained in [26].

The notion of ideal topological spaces was introduced in [14] and [34]. As generalizations of open sets, the notions of I-open sets, semi-I-open sets, pre-I-open sets, α-I-open sets, β-I-open sets and b-I-open sets are introduced and used to obtain new decomposition of continuity. The notion of I-continuous (resp. semi-I-continuous) multifunctions is introduced in [1](resp. [2]). Quite recently, the notion of nearly I-continuous multifunctions is introduced in [3].

The purpose of this paper is to extend the notion of m-continuous multifunctions to nearly m-continuous multifunctions in bitopological spaces. Moreover, we obtain a unified form of nearly continuous, C-continuous and S-continuous multifunctions on ideal bitopological spaces.

Throughout the present paper, (X, τ) and (Y, σ) always denote topological spaces and $F : (X, \tau) \to (Y, \sigma)$ presents a mutifunction. For a multifunction $F : (X, \tau) \to (Y, \sigma)$, we shall denote the upper and lower inverse of a subset B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is,

$$F^+(B) = \{x \in X : F(x) \subset B\} \text{ and } F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

2. Preliminaries

Definition 1 ([25]). *A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a* minimal structure *(briefly m-*structure*) on X if $\emptyset \in m_X$ and $X \in m_X$.*

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and call it an m-space. Each member of m_X is said to be *m_X-open* (briefly *m-open*) and the complement of an m_X-open set is said to be *m_X-closed* (briefly *m-closed*).

Definition 2 ([15]). *Let (X, m_X) be an m-space. For a subset A of X, the m_X-*closure *of A and the m_X-*interior *of A are defined as follows:*
(1) mCl(A) $= \bigcap\{F : A \subset F, X \setminus F \in m_X\}$,
(2) mInt(A) $= \bigcup\{U : U \subset A, U \in m_X\}$.

Definition 3 ([15]). *An m-structure m_X on a nonempty set X is said to have* property $\mathcal{B}$ *if the union of any family of subsets belonging to m_X belongs to m_X.*

Lemma 1 ([15]). *Let (X, m_X) be an m-space. For subsets A and B of X, the following properties hold:*
(1) $\mathrm{mCl}(X \setminus A) = X \setminus \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X \setminus A) = X \setminus \mathrm{mCl}(A)$,
(2) If $(X \setminus A) \in m_X$, then $\mathrm{mCl}(A) = A$ *and if $A \in m_X$, then* $\mathrm{mInt}(A) = A$,
(3) $\mathrm{mCl}(\emptyset) = \emptyset$ *and* $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,
(4) If $A \subset B$, then $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,

(5) $\mathrm{mInt}(A) \subset A \subset \mathrm{mCl}(A)$,
(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Lemma 2 ([29]). *Let (X, m_X) be an m-space and m_X have property $\mathcal{B}$. For a subset A of X, the following properties hold:*
(1) $A \in m_X$ if and only if $\mathrm{mInt}(A) = A$,
(2) A is m_X-closed if and only if $\mathrm{mCl}(A) = A$,
(3) $\mathrm{mInt}(A) \in m_X$ *and* $\mathrm{mCl}(A)$ *is m_X-closed.*

Let (X, τ_1, τ_2) be a bitopological space. A subset A of X is said to be $\tau_1\tau_2$-*open* [4] if $A = \tau_1\text{-Int}(\tau_2\text{-Int}(A))$. In this paper, we call a subset A (i,j)-*open* if $A = i\mathrm{Int}(j\mathrm{Int}(A))$, where $i \neq j, i, j = 1, 2$.

Definition 4. *A subset A of a bitopological space (Y, σ_1, σ_2) is said to be*
(1) (i, j)-N-closed *[31] if for every cover $\{U_\alpha : \alpha \in \Delta\}$ of A by σ_i-open sets, there exists a finite subset Δ_0 of Δ such that $A \subset \cup\{i\mathrm{Int}(j\mathrm{Cl}(U_\alpha)) : \alpha \in \Delta_0\}$,*
(2) (i, j)-Lindelöf *if every (i,j)-open cover of A has a countably subcover,*
(3) (i, j)-compact *if every (i,j)-open cover of A has a finite subcover,*
(4) (i, j)-connected *if A cannot be written as the union of two nonempty disjoint (i,j)-open sets.*

Remark 1. In the following, by BTP we denote the properties (i,j)-N-closed, (i,j)-Lindel$\ddot{o}f$, (i,j)-compact, and (i,j)-connected.

3. $mBTP$-continuous multifunctions

Definition 5. *Let (X, m_X) be an m-space and (Y, σ_1, σ_2) a bitopological space. A multifunction $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$ is said to be*
1) (i, j)-upper mBTP-continuous *at a point $x \in X$ if for each σ_i-open set V of Y containing $F(x)$ and having BTP complement, there exists $U \in m_X$ containing x such that $F(U) \subset V$,*
2) (i, j)-lower mBTP-continuous *at a point $x \in X$ if for each σ_i-open set V of Y meeting $F(x)$ and having BTP complement, there exists $U \in m_X$ containing x such that $F(u) \cap V \neq \emptyset$ for each $u \in U$,*
3) (i, j)-upper/lower mBTP-continuous *if F has this property at each point $x \in X$.*

Theorem 1. *For a multifunction $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*
(1) F is (i, j)-upper mBTP-continuous;
(2) $F^+(V) = \mathrm{mInt}(F^+(V))$ for each σ_i-open set V of Y having BTP complement;
(3) $F^-(K) = \mathrm{mCl}(F^-(K))$ for every BTP and σ_i-closed set K of Y;

(4) $\mathrm{mCl}(F^-(B)) \subset F^-(i\mathrm{Cl}(B))$ *for every subset B of Y having the BTP σ_i-closure;*

(5) $F^+(i\mathrm{Int}(B)) \subset \mathrm{mInt}(F^+(B))$ *for every subset B of Y such that* $Y \setminus i\mathrm{Int}(B)$ *is BTP.*

Proof. (1) ⇒ (2): Let V be any σ_i-open set of Y having BTP complement and $x \in F^+(V)$. Then $F(x) \subset V$ and there exists $U \in m_X$ containing x such that $F(U) \subset V$. Therefore, $x \in U \subset F^+(V)$ and hence $x \in \mathrm{mInt}(F^+(V)$. This shows that $F^+(V) \subset \mathrm{mInt}(F^+(V))$. Therefore, by Lemma 1 we obtain $F^+(V) = \mathrm{mInt}(F^+(V))$.

