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COINCIDENCE THEOREMS IN RANDOM NORMED SPACES*

ABSTRACT. A probabilistic version of Meir and Keeler fixed point theorem and a generalization of Park's coincidence theorem in random normed spaces are presented.

KEY WORDS: fixed point, contraction mapping, probabilistic metric space, random normed space, coincidence point.

1. INTRODUCTION

A probabilistic spaces (briefly PM-spaces) were first studied by Menger [11]. The notion of random normed spaces (briefly RN-spaces) were introduced by Sherstner [17]. A PM space is a space in which the concept of distance is considered to be probabilistic, rather than deterministic. These probabilistic spaces are assumed to satisfy axioms which are quite similar to the axioms satisfied in an ordinary metric space. The triangle inequality has been the subject of some controversy. Sehgal and Bharucha-Reid [16] obtained the Banach principle in the PM-spaces. In [2, 3, 14] was presented a new type of the contraction mapping in PM spaces. Some fixed point theorems for singlevalued and multivalued contraction mappings in PM-spaces and RN-spaces were obtained in [1, 5, 19].

The purpose of this paper is to present some fixed point theorems for Meir and Keeler's type of the contraction mapping in RN-spaces. In [3] probabilistic version of these theorems is given in PM-spaces without proofs. For detailed discussions of random metric spaces we refer to [7, 14].

2. PRELIMINARIES

Throughout this paper \mathfrak{R} denotes the real numbers, $\mathfrak{R}^+ = [0, \infty)$, D denote the set of all distribution functions $F: \mathfrak{R} \rightarrow \mathfrak{R}^+$ such that $F(0) = 0$, F is non-decreasing, left-continuous mapping such that $\sup F(x) = 1$. It will always

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denote the specific distribution function defined by $H(x) = 0$ for $x < 0$ and $H(x) = 1$ for $x > 0$. Let (Ω, \mathcal{B}, P) be a complete probability space and (\mathcal{R}, d) be a real metric space with euclidean metric d . An ordered pair (S, F) is a probabilistic metric space if S is a nonempty set and $F: S \times S \rightarrow \mathcal{D}$ satisfies the following conditions ($F(p, q)$ will be denoted by F_{pq} for $p, q \in S$):

$$(2.1) \quad F_{pq}(x) = 1 \quad \text{iff} \quad p = q, \quad p, q \in S \text{ and } x > 0.$$

$$(2.2) \quad F_{pq} = F_{qp}.$$

$$(2.3) \quad F_{pq}(x) = 1 \text{ and } F_{qr}(y) = 1 \text{ imply } F_{pr}(x+y) = 1, \quad p, q, r \in S \text{ and } x, y > 0.$$

A Menger space is a triple (S, F, Δ) , where (S, F) is a PM-space and Δ is a t -norm so that

$$(2.4) \quad F_{pq}(x+y) \geq \Delta(F_{pr}(x), F_{rq}(y)), \quad p, q, r \in F \text{ and } x, y > 0.$$

Recall that a mapping $\Delta: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a t -norm if the following conditions are satisfied:

$$(2.5) \quad \Delta(0, 0) = 0, \quad \Delta(a, 1) = a, \quad \text{for every } a \in [0, 1].$$

$$(2.6) \quad \Delta(a, b) = \Delta(b, a), \quad a, b \in [0, 1].$$

$$(2.7) \quad \text{if } a \geq b, \quad c \geq d, \text{ then } \Delta(a, c) \geq \Delta(b, d), \text{ for every } a, b, c, d \in [0, 1].$$

$$(2.8) \quad \Delta(a, \Delta(b, c)) = \Delta(\Delta(a, b), c), \quad a, b, c \in [0, 1].$$

Here are example t -norms [14]:

$$\Delta_1(a, b) = \max(a + b - 1, 0), \quad \Delta_2(a, b) = ab, \quad \Delta_3(a, b) = \min(a, b) \text{ and}$$

$$\Delta_4(a, b) = \begin{cases} a, & \text{if } b = 1, \\ b, & \text{if } a = 1, \\ 0, & \text{otherwise.} \end{cases}$$

These four functions are ordered in an increasing „strength” Δ is said to be stronger than Δ' (and Δ' weaker than Δ) if $\Delta''(a, b) \geq \Delta'(a, b)$ for all (a, b) on the unit square with strict inequality for at least one pair (a, b) .

