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IMMERSION OF A DIFFERENTIABLE MANIFOLD WITH SYMPLECTIC STRUCTURE

In this paper the theorems relate to immersion of a differentiable manifold with symplectic structure in symplectic space are proved

Key words: differentiable manifold, symplectic structure, immersion, symplectic space.

1. Basic definitions

Definition 1. *The affine space* A^{2n} *we call* 2n *dimensional symplectic space and denote by* Sc^{2n} *where, the bilinear antisymetric form* $\Omega(X,Y)$ *is defined for every* $X,Y \in A^{2n}$.

Definition 2. *The symplectic space* Sc^{2n} *we call proper symplectic space if for any (so also for each) system of 2n lineary independent vectors* $X_a, X_b \in Sc^{2n}$ *rang* $[\Omega(X_a, X_b)] = 2n$ $(a,b = 1,2,\ldots,n,\overline{1},\overline{2},\ldots,\overline{n})$.

Definition 3. *A subgroup of the linear group* $L(2n,R)$ *which preserver the forms* $\Omega(X,Y)$ *we call symplectic group and denote* $Sp(2n,R)$.

Definition 4. *A system of 2n linearly independent vectors* $e_i, e_{\overline{i}} \in Sc^{2n}$ $(i = 1,2,\ldots,n)$ *we call a symplectic basis of the space* Sc^{2n} *if there holds:*

$$\Omega(e_a,e_b)=\begin{cases} 1 & \text{if } a=1, b=\bar{i} \\ -1 & \text{if } a=\bar{i}, b=i \\ 0 & \text{in other cases} \end{cases} \quad (1.1)$$

The relation (1.1) one can write in the form

$$\begin{aligned} &\Omega(e_i,e_{\bar{j}})=-\Omega(e_{\bar{i}},e_j)=\delta_{ij} \\ &\Omega(e_i,e_j)=\Omega(e_{\bar{i}},e_{\bar{j}})=0 \end{aligned} \quad (1.1)'$$

During the cours of further considerations the following indices are running as follows:

$i,j,k=1,2,...,n;\quad a,b,c=1,2,...,n,\bar{1},\bar{2},...,\bar{n};$

$I,J,K=1,2,...,n,n+1,n+2,...,n+m;$

$A,B,C=1,...,n,n+1,...,n+m,\bar{1},...,\bar{n},\overline{n+1},...,\overline{n+m};$

$p,q,r=n+1,n+2,...,n+m;$

$P,Q,R=n+1,...,n+m,\overline{n+1},...,\overline{n+m}.$

Remark. In a proper symplectic space one can always find a symplectic basis.

Let $\{e_a\}$ be a symplectic basis. Then for any two vectors $X,Y\in Sc^{2n}$ there holds $X=X^a e_a, Y=Y^a e_a$. So employing (1.1)' one can calculate $\Omega(X,Y)$.

$$\Omega(X,Y)=\sum_{i=1}^{n} X^i Y^{\bar{i}} - Y^i X^{\bar{i}} \quad (1.2)$$

Definition 5. *The right hand side of the equation* (1.2) *we call a canonical form of the form* $\Omega(X,Y)$.

Remark. 1. The form $\Omega(X,Y)$ is also called a fundamental form of the symplectic space Sc^{2n}.

2. The group $Sp(2n,R)$ preserver the canonical form of the fundamental form. It follows from the fact a base of the symplectic space Sc^{2n} remains symplectic.

Let us denote $A=[A^a_{a'}]\in Sp(2n,R)$. From (1.1) there follows condition, which $A^a_{a'}$ have to satisfy

(1.3)
$$\sum_{i=1}^{n} A^i_{a'}A^{\bar{i}}_{b'} - A^{\bar{i}}_{a'}A^{\bar{i}}_{b'} = \begin{cases} 1 & \text{for } a=j, b=\bar{j} \\ -1 & \text{for } a=\bar{j}, b=j \\ 0 & \text{in other cases}. \end{cases}$$

Let M denotes $2n$ dimensional differentiable manifold of the class C^∞. By Mp we denote a tangent space to this manifold at the point $p\in M$.