(2) ⇒ (3): Let K be any BTP and σ_i-closed set of Y. Then, by Lemma 1 we have $X \setminus F^-(K) = F^+(Y \setminus K) = \mathrm{mInt}(F^+(Y \setminus K)) = \mathrm{mInt}(X \setminus F^-(K)) = X \setminus \mathrm{mCl}(F^-(K))$. Therefore, we obtain $F^-(K) = \mathrm{mCl}(F^-(K))$.

(3) ⇒ (4): Let B be any subset of Y having the BTP σ_i-closure. By Lemma 1, we have $F^-(B) \subset F^-(i\mathrm{Cl}(B)) = \mathrm{mCl}(F^-(i\mathrm{Cl}(B)))$. Hence $\mathrm{mCl}(F^-(B)) \subset \mathrm{mCl}(F^-(i\mathrm{Cl}(B))) = F^-(i\mathrm{Cl}(B))$.

(4) ⇒ (5): Let B be a subset of Y such that $Y \setminus i\mathrm{Int}(B)$ is BTP. Then by Lemma 1 we have

$$X \setminus \mathrm{mInt}(F^+(B)) = \mathrm{mCl}(X \setminus F^+(B)) = \mathrm{mCl}(F^-(Y \setminus B)) \subset$$
$$\subset F^-(i\mathrm{Cl}(Y \setminus B)) \subset F^-(Y \setminus i\mathrm{Int}(B)) = X \setminus F^+(i\mathrm{Int}(B)).$$

Therefore, we obtain $F^+(i\mathrm{Int}(B)) \subset \mathrm{mInt}(F^+(B))$.

(5) ⇒ (1): Let $x \in X$ and V be any σ_i-open set of Y containing $F(x)$ and having BTP complement. Then $x \in F^+(V) = F^+(i\mathrm{Int}(V)) \subset \mathrm{mInt}(F^+(V))$. There exists $U \in m_X$ containing x such that $U \subset F^+(V)$; hence $F(U) \subset V$. This shows that F is (i, j)-upper $mBTP$-continuous. ■

Theorem 2. *For a multifunction* $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F is (i, j)-lower mBTP-continuous;

(2) $F^-(V) = \mathrm{mInt}(F^-V))$ *for each σ_i-open set V of Y having BTP complement;*

(3) $F^+(K) = \mathrm{mCl}(F^+(K))$ *is for every BTP and σ_i-closed set K of Y;*

(4) $\mathrm{mCl}(F^+(B)) \subset F^+(i\mathrm{Cl}(B))$ *for every subset B of Y having BTP σ_i-closure;*

(5) $F^-(i\mathrm{Int}(B)) \subset \mathrm{mInt}(F^-(B))$ *for every subset B of Y such that* $Y \setminus i\mathrm{Int}(B)$ *is BTP.*

Proof. The proof is similar to that of Theorem 1. ■

Corollary 1. *Let* (X, m_X) *be an m-space and* m_X *have property* $\mathcal{B}$*. For a multifunction* $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F is (i, j)-upper mBTP-continuous;

(2) $F^+(V)$ *is* m_X*-open for each* σ_i*-open set* V *of* Y *having BTP complement;*

(3) $F^-(K)$ *is* m_X*-closed for every BTP and* σ_i*-closed set* K *of* Y.

Proof. This is an immediate consequence of Theorem 1 and Lemma 2. ■

Corollary 2. *Let* (X, m_X) *be an m-space and* m_X *have property* $\mathcal{B}$*. For a multifunction* $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F is (i, j)-lower mBTP-continuous;

(2) $F^-(V)$ *is* m_X*-open for each* σ_i*-open set* V *of* Y *having BTP complement;*

(3) $F^+(K)$ *is* m_X*-closed for every BPT and* σ_i*-closed set* K *of* Y.

Proof. This is an immediate consequence of Theorem 2 and Lemma 2. ■

Theorem 3. *For a multifunction* $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F is (i, j)-upper mBTP-continuous at $x \in X$*;*

(2) $x \in \mathrm{mInt}(F^+(V))$ *for every* σ_i*-open set* V *of* Y *containing* $F(x)$ *and having BTP complement;*

(3) $x \in F^-(i\mathrm{Cl}(B))$ *for every subset* B *of* Y *having BTP* σ_i*-closure such that* $x \in \mathrm{mCl}(F^-(B))$*;*

(4) $x \in \mathrm{mInt}(F^+(B))$ *for every subset* B *of* Y *such that* $x \in F^+(i\mathrm{Int}(B))$ *and* $Y - i\mathrm{Int}(B)$ *is BTP.*

Proof. (1) ⇒ (2): Let V be any σ_i-open set of Y containing $F(x)$ and having BTP complement. There exists an m_X-open set U containing x such that $F(U) \subset V$. Thus $x \in U \subset F^+(V)$. Since U is m_X-open, we have $x \in \mathrm{mInt}(F^+(V))$.

(2) ⇒ (3): Suppose that B is any subset of Y having BTP σ_i-closure such that $x \in \mathrm{mCl}(F^-(B))$. Then $i\mathrm{Cl}(B)$ is σ_i-closed and $Y - i\mathrm{Cl}(B)$ is a σ_i-open set having BTP complement. Let $x \notin F^-(i\mathrm{Cl}(B))$, then $x \in X - F^-(i\mathrm{Cl}(B)) = F^+(Y - i\mathrm{Cl}(B))$. This implies that $F(x) \subset Y - i\mathrm{Cl}(B)$. Since $Y - i\mathrm{Cl}(B)$ is a σ_i-open set having BTP complement, by (2) we have $x \in \mathrm{mInt}(F^+(Y - i\mathrm{Cl}(B))) = \mathrm{mInt}(X - F^-(i\mathrm{Cl}(B))) = X - \mathrm{mCl}(F^-(i\mathrm{Cl}(B))) \subset X - \mathrm{mCl}(F^-(B))$. Hence $x \notin \mathrm{mCl}(F^-(B))$.

