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THE DOUBLE SEQUENCE SPACE Γ^2

Abstract. Let Γ^2 denote the space of all prime sense double entire sequences and Λ^2 the space of all prime sense double analytic sequences. This paper is devoted to the general properties of Γ^2.

Key words: entire sequence, analytic sequence, double sequence, dual.

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1. Introduction

Throughout w, Γ and Λ denote the classes of all, entire and analytic scalar valued single sequences respectively.

We write w^2 for the set of all complex sequences (x_{mn}), where $m, n \in N$, the set of positive integers. Then w^2 is a linear space under the coordinatewise addition and scalar multiplication.

Some initial works on double sequence space is found in Bromwich[2]. Later on it was investigated by Hardy [3], Moricz [4], Moricz and Rhoades [5], Basarir and Solancan [1], Tripathy [6], Colak and Turkmenoglu [7] and many others.

We need the following inequality in the sequel of the paper.

For $a, b \geq 0$ and $0 < p < 1$, we have

$$(a+b)^p \leq a^p + b^p. \tag{1}$$

The double series $\sum\limits_{m,n=1}^{\infty} x_{mn}$ is convergent if and only if the double sequence (S_{mn}) is convergent, where $S_{mn} = \sum\limits_{i,j=1}^{m,n} x_{ij}$ $(m, n = 1, 2, 3, \ldots)$(see [9]).

A sequence $x = (x_{mn})$ is said to be double analytic if $\sup\limits_{m,n} |x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences will be denoted by Λ^2. A sequence $x = (x_{mn})$ is called double entire sequence if $|x_{mn}|^{\frac{1}{m+n}} \to 0$, as $m, n \to \infty$. The double entire sequences will be denoted by Γ^2. Let

ϕ = {all finite sequences}. Consider a double sequence $x = (x_{ij})$. The $(m,n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{\infty} x_{ij}\delta_{ij}$, for all $m, n \in N$.

$$\delta_{mn} = \begin{pmatrix} 0, & 0, & ...0, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \\ \vdots & & & & \\ 0, & 0, & ...1, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \end{pmatrix}$$

with 1 in the $(m,n)^{th}$ position and zero otherwise. An FK-space (or a metric space) X is said to have AK property if (δ_{mn}) is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$. An FDK-space is a double sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings $x = (x_k) \to (x_{mn})$ $(m, n \in N)$ are also continuous. If X is a sequence space, we give the following definitions:

(i) $X' =$ the continuous dual of X;

(ii) $X^{\alpha} = \{a = (a_{mn}) : \sum_{m,n=1}^{\infty} |a_{mn}x_{mn}| < \infty$, for each $x \in X$;

(iii) $X^{\beta} = \{a = (a_{mn}) : \sum_{m,n=1}^{\infty} a_{mn}x_{mn}$, is convergent, for each $x \in X$;

(iv) $X^{\gamma} = \{a = (a_{mn}) : \sup_{m,n\geq 1}|\sum_{m,n=1}^{\infty} a_{mn}x_{mn}| < \infty$, for each $x \in X$;

(v) let X be an FK-space $\supset \phi$, then $X^f = \{f(\delta_{mn}) : f \in X\}$;

(vi) $X^{\wedge} = \{a = (a_{mn}); \sup_{m,n} |a_{mn}x_{mn}|^{1/m+n} < \infty,$ for each $x \in X\}$;

X^{α}, X^{β}, X^{γ} are called $\alpha-$(or Kothe-Toeplitz) dual of X, β-(generalized- Kothe-Toeplitz) dual of X, γ-dual of X and $\wedge$-dual of X respectively.

2. Definitions and preliminaries

Let w^2 denote the set of all complex double sequences. A sequence $x = (x_{mn})$ is said to be analytic if $\sup_{(m,n)} |x_{mn}|^{1/m+n} < \infty$. The vector space of all prime sense double analytic sequences will be denoted by Λ^2. A sequence $x = (x_{mn})$ is called prime sense double entire sequence if $|x_{mn}|^{1/m+n} \to 0$ as $m, n \to \infty$.The double entire sequences will be denoted by Γ^2. The spaces $\wedge^2$ and Γ^2 are metric spaces with the metric

$$d(x,y) = \sup_{m,n} \{|x_{mn} - y_{mn}|^{1/m+n} : m, n = 1, 2, 3...\} \tag{2}$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in Γ^2.

Proposition 1. Γ^2 *has monotone metric.*

Proof. We know that

$$d(x,y) = \sup_{m,n}\left\{|x_{mn}-y_{mn}|^{1/m+n} : m,n = 1,2,3,...\right\}$$

$$\begin{aligned} d(x^n,y^n) &= \sup_{n,n}\left\{|x_{nn}-y_{nn}|^{1/2n}\right\} \quad \text{and} \\ d(x^m,y^m) &= \sup_{m,m}\left\{|x_{mm}-y_{mm}|^{1/2m}\right\}. \end{aligned}$$

Let $m > n$. Then $\sup_{m,m}\left\{|x_{mm}-y_{mm}|^{1/2m}\right\} \geq \sup_{n,n}\left\{|x_{nn}-y_{nn}|^{1/2n}\right\}$

$$d(x^m,y^m) \geq d(x^n,y^n), \quad m > n \tag{3}$$

Also $\{d(x^n,x^n) : n = 1,2,3,\ldots\}$ is monotonically increasing bounded by $d(x,y)$.