Definition 6. *If in the space Mp for every $p\in M$ there is defined bilinear antisymetric form $\Omega(X,Y)$ then the manifold* M *is called symplectic.*

Definition 7. *If the manifold M is symplectic, and for some (it means for every) system of lineary independent vectors $X_a, Y_b \in Mp$ the rang $[\Omega(X_a, Y_b)]=2n$, then this manifold we call the proper symplectic manifold.*

Let us denote by w^b_a the form of connection which defined on M. Infinitesimal transformation of the symplectic basis at the point $p\in M$, and denoted by $R_p=\{p,e_a\}$ we define by the formula

(1.4)
$$dp = w^a\, e_a$$
$$de_a = w^b_a\, e_b$$

Definition 8. *The connection determined by the forms* w_a^b *we call symplectic if an exterior differential (denoted by* $\bar{d}$*) of the fundamental form for a symplectic basis satisfies the conditions*

$$\bar{d}\,\Omega(e_a, e_b) = w_a^c\,\Omega(e_c, e_b) + w_b^c\,\Omega(e_a, e_c). \tag{1.5}$$

Theorem 1. *If* w_a^b *is a form of symplectic connection defined on the M, then*

$$\begin{aligned} & w_j^i + w_{\bar{i}}^{\bar{j}} = 0 \\ & w_{\bar{j}}^i - w_{\bar{i}}^j = 0, \qquad w_j^{\bar{i}} - w_i^{\bar{j}} = 0. \end{aligned} \tag{1.6}$$

In the proof of the theorem we employ the relation (1.5), where the side on the left is zero. The relation (1.1) is also employed.

Differentiating externaly (1.4) and denoting $\bar{d}(dp) = \Omega^a e_a$, and $\bar{d}(de_a) = \Omega_a^b e_b$ we get

$$\begin{aligned} \Omega^a &= \bar{d}w^a + w_b^a \wedge w^b \\ \Omega_a^b &= \bar{d}w_a^b + w_c^b \wedge w_a^c \\ \Omega_a^b &= \bar{d}w_a^b + w_c^b \wedge w_a^c. \end{aligned} \tag{1.7}$$

The equation (1.7) are called the structural equations, and the forms Ω^a and Ω_a^b are called the curvature form and the torsion form of the symplectic connection respectively.

Definition 9. *If the torsion forms of symplectic connection* $\Omega^a \equiv 0$, *on the manifold M, then M is called the manifold with symplectic structure.*

2.Condition I

Let M be $2n$ dimensional differentiable manifold with symplectic structure. Let us consider the neighbourhood $\{Uq\}$ which covers this manifold, then in each of these neighbourhoods we get the local coordinates. By the definition 9 and from the assumption on the manifold M to be with symplectic structure, the formula (1.7) takes the form

$$\bar{d}w^a + w^a_b \wedge w^b = 0 \qquad (2.1)$$
$$\Omega^b_a = \bar{d}w^b_a + w^b_c \wedge w^c_a$$

Condition I. *There exists such a number 2m, that each neighbourhood $Uq \subset M$ is immersed in the proper symplectic space Sc^{2n+2m} with the form Ω, which restricted to the space Mp coincides with a fundamental form of the manifold M in every point $p \in Uq$. It means that for every neighbourhood Uq there exist forms w^p_a, w^a_p, w^q_p of the class C^∞ that along with forms w^a, w^a_b satisfy the equations*:

$$w^p_a \wedge w^a = 0 \qquad \bar{d}w^p_a + w^p_A \wedge w^A_a = 0$$
$$(2.2) \qquad w^a_p \wedge w^p_b = -\Omega^a_b \qquad \bar{d}w^a_p + w^a_A \wedge w^A_p = 0$$
$$\bar{d}w^p_Q + w^p_A \wedge w^A_Q = 0.$$

The equations (2.1) and (2.2) taken all together constitute the structural equations of the form w^A_B in the space Sc^{2n+2m} calculated for $w^p = 0$.