(3) ⇒ (4): Let B be any subset of Y such that $x \notin \mathrm{mInt}(F^+(B))$ and $Y - i\mathrm{Int}(B)$ is BTP. Then we have $x \in X - \mathrm{mInt}(F^+(B)) = \mathrm{mCl}(X - F^+(B)) = \mathrm{mCl}(F^-(Y - B))$. By (3) we have $x \in F^-(i\mathrm{Cl}(Y - B)) = F^-(Y - i\mathrm{Int}(B)) = X - F^+(i\mathrm{Int}(B))$. Hence $x \notin F^+(i\mathrm{Int}(B))$.

(4) ⇒ (1): Let V be any σ_i-open set of Y containing $F(x)$ and having BTP complement. We have $x \in F^+(V) = F^+(i\mathrm{Int}(V))$. Then, by (4) $x \in$

$\mathrm{mInt}(F^+(V))$. Therefore, there exists $U \in m_X$ such that $x \in U \subset F^+(V)$. Thus $F(U) \subset V$. This shows that F is (i,j)-upper $mBTP$-continuous at $x \in X$. ■

Theorem 4. *For a multifunction $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is (i, j)-lower mBTP-continuous at $x \in X$;

(2) $x \in \mathrm{mInt}(F^-(V))$ for every σ_i-open set V of Y having BTP complement such that $F(x) \cap V \neq \emptyset$;

(3) $x \in F^+(i\mathrm{Cl}(B))$ for every subset B of Y having BTP σ_i-closure such that $x \in \mathrm{mCl}(F^+(B))$;

(4) $x \in \mathrm{mInt}(F^-(B))$ for every subset B of Y such that $x \in F^-(i\mathrm{Int}(B))$ and $Y - i\mathrm{Int}(B)$ is BTP.

Proof. The proof is similar to that of Theorem 3. ■

For a multifunction $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$, we define $D^+_{mBTP}(F)$ and $D^-_{mBTP}(F)$ as follows:

$D^+_{mBTP}(F) = \{x \in X : F$ is not (i,j)-upper $mBTP$-continuous at $x\}$,

$D^-_{mBTP}(F) = \{x \in X : F$ is not (i,j)-lower $mBTP$-continuous at $x\}$.

Theorem 5. *For a multifunction $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$, the following properties hold:*

$$\begin{aligned} D^+_{mBTP}(F) &= \textstyle\bigcup_{G \in \sigma_i BTP}\{F^+(G) - \mathrm{mInt}(F^+(G))\} \\ &= \textstyle\bigcup_{B \in iBTP}\{F^+(i\mathrm{Int}(B)) - \mathrm{mInt}(F^+(B))\} \\ &= \textstyle\bigcup_{B \in BTP}\{\mathrm{mCl}(F^-(B)) - F^-(i\mathrm{Cl}(B))\} \\ &= \textstyle\bigcup_{H \in \mathcal{F}}\ \{\mathrm{mCl}(F^-(H)) - F^-(H)\}, \end{aligned}$$

where $\sigma_i BTP$ is the family of all σ_i-open sets of Y having BTP complement,
$iBTP$ is the family of all subset B of Y such that $Y - i\mathrm{Int}(B)$ is BTP,
BTP is the family of all subsets of Y having BTP σ_i-closure,
$\mathcal{F}$ is the family of all subset H of Y which is BTP and σ_i-closed.

Proof. We shall show only the first equality and the last since the proofs of other are similar to the first.

Let $x \in D^+_{mBTP}(F)$. By Theorem 3, there exists a σ_i-open set V of Y having BTP complement such that $x \in F^+(V)$ and $x \notin \mathrm{mInt}(F^+(V))$. Therefore, we have $x \in F^+(V) - \mathrm{mInt}(F^+(V)) \subset \bigcup_{G \in \sigma_i BTP}\{F^+(G) - \mathrm{mInt}(F^+(G))\}$.

Conversely, let $x \in \bigcup_{G \in \sigma_i BTP}\{F^+(G) - \mathrm{mInt}(F^+(G))\}$. There exists a σ_i-open set V of Y having BTP complement such that $x \in F^+(V) - \mathrm{mInt}(F^+(V))$. By Theorem 3, we obtain $x \in D^+_{mBTP}(F)$.

We prove the last equality.

$$\bigcup_{H\in\mathcal{F}}\{\mathrm{mCl}(F^-(H)) - F^-(H)\} \\ \subset \bigcup_{B\in BTP}\{\mathrm{mCl}(F^-(i\mathrm{Cl}(B))) - F^-(i\mathrm{Cl}(B))\} = D^+_{mBTP}(F).$$

Conversely, we have

$$D^+_{mBTP}(F) = \bigcup_{B\in BTP}\{\mathrm{mCl}(F^-(i\mathrm{Cl}(B))) - F^-(i\mathrm{Cl}(B))\} \\ \subset \bigcup_{H\in\mathcal{F}}\{\mathrm{mCl}(F^-(H)) - F^-(H)\}.$$

■

Theorem 6. *For a multifunction $F : (X, m_X) \to (Y, \sigma_1, \sigma_2)$, the following properties hold:*

$$\begin{aligned} D^-_{mBTP}(F) &= \bigcup_{G\in\sigma_i BTP}\{F^-(G) - \mathrm{mInt}(F^-(G))\} \\ &= \bigcup_{B\in iBTP}\{F^-(i\mathrm{Int}(B)) - \mathrm{mInt}(F^-(B))\} \\ &= \bigcup_{B\in BTP}\{\mathrm{mCl}(F^+(B)) - F^+(i\mathrm{Cl}(B))\} \\ &= \bigcup_{H\in\mathcal{F}}\{\mathrm{mCl}(F^+(H)) - F^+(H)\}. \end{aligned}$$

Proof. The proof is similar to that of Theorem 5. ■

4. Ideal topological spaces

Let (X, τ) be a topological space. The notion of ideals has been introduced in [14] and [34] and further investigated in [12].