In [16] it was shown that there is a strongest t -norm, namely the function Min , and a weakest t -norm, namely the function Δ_4 . For Δ_2 the condition (2.4) can be interpreted as follows: the probability that the distance of p and q is less than $x + y$ is not less than the joint probability that, independently, the distance of p and r is less than x , and the distance of r and q is less than y . Similar interpretations can be given to the other presented t -norms.

A t -norm is strict if it is continuous on the closed unit square $[0, 1] \times [0, 1]$ and strictly increasing in each place on the half-open square $(0, 1] \times (0, 1]$. Abel-Aczel [9] proved the a strict t -norm admits the representation

$$(A) \quad \Delta(a, b) = f^*(f(a) + f(b)),$$

where f is a function which is defined, continuous, and strictly decreasing on the half-open interval $(0,1]$, $\lim_{x \rightarrow 0^+} f(x) = \infty$, $f(1) = 0$, and f^* is the inversion of f . Conversely, if f is any function with these properties, then the function Δ given by (A) is a strict t -norm. A function f with the properties mentioned above is called an additive generator of the strict t -norm Δ .

A random normed space (S, F, Δ) (briefly RN-space) is an ordered triple. Here S is a real or complex vector space, Δ is a t -norm which is stronger than t -norm Δ_1 [4, 18, 19] and the mapping $F: S \rightarrow D$ satisfies the following conditions:

$$(2.9) \quad F_p = H \text{ if and only if, } p = 0 \text{ (0 is neutral element of } S).$$

$$(2.10) \quad F_{kp}(x) = F_p(x/|k|) \text{ for every } p \in S, \text{ and every } k \in K/[0] \text{ (} K \text{ is the scalar field) and } x > 0.$$

$$(2.11) \quad \text{For every } p, q \in S \text{ and } x, y \geq 0$$

$$F_{p-r}(x+y) \geq \Delta(F_{p-q}(x), F_{q-r}(y)).$$

If Δ is a strict t -norm with additive generator f , then the Menger triangle inequality (the condition (2.11)) can be rewrite in the following form

$$(2.12) \quad f(F_{p-r}(x+y)) \leq f(F_{p-q}(x) + f(F_{q-r}(y))).$$

Every random normed space is a Menger space, if $F: S \times S \rightarrow D$ is defined by $F(p, q) = F_{p-q}$.

Here are examples of RN-spaces:

1. An ordered pair (S, F) is called an S -space over (\mathfrak{R}, d) (briefly an S -space) if S is a collection of all random variables from (Ω, \mathcal{B}, P) into (\mathfrak{R}, d) such that for every $p \in S$ and every real number t . the set

$$[\omega \in \Omega: |p(\omega)| < t] \in \mathcal{B},$$

and F is the mapping from S into D (the set of all distribution functions) defined by

$$F_p(t) = P[\omega \in \Omega: |p(\omega)| < t]$$

for every real t . As usual, the random variables in S which differ at most on a set of P -measure zero are identified.

Sherwood pointed out that if (S, F) is an S -space over (\mathfrak{R}, d) , then (S, F, Δ_1) is the RN-space.

2. Let $(S, \|\cdot\|)$ be a vector normed space and the norm $\|\cdot\|$ induces a mapping $F_p(x) = H(x - \|p\|)$, $x \in \mathfrak{R}$, than (S, F, Δ_3) is the RN-space [14].

The concept of neighbourhoods in a PM-space was introduced by Schweizer and Sklar [14]. An (ε, λ) -neighbourhood of p , $p \in S$, $\varepsilon > 0$ and $\lambda \in (0,1)$ is defined by

$$U_p(\varepsilon, \lambda) = \{q \in S : F_{p-q}(\varepsilon) > 1 - \lambda\}.$$

If $\sup[\Delta(a, a) : a < 1] = 1$, then a Menger space (S, F, Δ) is a Hausdorff space in the topology τ induced by the family $[U_p(\varepsilon, \lambda) : p \in S, \varepsilon > 0, \lambda \in (0, 1)]$.