From the theorem 1 follows that connection forms of the space Sc^{2n+2m} denoted by w^A_B satisfy the relations

$$w^I_J + w^{\bar{J}}_{\bar{I}} = 0$$
$$(2.3) \qquad w^I_{\bar{J}} - w^J_{\bar{I}} = 0$$
$$w^{\bar{I}}_J - w^{\bar{J}}_I = 0.$$

The formulas (2.3) one can write in another form

$$
(2.4)\qquad
\begin{array}{ll}
w^i_j + w^{\bar{j}}_{\bar{i}} = 0 & w^i_p + w^{\bar{p}}_{\bar{i}} = 0 \\
w^i_{\bar{j}} - w^j_{\bar{i}} = 0 & w^i_{\bar{p}} - w^p_{\bar{i}} = 0 \\
w^{\bar{i}}_j - w^{\bar{j}}_i = 0 & w^{\bar{i}}_p - w^{\bar{p}}_i = 0 \\
w^p_i + w^{\bar{i}}_{\bar{p}} = 0 & w^p_q + w^{\bar{q}}_{\bar{p}} = 0 \\
w^p_{\bar{i}} - w^i_{\bar{p}} = 0 & w^p_{\bar{q}} - w^q_{\bar{p}} = 0 \\
w^{\bar{p}}_i - w^{\bar{i}}_p = 0 & w^{\bar{p}}_q - w^{\bar{q}}_p = 0
\end{array}
$$

3. Condition II

Let there exists the symplectic group *Sp(2m,R)* for the point $p \in Uq$ such that the form w^A_B which changing the symplectic basis $\{p; e_1, e_2, \ldots, e_{n+m}, e_{\bar{1}}, e_{\bar{2}}, \ldots, e_{\overline{n+m}}\}$ in the sympletic basis $\{p; e_1, \ldots, e_{n+m}, e_{\bar{1}}, \ldots, e_{\overline{n+m}}\}$ is transformed according to the formulas, which when the relation

$$w^A_B\, \Omega(e_A, e_C) + w^A_C\, \Omega(e_B, e_A) = 0$$

in to account is taken, form the relations

$$
(3.1)\qquad
\begin{array}{l}
w^{a'}_{b'} = A^{a'}_a dA^a_{b'} + A^{a'}_a w^a_b A^b_{b'} \\
w^{P'}_{a'} = B^{P'}_P w^P_a A^a_{a'}, \quad w^{a'}_{P'} = A^{a'}_a w^a_P B^P_{p'} \\
w^{P'}_Q = B^{P'}_P w^P_Q B^Q_Q + B^{P'}_P dB^P_Q
\end{array}
$$

where $A^a_{a'} \in Sp(2n, R)$ and $B^{P'}_P \in Sp(2m, R)$.

Geometrically it denotes that in Sc^{2n+2m} the piece *Nq* of surface, which is an immersion of *Uq* is considered. In every point of the piece *Nq* we define *2n* tangential vectors which form the symplectic basis of a tangential space, and *2m* vectors which form the symplectic basis of the normal space to the piece *Nq*. The components of the infinitesimal translation of the basis which is defined as above are the identical with the forms w_B^A. If the point $\overline{P} \in Nq$ is an image of the point $p \in Uq$, then the change of the symplectic basis in the point $\overline{P}$ is defined by formulas:

$$e'_a = A_{a'}^a \, e_a$$
$$e_{P'} = B_{P'}^P \, e_P$$

where $\{e_a\}$ is a symplectic basis of the tangential space, $\{e_P\}$ is a symplectic basis of the space which is normal to the piece Nq $(P \in Nq)$, and $[A_{a'}^a] \in Sp(2n,R)$ and $[B_{P'}^P] \in Sp(2m,R)$.

The components of the infinitesimal transformation of the symplectic basis in Sc^{2n+2m}, with the above properties, are forms $w_{A'}^{B'}$, which one obtain forms w_A^B employing the forms (3.1).

Let us produce for every neighbourhood $Uq \subset M$ of the cartesian product $Pq = Uq \times Sc^{2n+2m} \times Sp(2n+2m,R)$, where *Sp(2n+2m,R)* is a symplectic group. Coordinates of any point *A* of this product *Pq* are of the form (U^a, X^A, G_B^A), where $[G_B^A] \in Sp(2n+2m)$.

