F A S C I C U L I   M A T H E M A T I C I

Nr 38 2007

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A MODIFIED CORRADO-MILLER IMPLIED VOLATILITY ESTIMATOR

Abstract. The implied volatility, i.e. volatility calculated on the basis of option price is a very important parameter in financial econometrics. Usually, it is calculated from the Black-Scholes option pricing formula, but it doesn't have any analytical solution. There are many ways to find it numerically. Unfortunately, all fast estimators give non rigorous results for deep-in or deep-out-of-the-money options. In this paper there are compared some estimators of implied volatility and there are estimated errors for many cases of option price, strike price and real volatility. Furthermore, to reduce error using least squares surface approximation, a new estimator basing on the Corrado-Miller estimator is constructed. There are shown some cases in which the modified Corrado-Miller estimator gives more exact results.

Key words: Black-Scholes formula, Corrado-Miller estimator, implied volatility, surface approximation.

AMS Mathematics Subject Classification: 41A10, 41A20, 41A30, 62P20.

1. Implied volatility

The volatility is one of the most important notions in financial market. Recall that volatility is often regarded as a measure of the risk in forecasts of returns of financial assets. An interest in volatility is manifested both on theoretical plane, for risk management models, and with practical causes e.g. decrease in risk of investment or achievement of larger incomes.

A special case of volatility is an implied volatility − a certain kind of volatility, determined from option values. In this section the method of determination of implied volatility on the basis of the Black-Scholes pricing formula for the call options will be presented. By the implied volatility of a financial instrument we mean the volatility implied by the market price of a derivative based on a theoretical pricing model. In our case, put and call options are the derivatives and to find value of implied volatility, we use Black-Scholes pricing formula. The implied volatility have a larger value than realized volatility, whenever participants of market forecast that future volatility of stocks are bigger than volatility observed in the past. There is the opposite situation in the case on a waiting lesser market fragility.

In the Black-Scholes model, the value of a European call option on a non-dividend paying stock is stated as

$$C_0^E = S\Phi(d_1) - Ke^{-rT}\Phi(d_2), \tag{1}$$

where

$$d_1 = \frac{\ln\frac{S}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{2}$$

and

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\ln\frac{S}{K} + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}. \tag{3}$$

The stock price, strike price, interest rate, and time to expiration are denoted by S, K, r, T, respectively and Φ is the standard normal distribution. The annualized variance of the continuously compounded return on the underlying stock is denoted by σ^2. The value of a put option is given by the following formula

$$P_0^E = -S\Phi(-d_1) + Ke^{-rT}\Phi(-d_2). \tag{4}$$

Since a closed-form solution for an implied standard deviation from Equation (1) is not possible, the implied volatility must be calculated numerically. One of the most popular algorithms is the Newton-Raphson algorithm. By this algorithm, we have the following formula

$$p_n - p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}.$$

In our case we have

$$\sigma_n = \sigma_{n-1} - \frac{C_0^E(\sigma_{n-1}) - C}{\mathrm{V}_C},$$

where C is known call option value and V_C is a V-sensitivity coefficient for call options that is given by the following formula

$$\mathrm{V}_C = \frac{\partial C_0^E}{\partial \sigma} = Sn(d_1)\sqrt{T}.$$

We can calculate implied volatility from Equation (4) analogously. We have

$$\sigma_n = \sigma_{n-1} - \frac{P_0^E(\sigma_{n-1}) - P}{\mathrm{V}_P},$$

where V_P is a V-sensitivity coefficient for put options and it is equal to V_C.

S. Manaster and G. Koehler proposed also the optimal initial approximation

$$\sigma^2 = \frac{2}{T}\left|\ln\frac{S}{K} + rT\right|$$

for this algorithm. Using this method, we can obtain very good approximation, but the algorithm is quite time consuming because in all iterations we must evaluate two integrals numerically. Moreover, very good precision is not necessary, because all models basing on the Black-Scholes formula have introduced bias into the implied volatility measure. The reason of it is assumption of constant volatility in the Black-Scholes model. Therefore, during last ten years there were constructed many estimators of implied volatility. Most popular are presented below. There are introduced estimators for call options only, because estimators for put options can be obtained by using the following put-call parity

$$C_0^E - P_0^E = S - Ke^{-rT}.$$

To simplify notation, we replace C_0^E by C.