Let (S, F, Δ) be a RN-space in the topology τ induced by the family $[U_p(\varepsilon, \lambda) : p \in S, \varepsilon > 0, \lambda > 0]$ of (ε, λ) -neighbourhoods. A sequence $\{S_n\}$ in S converges in τ to a point $s^* \in S$ ($s_n \rightarrow s^*$) if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$ there exists a positive integer $N = N(\varepsilon, \lambda)$ such that $F_{s_n - s^*}(\varepsilon) > 1 - \lambda$ whenever $n \geq N$. A sequence $\{S_n\}$ in S will be called τ -Cauchy sequence in S , if for each $\varepsilon > 0$, $\lambda > 0$ there exists a positive integer $N = N(\varepsilon, \lambda)$ such that $F_{s_n - s_m}(\varepsilon) > 1 - \lambda$ whenever $n, m \geq N$.

A RN-space (S, F, Δ) is called τ -complete, if each τ -Cauchy sequence in S converges in τ to an element in S .

Let (S, F, Δ_1) be a RN-space, where (S, F) is the S -space over (\mathfrak{R}, d) . Then the following facts are equivalent: (see Shin-sen Chang ([18] Prop. 1.)

(2.13) $\{S_n\}$ S converges in τ to a point $s^* \in S$ ($s_n \rightarrow s^*$).

(2.14) $\lim F_{s_n - s^*}(t) = H(t)$ for every $t \geq 0$ (pointwise).

(2.14) $\{S_n\}$ S converges in probability to $s^* \in S$ ($s_n \rightarrow s^*$).

Let (S, F) is an S -space over (\mathfrak{R}, d) . A mapping $T(\cdot, \cdot) : \Omega \times \mathfrak{R} \rightarrow \mathfrak{R}$ is said to be a random operator, if for any $x \in \mathfrak{R}$ $y(\omega) = T(\omega, x)$ is an \mathfrak{R} -valued random variable. A random operator $T(\cdot, \cdot) : \Omega \times \mathfrak{R} \rightarrow \mathfrak{R}$ is called d -continuous, if for each $\omega \in \Omega$, $T(\omega, \cdot)$ is continuous in the topology induced by the metric d .

A random variable $\xi(\omega) \in S$ is called a random fixed point of the random operator $T(\omega, \cdot)$ if for each $\omega \in \Omega$.

$$\xi(\omega) = T(\omega, \xi(\omega)).$$

3. MEIR AND KEELER FIXED POINT THEOREM IN RN-SPACES

Sehgal and Bharucha-Reid [16] initiated the study of contraction mappings on PM-spaces. Bocsan [1] defined contraction mappings on RN-spaces as follows:

Definition 1. A mapping T of RN-space (S, F) into it self will be called B -contraction if and only if there exists a constant k , with $0 < k < 1$ such that for each $p, q \in S$ the following condition holds

$$(*) \quad F_{T_p - T_q}(kx) \geq F_{p-q}(x) \quad \text{for all } x > 0.$$

Recently, Hicks (see also [15-16]) considered another notion of the contraction mapping in PM-spaces. In our paper we shall refer to it as *H-contraction*, and it is obtained by replacing (*) by

$$(**) \quad F_{T_p - T_q}(kx) > 1 - kx \quad \text{whenever} \quad F_{p-q}(x) > 1 - x.$$

Schweizer, Sherwood and Tardiff [15] showed by appropriate examples that the two definitions are independent in RN-spaces.

Theorem 2. ([8] and [15], [16]). Let (S, F, Δ) be a τ -complete Menger-space, where Δ is a continuous *t*-norm satisfying $\Delta(x, x) \geq x$ for each $x \in [0, 1]$. If T is *B-contraction* or *H-contraction* of S into itself, then there is a unique $p \in S$ such that $T_p = p$. Moreover, $T^n q \rightarrow p$ for each $q \in S$.

Now we state the Meir and Keeler theorem in RN-spaces.