We consider the system of equations on the product *Pq*:

(3.2)
$$dX^A = G_a^A w^a$$
$$dG_B^A = G_C^A w_B^C$$

Lemma 1. *The sytem* (3.2) *is unrestrictedly integrable and for the Nq 2n dimensional piece, which is a solution of the* (3.2), *the projection* π: $(U^a, X^A, G_B^A) \to U^a$ *is a homeomorphism.*

Proof. First let us consider the system (3.2) for $Pq' = Uq \times Sc^{2n+2m} \times S'$, where S' is a set of square matrices of the dimension *2n+2m*. It follows from the Frobenius theorem that the system (3.2) is unrestrictedly integrable on Pq'.

From the same theorem it follows that through each point $A \in Pq'$ here passes exactly one surface of the dimension *2n*, such that over every point $p \in Uq$ it has exactly one point in *Nq*. This relation is a homeomorphism and it defines a projection π. On the surface Nq the formulas (2.1) and (2.2) are valid. We shall prove that S' is a symplectic group, providing the element of symplectic group is the initial condition. For this aim let us present the equation (3.2) in the form:

$$(3.3)\quad \begin{aligned} d(G^I_J G^{\bar I}_K - G^{\bar I}_J G^I_K) &= (G^I_L G^{\bar I}_K - G^{\bar I}_L G^I_K) w^L_J + (G^I_{\bar L} G^{\bar I}_K - G^{\bar I}_{\bar L} G^I_K) w^{\bar L}_J \\ &\quad + (G^I_J G^{\bar I}_L - G^{\bar I}_J G^I_L) w^L_K + (G^I_J G^{\bar I}_{\bar L} - G^{\bar I}_J G^I_{\bar L}) w^{\bar L}_K \\ d(G^I_{\bar J} G^{\bar I}_{\bar K} - G^{\bar I}_{\bar J} G^I_K) &= (G^I_L G^{\bar I}_{\bar K} - G^{\bar I}_L G^I_{\bar K}) w^L_{\bar J} + (G^I_{\bar L} G^{\bar I}_{\bar K} - G^{\bar I}_{\bar L} G^I_{\bar K}) w^{\bar L}_{\bar J} \\ &\quad + (G^I_{\bar J} G^{\bar I}_L - G^{\bar I}_{\bar J} G^I_L) w^L_{\bar K} + (G^I_{\bar J} G^{\bar I}_{\bar L} - G^{\bar I}_{\bar J} G^I_{\bar L}) w^{\bar L}_{\bar K} \\ d(G^I_J G^{\bar I}_{\bar K} - G^{\bar I}_J G^I_{\bar K}) &= (G^I_L G^{\bar I}_{\bar K} - G^{\bar I}_L G^I_{\bar K}) w^L_J + (G^I_{\bar L} G^{\bar I}_{\bar K} - G^{\bar I}_{\bar L} G^I_K) w^{\bar L}_J \\ &\quad + (G^I_J G^{\bar I}_L - G^{\bar I}_J G^I_L) w^L_{\bar K} + (G^I_J G^{\bar I}_{\bar L} - G^{\bar I}_J G^I_{\bar L}) w^{\bar L}_{\bar K} . \end{aligned}$$

Let in the system (3.3) G^A_B be the functions of the parametr t (it is a line going through a group), then this system is a system of ordinary equations. The solution of the system (3.3) is a family of lines which run over a group. If we assume the condition: for $t = 0, G^A_B\ (0) = Go^A_B$, then we get the particular solution of the system (3.3). It suffices to show that $G^A_B\ (t)$ forms an element of a symplectic group it will imply $S' = Sp(2n+2m, R)$. From properties of the symplectic group (1.3) follows the left-hand sides of the system (3.3) to be zero while the right-hand sides are

$$(3.4)\quad \begin{aligned} 0 &= 0w^L_J - \delta_{LK} w^{\bar L}_J + \delta_{JL} w^{\bar L}_K + 0w^L_K \\ 0 &= \delta_{LK} w^L_{\bar J} + 0w^{\bar L}_{\bar J} - \delta_{JL} w^L_{\bar K} + 0w^{\bar L}_{\bar K} \\ 0 &= \delta_{LK} w^L_J + 0w^{\bar L}_J + \delta_{JL} w^{\bar L}_{\bar K} + 0w^L_{\bar K} . \end{aligned}$$

It is easy to observe that equations (3.4) are valid, since they are equivalent to (2.3). So the equations (3.4), (3.3) and (3.2) are satisfied.