C. Corrado and T. Miller in [2] provided an improved quadratic formula which is valid when stock prices deviate from discounted strike prices. Their formula is given as

$$\sigma_{CM} = \frac{1}{\sqrt{T}}\frac{\sqrt{2\pi}}{S+X}\left[C - \frac{S-X}{2} + \sqrt{\left(C - \frac{S-X}{2}\right)^2 - \frac{(S-X)^2}{\pi}}\,\right], \tag{5}$$

where $X = Ke^{-rT}$.

Another less time-consuming method was introduced by S. Li in [6]. The formulas presented by him approximate implied volatility for all cases with better precision than the Corrado-Miller estimator. It bases on the third order Taylor expansion of standard normal distributions. For at-the-money options, the problem come down to solving 3'rd order polynomial. In other situations they are polynomials of 4'th order. For facilitate very compound calculation he has considered three cases. There were obtained three formulas, which could be reduced to the following two formulas

$$\sigma_{LD} = \frac{\tilde{\alpha} + \sqrt{\tilde{\alpha}^2 - \frac{4(\eta-1)^2}{1+\eta}}}{2\sqrt{T}} \tag{6}$$

for deep in- or out-of-the-money options and

$$\sigma_{LN} = \frac{2\sqrt{2}}{\sqrt{T}}\tilde{z} - \frac{1}{\sqrt{T}}\sqrt{8\tilde{z}^2 - \frac{6\tilde{\alpha}}{\sqrt{2}\tilde{z}}} \tag{7}$$

for nearly at-the-money option options, where

$$\tilde{\alpha} = \frac{\sqrt{2\pi}}{1+\eta}\left[\frac{2C}{S} + \eta - 1\right],$$

$$\tilde{z} = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{3\tilde{\alpha}}{\sqrt{32}}\right)\right]$$

and

$$\eta = \frac{Ke^{-rT}}{S}.$$

M. Kelly in [5] proposed a method based on partial derivatives of $\sigma(C)$ and the 5'th order Taylor expansion of this function. His method gives very good approximation for nearly at-the-money options, even if the initial value is distant from the solution.

Unfortunately, as it was mentioned above, all models basing on the Black-Scholes formula have introduced bias into the implied volatility measure. J. Hull and A. White in [4] found that the magnitude of the bias in models basing on the Black-Scholes formula is the smallest for nearly at-the-money and close-to-maturity options. Therefore in empirical research only nearly at-the-money options should be used to computations.

2. Comparison of results

The following tables list absolute errors of implied volatility for some cases of σ, η and calculated them from formulas (5), (6) and (7). There is assumed 5% risk free-rate. The symbol dash „-" in a cell means that in this case estimator does not give any real result.

Table 1. The absolute estimation errors of implied volatility calculated from the Corrado-Miller formula and the formulas (6) and (7). There is assumed 5% risk free-rate and that the time to expiration is equaled to 1.

σ	0.15			0.3			0.45		
η	(5)	(6)	(7)	(5)	(6)	(7)	(5)	(6)	(7)
0.75	-	0.66520	0.17522	0.02242	0.00907	0.08691	0.01069	0.03464	0.04802
0.8	-	0.72035	0.10636	0.00743	0.00382	0.03977	0.00690	0.00623	0.01278
0.85	0.02818	0.00296	0.05086	0.00302	0.00083	0.00520	0.00509	0.002730	0.01249
0.9	0.00221	0.00025	0.01062	0.00158	0.00060	0.01797	0.00423	0.00359	0.02916
0.95	0.00025	0.00002	0.01324	0.00120	0.00108	0.03100	0.00387	0.00379	0.03844
1	0.00014	0.00014	0.02092	0.00112	0.00112	0.03508	0.00377	0.00377	0.04132
1.05	0.00024	0.00045	0.01397	0.00119	0.00129	0.03139	0.00387	0.00392	0.03871
1.1	0.00151	0.00346	0.00503	0.00147	0.00227	0.02106	0.00414	0.00466	0.03136
1.15	0.00842	0.02043	0.03321	0.00226	0.00504	0.00516	0.00466	0.00639	0.01995
1.20	-	0.71824	0.01995	0.00399	0.01108	0.01539	0.00555	0.00965	0.00508
1.25	-	0.71611	0.10636	0.00743	0.02350	0.03977	0.00690	0.01503	0.01278