Theorem 3. Let (S, F, Δ) be a complete RN-space, where Δ is continuous function and at least as strong as Δ_1 . If T is any mapping of S into itself and the following condition holds:

$$(***) \quad \text{for given } (\varepsilon, \lambda), \text{ there exists } \delta > 0 \text{ such that } F_{p-q}(\varepsilon) \leq 1 - \lambda \text{ and} \\ F_{p-q}(\varepsilon + \delta) > 1 - \lambda \text{ imply } F_{T_p - T_q}(\varepsilon) > 1 - \lambda, \text{ for all } p, q \in S.$$

Then T has a unique fixed point $u \in S$. Moreover, for any $p \in S$ $T^n p \rightarrow u$.

Proof. We observe that (***) condition implies T is strict contraction i.e.

$$(3.1) \quad F_{T_p - T_q}(x) > F_{p-q}(x) \text{ under the condition } p = q, \text{ for all } x > 0.$$

Thus T is continuous in the (ε, λ) -topology and has at most one fixed point.

Lemma 1. If $T: S \rightarrow S$ is a strict contraction in a RN-space (S, F, Δ) and for every $q \in S$, $p_n = T^n q$ is the Cauchy sequence, then T has the unique fixed point u and $T^n q \rightarrow u$.

Proof. Since (S, F, Δ) is complete RN-space, each Cauchy sequence $p_n = T^n q$ has limit $v(q)$. By the continuity of the random metric F_{pq} [18]

$$Tv(q) = T \lim T^n q = \lim T^{n+1} q = v(q).$$

Thus $v(q)$ is a fixed point of T and therefore all $v(q)$ are equal.

Now we will show that (***) condition implies that every sequence $p_n = T^n q$ of iterated mappings is a Cauchy sequence.

*Lemma 2. If the condition (***) holds, then*

$$\lim_{F_{p_n - p_{n+1}}}(x) = H(x) \quad \text{for } x > 0.$$

Proof. Let $c_n = F_{p_n - p_{n+1}}$, i.e. $c_n(x) = F_{p_n - p_{n+1}}(x)$ for $x > 0$. From (3.1), for $x > 0$ $c_n(x)$ is increasing with n . Assume $c_n(x) \rightarrow 1 - \lambda_0$ for some $\lambda_0 \in (0, 1)$. Let $c_n(x) = F_{p_n - p_{n+1}}(x) \leq 1 - \lambda_0$ and $c_n(x + \delta) > 1 - \lambda_0$, for some δ defined in (3.1). Thus $c_{n+1}(x) = F_{p_{n+1} - p_{n+2}}(x) > 1 - \lambda_0$.

Now we suppose that some sequence $p_n = T^n q$ is not a Cauchy sequence. Then there exist (ε, λ) such that for every $M = M(\varepsilon, \delta)$ there exist m and n such that $\limsup_{F_{p_m - p_n}}(2\varepsilon) \leq 1 - \lambda$. By hypothesis (***) there exists $\delta > 0$ such that

$$(3.2) \quad F_{p-q}(\varepsilon) \leq 1 - \lambda \text{ and } F_{p-q}(\varepsilon + \delta) > 1 - \lambda \text{ imply } F_{T^p - T^q}(\varepsilon) > 0.$$

Formula (3.2) remains true with δ replaced by $\delta^- = \max(\delta, \varepsilon)$. From Lemma 2 we can find M such that $c_M = F_{p_M - p_{M+1}}(\delta^-/3) > 1 - \lambda$

$$(3.3) \quad F_{p_m - p_{j+1}}(\varepsilon + \delta^-) \geq \Delta(F_{p_m - p_j}(\varepsilon + 2\delta^-/3), F_{p_j - p_{j+1}}(\delta^-/3))$$

By the definition of RN-space we have

$$(3.4) \quad F_{p_m - p_j}(\varepsilon + \delta^-) \geq F_{p_m - p_j}(\varepsilon + 2\delta^-/3) + F_{p_j - p_{j+1}}(\delta^-/3) - 1.$$

This implies, since $F_{p_m - p_{m+1}}(\varepsilon) \leq 1 - \lambda$ and $F_{p_m - p_n}(\varepsilon + \delta^-) < 1 - \lambda$, that there exists j in $[m, n]$,

$$(3.5) \quad F_{p_m - p_j}(\varepsilon + 2\delta^-/3) < 1 - \lambda \quad \text{and} \quad F_{p_m - p_n}(\varepsilon + \delta^-) < 1 - \lambda$$

On the hand, for all m and j ,

$$F_{p_m - p_j}(\varepsilon + 2\delta^-/3) \geq \Delta(F_{p_m - p_{m+1}}(\delta^-/3), \Delta(F_{p_{m+1} - p_{j+1}}(\varepsilon), F_{(p_{j+1} - p_j)}(\delta^-/3)))$$

Therefore by (3.2), (3.5) and the assumption of our theorem,

$$F_{p_m - p_j}(\varepsilon + 2\delta^-/3) > 1 - \lambda$$

which contradicts (3.5). This contradiction proves that p_n must be a Cauchy sequence and establishes our theorem.