Two points $P(U^a, X^A, G^A_B) \in Pq$ and $P'(U^{a'}, X^{A'}, G^{A'}) \in Pq'$ we consider to be identical, if for $A = A'$ there holds

$$(3.5)\quad X^{A'} = X^A \quad ; \quad G^{A'}_{a'} = G^A_a A^a_{a'} \quad ; \quad G^{A'}_{Q'} = G^A_Q B^Q_{Q'} \quad ;$$

where $[A^a_{a'}] \in Sp(2n, R)$, $[B^Q_{Q'}] \in Sp(2m, R)$.

Let $Tq'q$ denote the transformation, defined by the formulas (3.5). They satisfy the properties of a group so there also holds $Tq''q = Tq''q'\,Tq'q$.

The set of products the fibre space with the fibre $Sc^{2n+2m} \times Sp(2n+2m, R)$ and with the group of a structure compound of the all transformations $Tq'q$. This fibre space we denote by $E(M)$. For every product $Pq \in E(M)$ the system (3.2) defines a family of the Nq $2n$ dimensional integral surfaces.

Lemma 2. *The points of the selected piece of surface* $\overline{M}q$ *over a common part of the neighbourhoods* Uq *and* Up *under the identification* (3.5) *are identical with the points of the certain piece of surface* $\overline{M}p$. *Then we say the piece* $\overline{M}q$ *and* $\overline{M}p$ *are splined over* $Uq \cap Up$.

Proof. Let us consider the piece of surface $\overline{M}p$ and Pp. It is an integral of the system of equations

$$(3.6) \qquad dX^{A'} = G^{A'}_{a'} w^{a'}, \qquad dG^{A'}_{B'} = G^{A'}_{C'} w^{C'}_{B'}$$

For the points from $Uq \cap Up$ the form w and w' are connected with the equations (3.1). In order to show an existence of the piece $\overline{M}q$ in Pq, which is splined over $Up \cap Uq$ with the piece $\overline{M}p$, it suffices to show that $dX^{A'}$ and $dG^{A'}_{B'}$ calculated in (3.6) are equal to $dX^{A'}$ and $dG^{A'}_{B'}$ calculated through identification (3.5). For $A = A'$ we have

$$dX^{A'} = G^{A'}_{a'} w^{a'} = G^{A}_{a} A^{a}_{a'} A^{a'}_{b} w^{b} = G^{A}_{b} w^{b} = dX^{A},$$

$$dG^{A'}_{a'} = G^{A'}_{B'} w^{B'}_{a'} = G^{A'}_{b'} w^{b'}_{a'} + G^{A'}_{R'} w^{R'}_{a'} = G^{A}_{b} A^{b}_{b'} A^{b'}_{c} w^{c}_{a} A^{a}_{a} + G^{A}_{b} A^{b}_{b'} A^{b'}_{c} dA^{c}_{a'} +$$
$$+ G^{A}_{R} B^{R}_{R'} B^{R'}_{Q} w^{Q}_{a} A^{a}_{a'} + G^{A}_{b} w^{b}_{a} A^{a}_{a'} + G^{A}_{b} dA^{b}_{a'} + G^{A}_{R} w^{R}_{a} A^{a}_{a'} =$$
$$= G^{A}_{B} w^{B}_{a} A^{a}_{a'} + G^{A}_{a} dA^{a}_{a'} = dG^{A}_{a} A^{a}_{a'} + G^{A}_{a} dA^{a}_{a'} = d(G^{A}_{a} A^{a}_{a'}),$$

$$dG^{A'}_{R'} = G^{A'}_{B'} w^{B'}_{R'} = G^{A'}_{a'} w^{a'}_{R'} + G^{A'}_{p'} w^{p'}_{R'} = G^{A}_{a} A^{a}_{a'} A^{a'}_{b} w^{b}_{R} B^{R}_{R'} + G^{A}_{p} B^{p}_{p'} B^{p}_{Q} w^{Q}_{R} B^{R}_{R'} +$$
$$+ G^{A}_{p} B^{p}_{p'} B^{p'}_{Q} dB^{Q}_{R'} = G^{A}_{a} w^{a}_{R} B^{R}_{R'} + G^{A}_{p} w^{p}_{R} B^{R}_{R'} + G^{A}_{p} dB^{p}_{R'} =$$
$$= G^{A}_{B} w^{B}_{R} B^{R}_{R'} + G^{A}_{p} dB^{p}_{R'} = dG^{A}_{R} B^{R}_{R'} + G^{A}_{R} dB^{R}_{R'} = d(G^{A}_{R} B^{R}_{R'}).$$