Table 2. The absolute estimation errors of implied volatility calculated from the Corrado-Miller formula and the formulas (6) and (7). There is assumed 5% risk free-rate and that the time to expiration is equaled to 0.5.

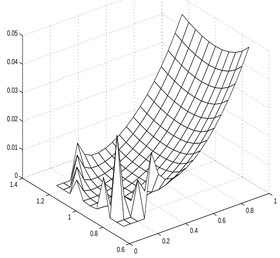
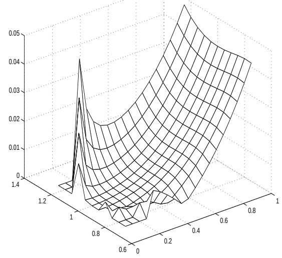
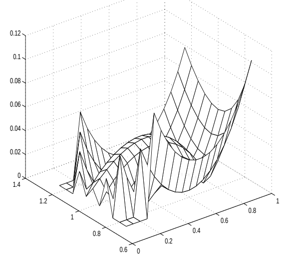
σ	0.15			0.3			0.45		
η	(5)	(6)	(7)	(5)	(6)	(7)	(5)	(6)	(7)
0.75	-	0.71309	0.01278	-	0.01083	0.18440	0.02567	0.01238	0.11410
0.8	-	0.74012	0.20058	0.04051	0.00356	0.10096	0.00962	0.00486	0.04989
0.85	-	0.73679	0.10967	0.00709	0.00225	0.03756	0.00432	0.00073	0.00304
0.9	0.01236	0.00385	0.03832	0.00169	0.00042	0.00620	0.00252	0.00121	0.02829
0.95	0.00046	0.00004	0.00680	0.00067	0.00044	0.03123	0.00199	0.00184	0.04586
1	0.00007	0.00007	0.02186	0.00056	0.00056	0.03914	0.00188	0.00188	0.05136
1.05	0.00039	0.00084	0.00821	0.00065	0.00085	0.03198	0.00198	0.00212	0.04638
1.1	0.00677	0.01297	0.02801	0.00131	0.00302	0.01207	0.00237	0.00344	0.03246
1.15	0.00842	0.73391	0.07905	0.00400	0.01057	0.01811	0.00338	0.00706	0.01098
1.20	-	0.73190	0.13810	0.01181	0.03378	0.05650	0.00551	0.01472	0.01684
1.25	-	0.72844	0.20058	0.04051	0.57844	0.10096	0.00962	0.02977	0.04989

Table 3. The absolute estimation errors of implied volatility calculated from the Corrado-Miller formula and the formulas (6) and (7). There is assumed 5% risk free-rate and that the time to expiration is equaled to 0.1.