A nonempty collection I of subsets of a set X is called an *ideal* on X [14], [34] if it satisfies the following two conditions:
(1) $A \in I$ and $B \subset A$ implies $B \in I$,
(2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

A topological space (X, τ) with an ideal I on X is called an *ideal topological space* and is denoted by (X, τ, I). Let (X, τ, I) be an ideal topological space. For any subset A of X, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$, is called the *local function* of A with respect to τ and I [12]. Hereafter $A^\star(I, \tau)$ is simply denoted by $A^\star$. It is well known that $\mathrm{Cl}^\star(A) = A \cup A^\star$ defines a Kuratowski closure operator on X and the topology generated by $\mathrm{Cl}^\star$ is denoted by $\tau^\star$.

Lemma 3 ([12]). *Let (X, τ, I) be an ideal topological space and A, B be subsets of X. Then the following properties hold:*
(1) $A \subset B$ implies $\mathrm{Cl}^\star(A) \subset \mathrm{Cl}^\star(B)$,
(2) $\mathrm{Cl}^\star(X) = X$ and $\mathrm{Cl}^\star(\emptyset) = \emptyset$,
(3) $\mathrm{Cl}^\star(A) \cup \mathrm{Cl}^\star(B) \subset \mathrm{Cl}^\star(A \cup B)$.

A) $(i,j)mIO(X)$

Let (X,τ_1,τ_2,I) be an ideal bitopological space. For any subset A of X, $A_j^\star(I,\tau_j) = \{x \in X : U \cap A \notin I$ for every $U \in \tau_j(x)\}$, where $\tau_j(x) = \{U \in \tau_j : x \in U\}$. We put $j\mathrm{Cl}^\star(A) = A \cup A_j^\star$ for every subset A of X.

Definition 6. *Let (X,τ_1,τ_2,I) be an ideal bitopological space. A subset A of X is said to be*

(1) (i, j)-α-I-open [9] if $A \subset i\mathrm{Int}(j\mathrm{Cl}^\star(i\mathrm{Int}(A)))$, for $i \neq j$, $i,j = 1,2$,

(2) (i, j)-semi-I-open [6] if $A \subset j\mathrm{Cl}^\star(i\mathrm{Int}(A))$, for $i \neq j$, $i,j = 1,2$,

(3) (i, j)-pre-I-open [5] if $A \subset i\mathrm{Int}(j\mathrm{Cl}^\star(A))$, for $i \neq j$, $i,j = 1,2$,

(4) (i, j)-b-I-open [32] if $A \subset i\mathrm{Int}(j\mathrm{Cl}^\star(A)) \cup j\mathrm{Cl}^\star(i\mathrm{Int}(A))$, for $i \neq j$, $i,j = 1,2$,

(5) (i, j)-β-I-open [6] if $A \subset j\mathrm{Cl}(i\mathrm{Int}(j\mathrm{Cl}^\star(A)))$, for $i \neq j$, $i,j = 1,2$,

(6) (i j)-weakly semi-I-open if $A \subset j\mathrm{Cl}^\star(i\mathrm{Int}(j\mathrm{Cl}(A)))$, for $i \neq j$, $i,j = 1,2$,

(7) (i, j)-weakly b-I-open [33] if $A \subset j\mathrm{Cl}(i\mathrm{Int}(j\mathrm{Cl}^\star(A))) \cup j\mathrm{Cl}^\star(i\mathrm{Int}(j\mathrm{Cl}(A)))$, for $i \neq j$, $i,j = 1,2$,

(8) (i, j)-strongly β-I-open if $A \subset j\mathrm{Cl}^\star(i\mathrm{Int}(j\mathrm{Cl}^\star(A)))$, for $i \neq j$, $i,j = 1,2$.

The family of all (i,j)-α-I-open (resp. (i,j)-semi-I-open, (i,j)-pre-I-open, (i,j)-b-I-open, (i,j)-β-I-open, (i,j)-weakly semi-I-open, (i,j)-weakly b-I-open, (i,j)-strongly β-I-open) sets in an ideal bitopological space (X,τ_1,τ_2,I) is denoted by $(i,j)\alpha IO(X)$ (resp. $(i,j)SIO(X)$, $(i,j)PIO(X)$, $(i,j)BIO(X)$, $(i,j)\beta IO(X)$, $(i,j)WSIO(X)$, $(i,j)WBIO(X)$, $(i,j)S\beta IO(X)$).

Remark 2. By $(i,j)mIO(X)$, we denote each one of the families $(i,j)\alpha IO(X)$, $(i,j)SIO(X)$, $(i,j)PIO(X)$, $(i,j)BIO(X)$, $(i,j)\beta IO(X)$, $(i,j)WSIO(X)$, $(i,j)WBIO(X)$, $(i,j)S\beta IO(X)$.

Lemma 4. *Let (X,τ_1,τ_2,I) be an ideal bitopological space. Then $(i,j)mIO(X)$ is a minimal structure and has property $\mathcal{B}$.*

Proof. By Lemmas 1(3) and 3(2), $(i,j)mIO(X)$ is a minimal structure on X. It follows from Lemmas 1(4) and 3(1) that $(i,j)mIO(X)$ has property $\mathcal{B}$. ■

Definition 7. *Let (X,τ_1,τ_2,I) be an ideal bitopological space. For a subset A of X, we define $(i,j)m\mathrm{Cl}_I(A)$ and $(i,j)m\mathrm{Int}_I(A)$ as follows:*

(1) $(i,j)m\mathrm{Cl}_I(A) = \cap\{F : A \subset F, X \setminus F \in (i,j)mIO(X)\}$,

(2) $(i,j)m\mathrm{Int}_I(A) = \cup\{U : U \subset A, U \in (i,j)mIO(X)\}$.

If $(i,j)mIO(X) = (i,j)\alpha IO(X)$ (resp. $(i,j)SIO(X)$, $(i,j)PIO(X)$, $(i,j)BIO(X)$, $(i,j)\beta IO(X)$, $(i,j)WSIO(X)$, $(i,j)WBIO(X)$, $(i,j)S\beta IO(X)$), then we have

(1) $(i,j)m\mathrm{Cl}_I(A) = (i,j)\alpha\mathrm{Cl}_I(A)$ (resp. $(i,j)s\mathrm{Cl}_I(A)$, $(i,j)p\mathrm{Cl}_I(A)$, $(i,j)b$ $\mathrm{Cl}_I(A)$, $(i,j)\beta\mathrm{Cl}_I(A)$, $(i,j)ws\mathrm{Cl}_I(A)$, $(i,j)wb\mathrm{Cl}_I(A)$, $(i,j)s\beta\mathrm{Cl}_I(A)$).