Goebel [6] proved a coincidence theorem which is a generalization of the Banach principle. Park [12] obtained a generalization of Goebel's coincidence theorem which also extends the known fixed point theorem of Meir and Keeler. Now we state a probabilistic version of Park's coincidence theorem. This extends our Theorem 3.

Theorem 4. Let A be a set and (S, F, Δ) be a RN-space such that Δ is a continuous t -norm and Δ is at least as strong as Δ_1 . Let $T, G: A \rightarrow S$ be such that $T[A] \subset G[A]$ and $(G[A], F, \Delta)$ is a complete subspace of the RN-space (S, F, Δ)

Moreover suppose that:

$$(3.6) \quad \text{For a given } (\varepsilon, \lambda) \text{ there exists a } \delta = \delta(\varepsilon, \lambda) \text{ such that for } p, q \in A \\ F_{GpGq}(\varepsilon) \leq 1 - \lambda \text{ and } F_{GpGq}(\varepsilon + \delta) > 1 - \lambda \text{ imply } F_{TpTq}(\varepsilon) > 1 - \lambda$$

$$(3.7) \quad Gp = Gq \text{ implies } Tp = Tq.$$

Then the following facts are true:

$$(3.8) \quad \text{There exists } v \in A \text{ such that } Tv = Gv \text{ and } (T_0G^{-1})^n u \rightarrow Gv \text{ as } n \rightarrow \infty \text{ for} \\ \text{all } u \in G[A].$$

$$(3.9) \quad \text{If } Gu = Gv \text{ (} v \in A \text{), } Gu = Tu = Gv = Tv.$$

$$(3.10) \quad \text{If } Gw = Tw \text{ (} w \in A \text{), then } Gw = Gu.$$

Proof. For each $p \in G[A]$, let $\Phi p = (T_0G^{-1})p$. Then for $q, q' \in \Phi p T[A]$, $G[A]$ there exist $r, r' \in G^{-1}p$ such that $q = Tr$, $q' = Tr'$. Since $Gr = gr'$ using (3.7) we have $q = Tr = Tr' = q'$. Therefore, $\Phi: G[A] \rightarrow G[A]$ is a well-defined map. Now for any $p, q \in G[A]$ and $r \in G^{-1}p$, $s \in G^{-1}q$ if $F_{pq}(\varepsilon) = F_{GrGs}(\varepsilon) \leq 1 - \lambda$ and $F_{pq}(\varepsilon + \delta) = F_{GrGs}(\varepsilon + \delta) > 1 - \delta$, then we have $F_{\Phi p \Phi q}(\varepsilon) = F_{TrTs}(\varepsilon) > 1 - \lambda$ by (3.6). Therefore, Φ satisfies the hypothesis of a fixed point Theorem 3. Hence there exists a unique $a \in G[A]$, we have $\Phi^n p \rightarrow a$ as $n \rightarrow \infty$. Hence, there exists a $u \in G^{-1}a$ such that $Tu = (T_0G^{-1})a = \Phi a = a = Gu$. This shows (3.8). Since $Gu = Gv$ implies $Tu = Tv$ by (3.7) we obtain (3.9); $Gu = Gv = Tu = Tv$. (iii), suppose $G_{GuGv}(\varepsilon) = 1 - \lambda$, that is, $F_{GuGv}(\varepsilon) > 1 - \lambda$, a contradiction.