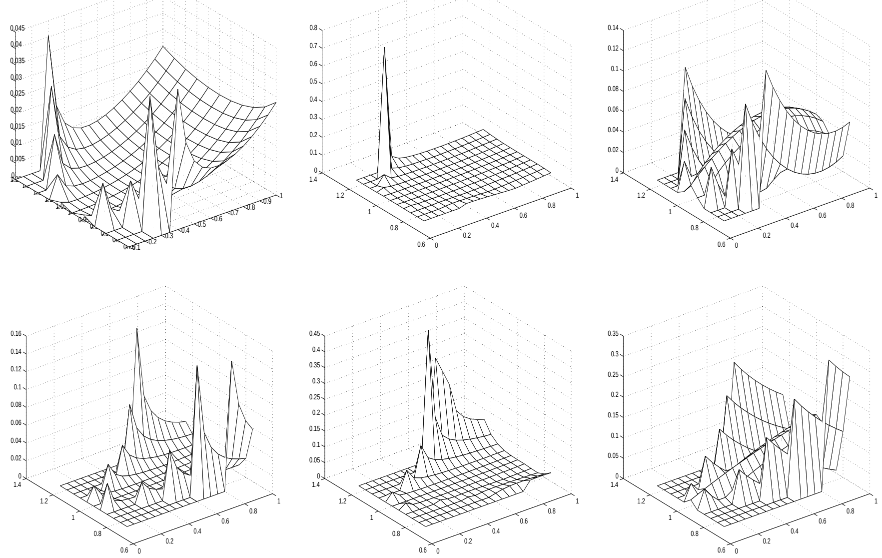
σ	0.15			0.3			0.45		
η	(5)	(6)	(7)	(5)	(6)	(7)	(5)	(6)	(7)
0.75	-	-	0.86845	-	-	0.71868	-	-	0.57701
0.8	-	0.77898	0.62900	-	0.62898	0.48113	-	0.47898	0.35499
0.85	-	0.76291	0.41019	-	0.61291	0.27203	-	0.01859	0.17410
0.9	-	0.75342	0.21041	-	0.02159	0.10314	0.00826	0.00031	0.04156
0.95	0.02737	0.01521	0.045717	0.00145	0.00016	0.00692	0.00079	0.00000	0.03768
1	0.00001	0.00135	0.02306	0.00011	0.00011	0.04418	0.00038	0.00012	0.06327
1.05	0.01571	0.02371	0.03966	0.00111	0.00227	0.01033	0.00072	0.00143	0.04010
1.1	-	0.74644	0.17670	0.02666	0.05610	0.07818	0.00543	0.01221	0.02310
1.15	-	0.74314	0.32961	-	0.59314	0.20027	0.03490	0.09756	0.11611
1.20	-	0.73359	0.48103	-	0.58359	0.33811	-	0.43359	0.22943
1.25	-	0.71211	0.62900	-	0.56211	0.48113	-	0.41211	-0.35499

The following graphs show estimator errors for wider range of σ and η.

Table 4. Graphs of absolute estimation errors of implied volatility for $\eta \in [0.75, 1.25]$, $\sigma \in [0.1, 1]$ **and** $T \in \{0.1, 0.5, 1\}$. **There is assumed** 5% **risk free-rate.**

T	Estimator (5)	Estimator (6)	Estimator (7)
1			

T Estimator (5) Estimator (6) Estimator (7)



0.5

0.1

The above results confirm that all considered estimators give good approximation only for nearly at-the-money options and close-to-maturity options. We see also that for small values of T, the error is also big. Similar result is obtained in [3]. In practice, e.g. in Polish stock market, there are concluded few option contracts, therefore to find implied volatility we have to use also deep-in or deep-out-of-the-money options. In the next paragraph there is presented a modified Corrado-Miller estimator which has relatively small errors for deep-in or deep-out-of-the-money options with long time to maturity.

3. A modified Corrado-Miller estimator

As we have noticed, the Corrado-Miller estimator gives usually too small values of implied volatility for deep-in or deep-out-of-the-money options. The values are also understate for big value of volatility. Therefore, an ε-function is added to the estimator. By the ε-function we mean an approximated function of absolute error. We can assume that σ_{CM} depends on two variables

$$x = \frac{1}{\sqrt{T}}\frac{\sqrt{2\pi}}{S+X} \tag{8}$$

$$\text{and} \quad y = \left[C - \frac{S-X}{2} + \sqrt{\left(C - \frac{S-X}{2}\right)^2 - \frac{(S-X)^2}{\pi}}\right],$$

where $X = Ke^{-rT}$. To determine the function $\varepsilon(x, y)$, we use the discrete set of values of function $\varepsilon_d(x_j, y_i)$ being the accurate absolute error function. Their values we can get as a difference between the estimator σ_{CM} and the implied volatility σ calculated for some set of pairs (x_i, y_j) from the Newton-Raphson algorithm. Now, the function $\varepsilon(x, y)$ can be determined by using the least-squares surface approximation. To this end we apply ZunZun program to find the best surface fit for our data. Since a relatively simple function $\varepsilon(x, y)$.with small number of parameters is expected, we decide to take into account the following families of ε-functions.