(2) $(i,j)m\mathrm{Int}_I(A) = (i,j)\alpha\mathrm{Int}_I(A)$ (resp. $(i,j)s\mathrm{Int}_I(A)$, $(i,j)p\mathrm{Int}_I(A)$, $(i,j)b\mathrm{Int}_I(A)$, $(i,j)\beta\mathrm{Int}_I(A)$, $(i,j)ws\mathrm{Int}_I(A)$, $(i,j)wb\mathrm{Int}_I(A)$, $(i,j)s\beta\mathrm{Int}_I(A)$).

B) $(1,2)mIO(X)$

Let (X,τ_1,τ_2) be a bitopological space. A subset A of X is said to be $\tau_1\tau_2$-*open* [22], [13] if $A \in \tau_1 \cup \tau_2$. The complement of a $\tau_1\tau_2$-*open* set is said to be $\tau_1\tau_2$-*closed*. The collection of $\tau_1\tau_2$-*open* sets is denoted by $\tau_1\tau_2O(X)$ or *(1, 2)O(X)*.

Definition 8. *Let (X,τ_1,τ_2,I) be an ideal bitopological space. For any subset A of X, $A^\star(I,\tau_1,\tau_2) = \{x \in X : U \cap A \notin I$ for every $U \in \tau_1\tau_2O(x)\}$, where $\tau_1\tau_2O(x) = \{U \in \tau_1\tau_2O(X) : x \in U\}$, is called the* local function *of A with respect to τ_1, τ_2 and I [22]. Hereafter $A^\star(I,\tau_1,\tau_2)$ is simply denoted by $A^\star_{\tau_1\tau_2}$.*

Definition 9 ([22])**.** *Let (X,τ_1,τ_2,I) be an ideal bitopological space. For any subset A of X, we put $\mathrm{Cl}^\star_{\tau_1\tau_2}(A) = A \cup A^\star_{\tau_1\tau_2}$. Then $\mathrm{Cl}^\star_{\tau_1\tau_2}(A)$ defines a Kuratowski closure operator on X and the topology generated by $\mathrm{Cl}^\star_{\tau_1\tau_2}(A)$ is denoted by $\tau^\star_{\tau_1\tau_2}$.*

Hereafter, we denote $\mathrm{Cl}^\star_{\tau_1\tau_2}(A)$ by $(1,2)\mathrm{Cl}_I(A)$ and $X - \mathrm{Cl}^\star_{\tau_1\tau_2}(X-A)$ by $(1,2)\mathrm{Int}_I(A)$.

Lemma 5. *Let (X,τ_1,τ_2,I) be an ideal bitopological space. For subsets A and B of X, the following properties hold:*

(1) $A \subset (1,2)\mathrm{Cl}_I(A)$,

(2) $(1,2)\mathrm{Cl}_I(\emptyset) = \emptyset$ *and* $(1,2)\mathrm{Cl}_I(X) = X$,

(3) If $A \subset B$, *then* $(1,2)\mathrm{Cl}_I(A) \subset (1,2)\mathrm{Cl}_I(B)$,

(4) $(1,2)\mathrm{Cl}_I(A) \cup (1,2)\mathrm{Cl}_I(B) \subset (1,2)\mathrm{Cl}_I(A \cup B)$.

Definition 10 ([22])**.** *Let (X,τ_1,τ_2,I) be an ideal bitopological space. A subset A of X is said to be*

(1) (1, 2)-α-I-open if $A \subset \tau_1\mathrm{Int}((1,2)\mathrm{Cl}_I(\tau_1\mathrm{Int}(A)))$,

(2) (1, 2)-semi-I-open if $A \subset (1,2)\mathrm{Cl}_I(\tau_1\mathrm{Int}(A))$,

(3) (1, 2)-pre-I-open if $A \subset \tau_1\mathrm{Int}((1,2)\mathrm{Cl}_I(A))$,

(4) (1, 2)-b-I-open if $A \subset (1,2)\mathrm{Cl}_I(\tau_1\mathrm{Int}(A)) \cup \tau_1\mathrm{Int}((1,2)\mathrm{Cl}_I(A))$,

(5) (1, 2)-β-I-open if $A \subset \tau_1\mathrm{Cl}(\tau_1\mathrm{Int}((1,2)\mathrm{Cl}_I(A)))$.

Tha family of $(1,2)$-α-I-open (resp. $(1,2)$-*semi*-I-open, (1, 2)-*pre*-I-open, (1, 2)-b-I-open, (1, 2)-β-I-open) sets is denoted by $(1,2)\alpha IO(X)$ (resp. $(1,2)SIO(X)$, $(1,2)PIO(X)$, $(1,2)BIO(X)$, $(1,2)\beta IO(X)$).

Remark 3. By $(1,2)mIO(X)$ we denote each one of the families $(1,2)\alpha$ $IO(X)$, $(1,2)SIO(X)$, $(1,2)PIO(X)$, $(1,2)BIO(X)$, and $(1,2)\beta IO(X)$.

Lemma 6. *Let (X, τ_1, τ_2, I) be an ideal bitopological space. Then $(1,2)mIO(X)$ is a minimal structure which has property $\mathcal{B}$.*

Proof. By Lemmas 1 and 5, $(1,2)mIO(X)$ is a minimal structure. It follows from Proposition 3.27 of [22] that $(1,2)mIO(X)$ has property $\mathcal{B}$. It is similarly proved that $(1,2)BIO(X)$ has property $\mathcal{B}$. ■

Definition 11. *Let (X, τ_1, τ_2, I) be an ideal bitopological space. For a subset A of X, we define $(1,2)m\mathrm{Cl}_I(A)$ and $(1,2)m\mathrm{Int}_I(A)$ as follows:*
(1) $(1,2)m\mathrm{Cl}_I(A) = \cap\{F : A \subset F, X \setminus F \in (1,2)mIO(X)\}$,
(2) $(1,2)m\mathrm{Int}_I(A) = \cup\{U : U \subset A, U \in (1,2)mIO(X)\}$.