The solutions of the system (3.2) in virtue of the identity (3.5) are defined on the whole fibre space *E(M)*. Let us consider integral surface $\overline{M}$ with maximal dimension, which pas through $P \in E(M)$. One can show that $\overline{M}$ covers whole manifold *M*.

Let us define the projection f of the surface $\overline{M}$ into the space Sc^{2n+2m}. The coordinates of the point $P \in \overline{M}$ are of the form $(U^a, X^A(U^a), G_B^A(U^a))$. We shall prove the surface piece *Np* being the projection of $\overline{M}p$ into Sc^{2n+2m} is of the dimension *2n+2m*, i.e. in the point *P* at least one jacobian

$$J = \left| \frac{\partial x^z}{\partial U^a} \right|$$

differs from zero (z runs over a set $1,...,n+m,\overline{1},...,\overline{n+m}$).
For this aim, for the system of equations (3.2) $w^a = a_b^a dU^b$ $(\det[a_b^a] \neq 0)$ we get :

$$\left| \frac{\partial x^z}{\partial U^a} \right| = \left| G_b^z a_a^b \right| = \left| G_b^z \right| \cdot \left| a_a^b \right| \neq 0$$

because det $[G_B^A] \neq 0$ (so there exists a minor of degree 2n different from zero).

Let *O* be the neighbourhood of the point *P*, where $J \neq 0$.
The projection

$$f\colon (U^a, X^A, G_B^A) \to X^A$$

of the surroundings *O* is a surroundings for which we get some piece of the surface Sc^{2n+2m}. This way $f(\overline{M}p) = Np$ is a piece of surface in Sc^{2n+2m} defined by *Up*.

Instead of the point from *Np* let us take, the basis pined in this point and compound of the basis vectors e_A with coordinates G_B^A . According to the formulas (3.1) vectors e_a are these ones which form the symplectic basis of the tangential space to *Np*, vectors e_R in turn are these ones which form the symplectic basis normal to *Np*. The form w_B^A , contains the forms w_b^a which define the connection in the surroundings *Up*, define the connection *Np*. Taking into account the projection of the whole space $\overline{M}$ into the symplectic space Sc^{2n+2m} we get the proper symplectic surface *N*, for which the forms of connection are induced by the forms of connection defined on the manifold *M*.

4. Theorem on immersion

Theorem 2. *If the conditions* I and II *on the differentiable manifold M of the class* C^{∞} *with symplectic structure and connection without torsion hold, then there exist the proper symplectic space* Sc^{2n+2m} *such that the surface N defined by the manifold M is immersed in this space. At the same moment the forms of connection on N for the point* $P \in N$, *which is an image of* $p \in M$, *are the forms of connection from the manifold M in the point p. It means, that the object of connection given in the point* $P \in N$ *is identical to the object of connection in the point* $p \in M$. *The fundamental form defined in* Sc^{2n+2m} *restricted to the tangential space to N in the point P coincides with the fundamental form defined on the space tangential to M in the point p.*

Proof. The surface *N* is an image of the surface $\overline{M}$ received throught the projection f. $\overline{M}$ is a surface which covers *M*. If the manifold *M* has a symplectic connection without torsion then it is homeomorphic to $\overline{M}$. Moreover if the surface *N* does not contains the point of selfcrossing and selftangentiality, then it is an immersion of the manifold *M*. The surface *N* is determined by the insertion to Sc^{2n+2m} and therefore the system (3.2) is unrestrictedly integrable.

The surface $\overline{M}$ is determined by the point $Po(Uo^{a}, Xo^{a}, Go_{B}^{A})$, so the surface *N* is determined by (Xo^{A}) and the vectors which form the symplectic basis of the space Sc^{2n+2m} $I_{A}(Go_{A}^{B})$.

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