I. Polynomial Equations:
- A) Simplified Linear: $z = a + bx + cy$
- B) Simplified Quadratic: $z = a + bx + cy + dx^2 + ey^2$
- C) Simplified Cubic: $z = a + bx + cy + dx^2 + ey^2 + fx^3 + gy^3$

II. Taylor Equations:
- A) $z = a + bx + cy + dx^2 + ey^2 + fxy$
- B) $z = a + b/x + cy + d/x^2 + ey^2 + fy/x$
- C) $z = a + bx + c/y + dx^2 + e/y^2 + fx/y$
- D) $z = a + b/x + c/y + d/x^2 + e/y^2 + f/(xy)$

III. Rational Equations:
- A) $z = (a + bx + cy)/(1 + dx + ey)$
- B) $z = (a + b\ln(x) + c\ln(y))/(1 + dx + ey)$
- C) $z = (a + bx + cy)/(1 + d\ln(x) + e\ln(y))$
- D) $z = (a + b\ln(x) + c\ln(y))/(1 + \ln(x) + eln(y))$
- E) $z = (a + bx + cy)/(1 + dx + ey) + f$
- F) $z = (a + b\ln(x) + c\ln(y))/(1 + dx + ey) + f$
- G) $z = (a + bx + cy)/(1 + d\ln(x) + e\ln(y)) + f$
- H) $z = (a + b\ln(x) + c\ln(y))/(1 + d\ln(x) + e\ln(y)) + f$

IV. Power Equations:
- A) $z = a(x^b + y^c)$
- B) $z = a + x^b + y^c$
- C) $z = a + x^b y^c$
- D) $z = ax^b + cy^d$
- E) $z = a(x^b + y^c) + d$
- F) $z = ax^b + cy^d + e$

V. Sigmoid Equations:
- A) Sigmoid: $z = a/((1 + \exp(b - cx))(1 + \exp(d - ey)))$
- B) Sigmoid With Offset: $z = a/((1 + \exp(b - cx))(1 + \exp(d - ey))) + f$

VI. Optical Equations:
- A) Sag For Asphere 0: $z = \left(s^2/r\right) / \left(1 + \sqrt{1 - (k+1)(s/r)^2}\right)$, where $s^2 = x^2 + y^2$

B) Sag For Asphere 0 Borisovsky: $z = \left(s^2/r\right) / \left(1 + \sqrt{1-(k+1)(s/r)^2}\right)$, where $s^2 = (x-a)^2 + (y-b)^2$.

The lowest SSQ of absolute error for 375 points is equal to 0.00179143 and it is found for equation $2(b)$ with parameters:
$a =$ 4.62627532e-01,
$b =$ -1.16851917e-02,
$c =$ 9.63541838e-04,
$d =$ 7.53502261e-05,
$e =$ 1.42451646e-05,
$f =$ -2.10237683e-05.
Therefore a modified Corrado-Miller estimator is stated as

$$(9)\quad \sigma_{MCM} = \frac{1}{\sqrt{T}}\frac{\sqrt{2\pi}}{S+X}\left[C - \frac{S-X}{2} + \sqrt{\left(C - \frac{S-X}{2}\right)^2 - \frac{(S-X)^2}{\pi}}\right] + a + \frac{b}{x} + cy + \frac{d}{x^2} + ey^2 + f\frac{y}{x}.$$

where $X = Ke^{-rT}$, x, y are defined by the formulas (8) and values of parammeters are presented above.

4. Error of the modified Corrado-Miller estimator and conclusions

The following tables list absolute errors of implied volatility calculated from the formula (9). There is assumed 5% risk free-rate. The symbol dash-" in a cell means that in this case estimator does not give any real result.

Table 6. The absolute estimation errors of implied volatility calculated from the modified Corrado-Miller estimator. There is assumed 5% risk free-rate and time to expiration T $\in\{0.1, 0.5, 1\}$.