If $(1,2)mIO(X) = (1,2)\alpha IO(X)$ (resp. $(1,2)SIO(X)$, $(1,2)PIO(X)$, $(1,2)BIO(X)$, $(1,2)\beta IO(X)$), then we have
(1) $(1,2)m\mathrm{Cl}_I(A) = (1,2)\alpha\mathrm{Cl}_I(A)$ (resp. $(1,2)s\mathrm{Cl}_I(A)$, $(1,2)p\mathrm{Cl}_I(A)$, $(1,2)b\mathrm{Cl}_I(A)$, $(1,2)\beta\mathrm{Cl}_I(A)$),
(2) $(1,2))m\mathrm{Int}_I(A) = (1,2)\alpha\mathrm{Int}_I(A)$ (resp. $(1,2)s\mathrm{Int}_I(A)$, $(1,2)p\mathrm{Int}_I(A)$, $(1,2)b\mathrm{Int}_I(A)$, $(1,2)\beta\mathrm{Int}_I(A)$).

C) $(1,2)^\star mIO(X)$

Definition 12. *Let (X, τ_1, τ_2, I) be an ideal bitopological space. A subset A of X is said to be*
(1) (1, 2)*-α-I-open if $A \subset (1,2)\mathrm{Int}_I((1,2)\mathrm{Cl}_I((1,2)\mathrm{Int}_I(A)))$,
(2) (1, 2)*-semi-I-open if $A \subset (1,2)\mathrm{Cl}_I((1,2)\mathrm{Int}_I(A))$,
(3) (1, 2)*-pre-I-open if $A \subset (1,2)\mathrm{Int}_I((1,2)\mathrm{Cl}_I(A))$,
(4) (1, 2)*-b-I-open if $A \subset (1,2)\mathrm{Cl}_I((1,2)\mathrm{Int}_I(A)) \cup (1,2)\mathrm{Int}_I((1,2)\mathrm{Cl}_I(A))$,
(5) (1, 2)*-β-I-open if $A \subset (1,2)\mathrm{Cl}_I((1,2)\mathrm{Int}_I((1,2)\mathrm{Cl}_I(A)))$.

The family of (1, 2)*-α-I-open (resp. (1, 2)*-semi-I-open, (1, 2)*-pre-I-open, (1, 2)*-b-I-open, (1, 2)*-β-I-open) sets is denoted by $(1,\ 2)^\star\alpha IO(X)$ (resp. $(1,\ 2)^\star SIO(X)$, $(1,\ 2)^\star PIO(X)$, $(1,\ 2)^\star BIO(X)$, $(1,\ 2)^\star\beta IO(X)$).

Remark 4. By $(1,\ 2)^\star mIO(X)$, we denote each one of the families $(1,\ 2)^\star\alpha IO(X)$, $(1,\ 2)^\star SIO(X)$, $(1,\ 2)^\star PIO(X)$, $(1,\ 2)^\star BIO(X)$, and $(1,\ 2)^\star\beta IO(X)$.

Lemma 7. *Let (X, τ_1, τ_2, I) be an ideal bitopological space. Then $(1,\ 2)^\star mIO(X)$ is a minimal structure which has property $\mathcal{B}$.*

Proof. The proof is similar to the proof of Lemma 6. ■

Definition 13. *Let (X, τ_1, τ_2, I) be an ideal bitopological space. For a subset A of X, we define (1, 2)$^\star m\mathrm{Cl}_I(A)$ and (1, 2)$^\star m\mathrm{Int}_I(A)$ as follows:*
(1) $(1,2)^\star m\mathrm{Cl}_I(A) = \cap\{F : A \subset F, X \setminus F \in (1,2)^\star mIO(X)\}$,
(2) $(1,2)^\star m\mathrm{Int}_I(A) = \cup\{U : U \subset A, U \in (1,2)^\star mIO(X)\}$.

If $(1,2)^{\star}mIO(X) = (1,2)^{\star}\alpha IO(X)$ (resp. $(1,2)^{\star}SIO(X)$, $(1,2)^{\star}PIO(X)$, $(1,2)^{\star}BIO(X)$, and $(1,2)^{\star}\beta IO(X)$), then we have the following:

(1) $(1,2)^{\star}m\mathrm{Cl}_I(A) = (1,2)^{\star}\alpha\mathrm{Cl}_I(A)$ (resp. $(1,2)^{\star}s\mathrm{Cl}_I(A)$, $(1,2)^{\star}p\mathrm{Cl}_I(A)$, $(1,2)^{\star}b\mathrm{Cl}_I(A)$, $(1,2)^{\star}\beta\mathrm{Cl}_I(A)$),

(2) $(1,2)^{\star}m\mathrm{Int}_I(A) = (1,2)^{\star}\alpha\mathrm{Int}_I(A)$ (resp. $(1,2)^{\star}s\mathrm{Int}_I(A)$, $(1,2)^{\star}p\mathrm{Int}_I(A)$, $(1,2)^{\star}b\mathrm{Int}_I(A)$, $(1,2)^{\star}\beta\mathrm{Int}_I(A)$).

Remark 5. Let (X,τ_1,τ_2,I) be an ideal bitopological space. By $m_{ij}IO(X)$, we denote each one of the families A) $(i,j)mIO(X)$, B) $(1,2)mIO(X)$ and C) $(1,2)^{\star}mIO(X)$. Then $m_{ij}IO(X)$ is an m-structure having proprty $\mathcal{B}$.

Definition 14. *Let (X,τ_1,τ_2,I) be an ideal bitopological space. For a subset A of X, we define $m_{ij}\mathrm{Cl}_I(A)$ and $m_{ij}\mathrm{Int}_I(A)$ as follows:*

(1) $m_{ij}\mathrm{Cl}_I(A) = \cap\{F : A \subset F, X \setminus F \in m_{ij}IO(X)\}$,

(2) $m_{ij}\mathrm{Int}_I(A) = \cup\{U : U \subset A, U \in m_{ij}IO(X)\}$.