Remark 1. Condition (3.6), in Theorem 4 can be replaced by any of the following:

$$(3.11) \quad \text{There exists a nondecreasing map } \Phi: [0, \infty) \rightarrow [0, \infty) \text{ such that } \lim \\ \Phi^n(t) = \infty \text{ for } t > 0 \text{ and}$$

$$F_{TpTq}(x) \geq F_{GpGq}(\Phi(x)) \quad \text{for all } x > 0$$

$$(3.12) \quad \text{There exists } k \in [0, 1) \text{ such that}$$

$$F_{TpTq}(kx) \geq F_{GpGq}(x) \quad \text{for all } x > 0.$$

Note that (3.12) implies (3.11) see [2], (3.12) implies (3.6) and that Theorem 4 instead of (3.6) is true and it is probability version of Goebel's coincidence theorem [6].

By setting $A = S$ and $G = I_S$ (the identity map defined on S) theorem 4 with one of the condition: (3.6), (3.11) and (3.12) respectively reduces to a fixed point theorem of Meir and Keeler, Matkowski [2] and Barucha-Reid, respectively.

The following questions: if (3.6) implies (3.11) or if (3.11) implies (3.6) are open problems.

Remark 2. Condition (3.12) from Remark 1 is given as Definition 2.3 in [4] and can be replaced by any of the following: (***) from Theorem 3 or (3.11) from Remark 1.

From Theorem 4, we get the following probabilistic version of Park's theorem [12]:

Corollary 1. Let (S, \mathcal{F}, Δ) be a complete RN-space and where Δ is a continuous t -norm and Δ is at least as strong as Δ_1 . Let Φ be a selfmap of the S , α, β numbers with $|\alpha| \neq |\beta|$, and $T = \alpha I_S + \beta \Phi$ $G = \beta I_S + \alpha \Phi$, where I_S is the identity map. Suppose that $TS \subset GS = S$, and for a given $\varepsilon > 0$, $\lambda \in (0, 1)$, there exists $\delta(\varepsilon) > 0$ such that for $p, q \in S$ $F_{Gp-Gq}(\varepsilon) \leq 1 - \lambda$ and $F_{Gp-Gq}(\varepsilon + \delta) > 1 - \lambda$ implies $F_{Tp-Tq}(\varepsilon) > 1 - \lambda$, and

$$Gp = Gq \quad \text{implies} \quad Tp = Tq.$$

Then Φ has a unique fixed point in S .

Proof. First of all the condition $Tu = Gu$ implies $\Phi(u) = u$. Now we observe the condition (3.7) implies existence of fixed point of Φ and the conditions (3.9), (3.10) imply that such point is a unique.

Also from Theorem 3, we get the following

Corollary 2. (see example 1) Let $(S, \mathcal{F}, \Delta_1)$ a complete RN-space where (S, \mathcal{F}) is a S space over euclidean metric space (\mathcal{R}, d) . suppose that $T(\omega, \cdot): \Omega \times \mathcal{R} \rightarrow \mathcal{R}$ is a d -continuous random operator.

Moreover for given (ε, λ) there exists $\delta > 0$ such that

$$F_{p(\omega)-q(\omega)}(\varepsilon) \leq 1 - \lambda \quad \text{and} \quad F_{p(\omega)-q(\omega)}(\varepsilon + \delta) > 1 - \lambda \quad \text{implies}$$

$$F_{T(\omega, p(\omega))-T(\omega, q(\omega))}(\varepsilon) > 1 - \lambda \quad \text{for all } p(\omega), q(\omega) \in S.$$

then for any $q(\omega) \in S$ The sequence $\{T^n(\omega, q(\omega))\}$ of X -valued random variables converges in the topology τ to the unique random fixed point $\xi(\omega)$ of $t(\omega, \cdot)$, i.e. $T(\omega, \xi(\omega)) = \xi(\omega)$ (We defined the symbol $T^n(\omega, q(\omega))$ in the following way:

$$\begin{aligned}T^0(\omega, q(\omega)) &= q(\omega), \\T^1(\omega, q(\omega)) &= T(\omega, q(\omega)), \\T^2(\omega, q(\omega)) &= T(\omega, T(\omega, q(\omega))) \text{ and so on.}\end{aligned}$$

Proof. We define $\Phi: S \rightarrow S$ by

$$\Phi(p(\omega)) = T(\omega, p(\omega)) \quad \text{for } p(\omega) \in S,$$

and we observe the function Φ satisfies the assumptions of Theorem 3.

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