σ	0.15			0.3			0.45		
η	T=1	T=0.5	T=0.1	T=1	T=0.5	T=0.1	T=1	T=0.5	T=0.1
0.75	-	-	-	0.00043	-	-	0.00015	0.02154	-
0.8	-	-	-	0.00042	0.00141	-	0.00622	0.01984	-
0.85	0.00193	-	-	0.00402	0.01733	-	0.01072	0.01237	-
0.9	0.01626	0.03479	-	0.00761	0.01215	-	0.01324	0.00628	0.03095
0.95	0.01235	0.03439	0.00986	0.00944	0.00770	0.02035	0.01406	0.00283	0.01516
1	0.01086	0.03013	0.01606	0.00952	0.00609	0.01269	0.01353	0.00202	0.00933
1.05	0.01230	0.03085	0.01263	0.00824	0.01925	0.01936	0.01198	0.00328	0.03687
1.1	0.01491	0.02974	-	0.00605	0.00954	0.01352	0.00978	0.00590	0.02865
1.15	0.01359	-	-	0.00357	0.01151	-	0.00722	0.00897	0.01651
1.20	-	-	-	0.00143	0.00948	-	0.00468	0.01155	-
1.25	-	-	-	0.00059	0.01263	-	0.00239	0.01224	-

For better comparison we present tables with Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE) of implied volatility calculated from formulas (5). (6), (7) and (9) for η from some given intervals and for $\sigma \in [0.1, 1]$, and $T = 1$. There is assumed 5% risk free rate.

Table 7. Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE) of implied volatility calculated from formulas (5). (6), (7) and (9) for η from given intervals and for $\sigma \in [0.1, 1]$, and $T = 1$.

	(5)		(6)		(7)		(9)	
η	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE
[0.75, 0.8)	0.03430	0.02147	0.01969	0.01426	0.09221	0.04937	0.00707	0.00290
[0.8, 0.85)	0.02674	0.01725	0.01781	0.01348	0.04890	0.02394	0.00459	0.00196
[0.85, 0.9)	0.02604	0.01484	0.01662	0.01253	0.04578	0.01918	0.00686	0.00184
[0.9, 0.95)	0.02061	0.01266	0.01651	0.01196	0.05388	0.02248	0.00541	0.00110
[0.95, 1)	0.01560	0.01186	0.01536	0.01181	0.06511	0.02705	0.00515	0.00094
(1, 1.05]	0.01559	0.01185	0.01579	0.01190	0.06586	0.02720	0.00593	0.00126
(1.05, 1.1]	0.01836	0.01237	0.02157	0.01294	0.05337	0.02303	0.00469	0.00115
(1.1, 1.15]	0.02056	0.01375	0.02717	0.01548	0.04156	0.01949	0.00240	0.00083
(1.15, 1.2]	0.02715	0.01575	0.18710	0.04175	0.04482	0.01897	0.00466	0.00115
(1.2, 1.25]	0.02811	0.01764	0.15584	0.04641	0.05263	0.02518	0.00351	0.00126
[0.75, 1.25]	0.022555	0.01460	0.01187	0.01856	0.05702	0.02549	0.00505	0.00140

Table 8. Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE) of implied volatility calculated from formulas (5). (6), (7) and (9) for η from given intervals and for $\sigma \in [0.1, 1]$, and $T = 0.5$.

	(5)		(6)		(7)		(9)	
η	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE
[0.75, 0.8)	0.03318	0.02003	0.01254	0.00798	0.11018	0.06100	0.09453	0.06256
[0.8, 0.85)	0.02441	0.01393	0.00978	0.00687	0.05761	0.02406	0.10289	0.06180
[0.85, 0.9)	0.01643	0.00955	0.00852	0.00646	0.04730	0.02215	0.11288	0.0595
[0.9, 0.95)	0.01262	0.00734	0.00860	0.00628	0.06379	0.03317	0.12258	0.05540
[0.95, 1)	0.00839	0.00608	0.00786	0.00597	0.08557	0.04213	0.13277	0.05100
(1, 1.05]	0.00828	0.00606	0.00877	0.00616	0.08604	0.04238	0.11330	0.04390
(1.05, 1.1]	0.01341	0.00709	0.01904	0.00822	0.06987	0.03445	0.08982	0.03875
(1.1, 1.15]	0.01511	0.00871	0.02688	0.01233	0.05142	0.02574	0.06416	0.03389
(1.15, 1.2]	0.02015	0.01120	0.12239	0.03943	0.04762	0.01938	0.04693	0.02821
(1.2, 1.25]	0.02565	0.01443	0.19053	0.06854	0.06162	0.02536	0.03212	0.02211
[0.75, 1.25]	0.00706	0.00978	0.03744	0.02005	0.06974	0.03368	0.00300	0.00141