5. m_{ij}-I-BTP-continuous multifunctions

Definition 15. *A multifunction $F : (X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is said to be*

1) upper m_{ij}-I-BTP-continuous *at a point $x \in X$ if for each σ_i-open set V of Y containing $F(x)$ and having BTP complement, there exists $U \in m_{ij}IO(X)$ containing x such that $F(U) \subset V$,*

2) lower m_{ij}-I-BTP-continuous *at a point $x \in X$ if for each σ_i-open set V of Y meeting $F(x)$ and having BTP complement, there exists $U \in m_{ij}IO(X)$ containing x such that $F(u) \cap V \neq \emptyset$ for each $u \in U$,*

3) upper/lower m_{ij}-I-BTP-continuous *if F has this property at each point $x \in X$.*

Remark 6. It follows from Definition 15 that a multifunction $F : (X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is upper/lower m_{ij}-I-BTP-continuous at $x \in X$ (on X) if and only if $F : (X, m_{ij}IO(X)) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-upper/(i,j)-lower $m_{ij}BTP$-continuous at $x \in X$ (on X). Therefore, by Theorems 1, 2 and Corollaries 1, 2, we obtain the following theorems:

Theorem 7. *For a multifunction $F : (X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is upper m_{ij}-I-BTP-continuous;

(2) $F^+(V) \in m_{ij}IO(X)$ for each σ_i-open set V of Y having BTP complement;

(3) $F^-(K)$ is m_{ij}-I-closed for every BTP and σ_i-closed set K of Y;

(4) $m_{ij}\mathrm{Cl}_I(F^-(B)) \subset F^-(i\mathrm{Cl}(B))$ for every subset B of Y having the BTP σ_i-closure;

(5) $F^+(i\mathrm{Int}(B)) \subset m_{ij}\mathrm{Int}(F^+(B))$ *for every subset* B *of* Y *such that* $Y \setminus i\mathrm{Int}(B)$ *is BTP.*

Theorem 8. *For a multifunction* $F : (X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is lower* m_{ij}*-*I*-BTP-continuous;*

(2) $F^-(V) \in m_{ij}IO(X)$ *for each* σ_i*-open set* V *of* Y *having BTP complement;*

(3) $F^+(K)$ *is* m_{ij}*-*I*-closed for every BTP and* σ_i*-closed set* K *of* Y*;*

(4) $m_{ij}\mathrm{Cl}_I(F^+(B)) \subset F^+(i\mathrm{Cl}(B))$ *for every subset* B *of* Y *having BTP* σ_i*-closure;*

(5) $F^-(i\mathrm{Int}(B)) \subset m_{ij}\mathrm{Int}(F^-(B))$ *for every subset* B *of* Y *such that* $Y \setminus i\mathrm{Int}(B)$ *is BTP.*

Theorem 9. *For a multifunction* $F : (X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper* m_{ij}*-*I*-BTP-continuous at* $x \in X$*;*

(2) $x \in m_{ij}\mathrm{Int}(F^+(V))$ *for every* σ_i*-open set* V *of* Y *containing* $F(x)$ *and having BTP complement;*

(3) $x \in F^-(i\mathrm{Cl}(B))$ *for every subset* B *of* Y *having BTP* σ_i*-closure such that* $x \in m_{ij}\mathrm{Cl}(F^-(B))$*;*

(4) $x \in m_{ij}\mathrm{Int}(F^+(B))$ *for every subset* B *of* Y *such that* $x \in F^+(i\mathrm{Int}(B))$ *and* $Y - i\mathrm{Int}(B)$ *is BTP.*

Proof. The proof follows from Theorem 3. ■

Theorem 10. *For a multifunction* $F : (X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is lower* m_{ij}*-*I*-BTP-continuous at* $x \in X$*;*

(2) $x \in m_{ij}\mathrm{Int}(F^-(V))$ *for every* σ_i*-open set* V *of* Y *having BTP complement such that* $F(x) \cap V \neq \emptyset$*;*

(3) $x \in F^+(i\mathrm{Cl}(B))$ *for every subset* B *of* Y *having BTP* σ_i*-closure such that* $x \in m_{ij}\mathrm{Cl}(F^+(B))$*;*

(4) $x \in m_{ij}\mathrm{Int}(F^-(B))$ *for every subset* B *of* Y *such that* $x \in F^-(i\mathrm{Int}(B))$ *and* $Y - i\mathrm{Int}(B)$ *is BTP.*

Proof. The proof is similar to that of Theorem 9. ■

For a multifunction $F : (X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$, we define $D^+_{m_{ij}BTP}(F)$ and $D^-_{m_{ij}BTP}(F)$ as follows:

$D^+_{m_{ij}BTP}(F) = \{x \in X : F$ is not upper m_{ij}-I-BTP-continuous at $x\}$,

$D^-_{m_{ij}BTP}(F) = \{x \in X : F$ is not lower m_{ij}-I-BTP-continuous at $x\}$.

Theorem 11. *For a multifunction* $F : (X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$*, the following properties hold:*

$$\begin{aligned} D^{+}_{m_{ij}BTP}(F) &= \bigcup_{G\in\sigma_i BTP}\{F^{+}(G) - m_{ij}\mathrm{Int}(F^{+}(G))\} \\ &= \bigcup_{B\in iBTP}\{F^{+}(i\mathrm{Int}(B)) - m_{ij}\mathrm{Int}(F^{+}(B))\} \\ &= \bigcup_{B\in BTP}\{m_{ij}\mathrm{Cl}(F^{-}(B)) - F^{-}(i\mathrm{Cl}(B))\} \\ &= \bigcup_{H\in\mathcal{F}}\{m_{ij}\mathrm{Cl}(F^{-}(H)) - F^{-}(H)\}, \end{aligned}$$

where $\sigma_i BTP$ is the family of all σ_i-open sets of Y having BTP complement, $iBTP$ is the family of all subset B of Y such that $Y - i\mathrm{Int}(B)$ is BTP, BTP is the family of all subsets of Y having BTP σ_i-closure, $\mathcal{F}$ is the family of all subset H of Y which is BTP and σ_i-closed.

Proof. This is obvious by Theorem 5. ■

Theorem 12. *For a multifunction $F : (X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$, the following properties hold:*

$$\begin{aligned} D^{-}_{m_{ij}BTP}(F) &= \bigcup_{G\in\sigma_i BTP}\{F^{-}(G) - m_{ij}\mathrm{Int}(F^{-}(G))\} \\ &= \bigcup_{B\in iBTP}\{F^{-}(i\mathrm{Int}(B)) - m_{ij}\mathrm{Int}(F^{-}(B))\} \\ &= \bigcup_{B\in BTP}\{m_{ij}\mathrm{Cl}(F^{+}(B)) - F^{+}(i\mathrm{Cl}(B))\} \\ &= \bigcup_{H\in\mathcal{F}}\{m_{ij}\mathrm{Cl}(F^{+}(H)) - F^{+}(H)\}. \end{aligned}$$

Proof. The proof is similar to that of Theorem 11. ■

References

[1] Akdag M., On upper and lower I-continuous multifunctions, *Far East J. Math. Sci.,* 25(2007), 48-57.

[2] Akdag M., Canan S., Upper and lower semi-I-continuous multifunctions, *J. Adv. Res. Pure Math.,* 6(2014), 78-88.

[3] Arivazhagi C., Rajesh N., Nearly I-continuous multifunctions, *Bol. Soc. Paran. Mat.,* 37(2019), 33-38.

[4] Boonpok C., Viriyapong C., Thongmoon M., On upper and lower (τ_1, τ_2)-precontinuous multifunctions, *J. Math. Computer Sci.,* 18(2018), 282-293.

[5] Caldas M, Jafari S., Rajesh N., Preopen sets in ideal bitopological spaces, *Bol. Soc. Paran. Mat.* (3), 29(2)(2011), 61-68.

[6] Caldas M, Jafari S., Rajesh N., Semiopen sets in ideal bitopological spaces, *An. Univ. Sci. Budapest. Math.,* (to appear).

[7] Ekici E., Nearly continuous multifunctions, *Acta Math. Univ. Comenianae,* 72(2003), 229-235.

[8] Ekici E., Almost nearly continuous multifunctions, *Acta Math. Univ. Comenianae,* 73(2004), 175-186.

[9] El-Maghrabi A.I., Caldas M., Jafari S., Latif R.M., Nasef A., Rajesh N., Shanthi S., Properties of ideal bitopological α-open sets, *Sci. Stud. Res. Ser. Math. Inform.,* 27(2) (2017), 15-36.

[10] Ewert J., Lipski T., On s-quasi-continuous multivalued maps, *Univ. u. Novom Sadu Zb. Rad. Prirod.-Mat. Fak. Ser. Mat.,* 20(1990), 167-183.

[11] Gentry K.R., Hoyle H.B., C-continuous functions, *Yokohama Math. J.,* 18(1970), 71-76.

[12] Janković D., Hamlett T.R., New topologies from old via ideals, *Amer. Math. Monthly,* 97(1990), 295-310.

[13] Jafari S., Thivagar M.L., Ponmani S.A., (1, 2)α-open sets based on bitopological separation axioms, *Soochow J. Math.,* 33(3)(2007), 375-381.

[14] Kuratowski K., *Topology,* Vol. I, Acadmic Press, New York, 1966.

[15] Maki H., Rao' C.K., Nagoor Gani A., On generalizing semi-open and preopen sets, *Pure Appl. Math. Sci.,* 49(1999), 17-29.

[16] Neubrunn T., C-continuity and closed graphs, Č*asopis P*ě*st. Mat.,* 110(1985), 172-178.

[17] Noiri T., N-closed sets and some separation axioms, *Ann. Soc. Sci. Bruxelles,* 88(1974), 195-199.

[18] Noiri T., Ergun N., Notes on N-continuous functions, *Res. Rep. Yatsushiro Coll. Tech.,* 11(1989), 65-68.

[19] Noiri T., Popa V., On upper and lower M-continuous multifunctions, *Filomat,* 14(2000), 73-86.

[20] Noiri T., Popa V., A generalization of nearly continuous multifunctions, *Ann. Univ. Sci. Budapest.,* 50(2007), 59-74.

[21] Noiri T., Popa V., Some forms of open sets and continuity in ideal topological spaces, *Fasc. Math.,* 65(2021), 57-66.

[22] Paul N.R., Weakly open sets in ideal bitopological spaces, *Int. J. Eng. Res. Techn.,* 2(7)(2013), 124-134.

[23] Popa V., Noiri T., On s-β-continuous multifunctions, *J. Egypt. Math. Soc.,* 8(2000), 127-137.

[24] Popa V., Noiri T., On s-precontinuous multifunctions, *Demonstr. Math.,* 33(2000), 679-687.

[25] Popa V., Noiri T., On M-continuous functions, *Anal. Univ. "Dunărea de Jos" Galąti, Ser. Mat. Fiz. Mec. Teor.,* Fasc. II 18(23)(2000), 31-41.

[26] Popa V., Noiri T., A unified theory for S-continuity of multifunctions, *J. Math. Fac. Sci. Univ. Istanbul,* 59(2000), 1-15.

[27] Popa V., Noiri T., On the definitions of some generalized forms of continuity under minimal conditions, *Mem. Fac. Sci. Kochi Univ. Ser. Math.,* 22(2001), 9-18.

[28] Popa V., Noiri T., On m-continuous multifunctions, *Bul. St. Univ. Politeh. Timisoara, Ser. Mat. Fiz.,* 46(60)(2)(2001), 1-12.

[29] Popa V., Noiri T., A unified theory of weak continuity for functions, *Rend. Circ. Mat. Palermo* (2)*,* 51(2002), 439-464.

[30] Popa V., Noiri T., Characterizations of C-quasi-continuous multifunctions, *Math. Balkan.,* 20(2006), 265-274.

[31] Rychlewicz A., Almost nearly continuity with reference to multifunctions in bitopological spaces, *Int. J. Pure Appl. Math.,* 105(1) (2015), 7-18.

[32] Sarma D.J., bI-open sets in ideal bitopological spaces, *Int. J. Pure Appl. Math.,* 105(1)(2015), 7-18.

[33] Sarma D.J., Kilicman A., Mishra L.N., A new type of weak open sets via idealization in bitopological spaces, *Malays. J. Math. Math. Sci.,* 15(2)(2021), 189-197.

[34] VAIDYANATHASWANI R., The localization theory in set-topology, *Proc. Indian Acad. Sci.,* 20(1945), 51-61.

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