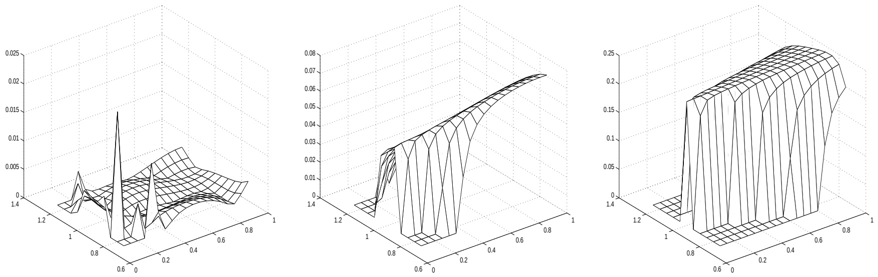
Table 9. Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE) of implied volatility calculated from formulas (5). (6), (7) and (9) for η from given intervals and for $\sigma \in [0.1, 1]$, and $T = 0.1$.

	(5)		(6)		(7)		(9)	
η	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE
[0.75, 0.8)	0.08781	0.07573	0.025337	0.022669	0.282240	0.24903	0.01940	0.17828
[0.8, 0.85)	0.05110	0.03733	0.011080	0.008993	0.180943	0.13780	0.26149	0.21096
[0.85, 0.9)	0.02741	0.01568	0.004914	0.003025	0.095038	0.05491	0.32852	0.22625
[0.9, 0.95)	0.01194	0.00543	0.003425	0.001575	0.071510	0.04242	0.40312	0.23033
[0.95, 1)	0.00867	0.00244	0.005132	0.001694	0.106737	0.06068	0.56486	0.22692
(1, 1.05]	0.00622	0.00205	0.008901	0.002654	0.106262	0.06117	0.54939	0.22014
(1.05, 1.1]	0.01101	0.00482	0.021477	0.009173	0.074208	0.04485	0.38482	0.21405
(1.1, 1.15]	0.02192	0.01138	0.110307	0.047473	0.076917	0.04145	0.31197	0.20406
(1.15, 1.2]	0.03281	0.02130	0.146392	0.087993	0.123252	0.08122	0.25635	0.19077
(1.2, 1.25]	0.05844	0.04225	0.196866	0.141562	0.196592	0.14975	0.20359	0.16605
[0.75, 1.25]	0.02207	0.01387	0.041509	0.024001	0.054209	0.07436	0.40155	0.21295

In the following graphs there are presented absolute errors values of estimators (9) for η and σ intervals taken as above.

Table 10. Graphs of absolute errors of implied volatility calculated by using the modified Corrado-Miller estimator for $\eta \in [0.75, 1.25]$, $\sigma \in [0.1, 1]$ and $T \in \{0.1, 0.5, 1\}$. There is assumed 5% risk free-rate.

$T = 1$ $T = 0.5$ $T = 0.1$



The formula (9) gives the smallest mean of relative errors and mean absolute errors for big T. The smallest value of T, the highest value of absolute error. The modified form of this estimator can be used for options with long time to maturity. The reason of that situation is a big fluctuation of error for a small difference between arguments. A good approximation of error function in such a case is possible only by using high order polynomials or high order Taylor equations. In such situation we would minimize values of functions of several dozen variables. Obviously, that procedure is extremely time consuming. It is the reason why we have not chosen that way.

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Received on 01.08.2006 and, in revised form, on 09.02.2007.