For such a sequence

$$\sup_{n,n}\left\{|x^{nn}-y^{nn}|^{1/2n}\right\} = \lim_{n\to\infty} d(x^n,y^n) = d(x,y) \tag{4}$$

From(3) and (4) it follows that $d(x,y) = \sup_{m,n}\left\{|x_{mn}-y_{mn}|^{1/m+n}\right\}$ is a monotone metric for Γ^2. This completes the proof. ■

Proposition 2. *The dual space of* Γ^2 *is* Λ^2. *In other words* $(\Gamma^2)^* = \Lambda^2$.

Proof. The proof is easy, so omitted. ■

Proposition 3. Γ^2 *is separable.*

Proof. The proof is easy, so omitted. ■

Proposition 4. Λ^2 *is not separable.*

Proof. Since $|x_{mn}|^{1/m+n} \to 0$ as $m,n \to \infty$, so it may so happen that first row or column may not be convergent, even may not be bounded. Let S be the set that has double sequences such that the first row is built up of sequences of zeros and ones. Then S will be uncountable. Consider open balls of radius 3^{-1} units. Then these open balls will not cover Λ^2.

Hence Λ^2 is not separable. This completes the proof. ■

Proposition 5. Γ^2 *is not reflexive.*

Proof. Γ^2 is separable by Proposition 3. But $(\Gamma^2)^* = \Lambda^2$, by Proposition 2. Since Λ^2 is not separable, by Proposition 4. Therefore Γ^2 is not reflexive. This completes the proof. ■

Proposition 6. Γ^2 *is not an inner product space and hence not a Hilbert space.*

Proof. Let us take

$$x = (x_{mn}) = \begin{pmatrix} 1 & 1/2 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \vdots & & & & \end{pmatrix} \text{ and } y = (y_{mn}) = \begin{pmatrix} 1 & -1/2 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \vdots & & & & \end{pmatrix}$$

$$\begin{aligned} d(x,\theta) &= \sup \begin{pmatrix} |x_{11}-0|^{1/2}, & |x_{12}-0|^{1/3}, & \ldots \\ |x_{21}-0|^{1/2}, & |x_{22}-0|^{1/4}, & \ldots \\ \vdots & & \end{pmatrix} \\ &= \sup \begin{pmatrix} |1-0|^{1/2}, & |1/2-0|^{1/3}, & \ldots \\ 0, & 0, & \ldots \\ \vdots & & \end{pmatrix} \\ &= \sup \begin{pmatrix} |1|^{1/2}, & |1/2|^{1/3}, & 0, & \ldots \\ 0, & 0, & 0, & \ldots \\ \vdots & & & \end{pmatrix} \end{aligned}$$

Here and later on in the paper sup will represent the supremum of the elements inside the matrix. We get $d(x,\theta) = 1$.

Similarly $d(y,\theta) = 1$. Hence $d(x,\theta) = d(y,\theta) = 1$

$$\begin{aligned} x+y &= \begin{pmatrix} 1 & 1/2 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \vdots & & & & \\ 0 & 0 & 0 & 0 & \ldots \end{pmatrix} + \begin{pmatrix} 1 & -1/2 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \vdots & & & & \\ 0 & 0 & 0 & 0 & \ldots \end{pmatrix} \\ &= \begin{pmatrix} 2 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \vdots & & & & \\ 0 & 0 & 0 & 0 & \ldots \end{pmatrix} \end{aligned}$$

$$d(x+y, x-y) = \sup\{(|x_{mn}+y_{mn}| - |x_{mn}-y_{mn}|)^{1/m+n} : m,n = 1,2,3,\ldots\}.$$

$$d(x_{mn}+y_{mn},\theta) = \sup\begin{pmatrix}|x_{11}+y_{11}|^{1/2}, & |x_{12}+y_{12}|^{1/3}, & \dots \\ \vdots & & \end{pmatrix}$$
$$= \sup\begin{pmatrix}|1+1|^{1/2}, & |1/2-1/2|^{1/3}, & \dots \\ \vdots & & \end{pmatrix}$$
$$= \sup\begin{pmatrix}|2|^{1/2}, & 0, & \dots \\ 0, & 0, & \dots \\ \vdots & & \end{pmatrix} = \sup\begin{pmatrix}1.414, & 0, & \dots \\ 0, & 0, & \dots \\ \vdots & & \end{pmatrix} = 1.414$$

Therefore $d(x+y,\theta) = 1.414$. Similarly $d(x-y,\theta) = 1$.

By parallelogram law, $[d(x+y,\theta)]^2 + [d(x-y,\theta)]^2 = 2[(d(x,\theta))^2 + (d(\theta,y))^2]$.

$\Longrightarrow (1.414)^2 + 1^2 = 2[1^2+1^2] \Longrightarrow 2.999396 = 4.$

Hence the parallelogram law is not satisfied. Therefore Γ^2 is not an inner product space. Assume that Γ^2 is a Hilbert space. But then Γ^2 would satisfy reflexivity condition. [Theorem 4.6.6 [10]]. Proposition 5, Γ^2 is not reflexive. Thus Γ^2 is not a Hilbert space. This completes the proof. ∎

Proposition 7. Γ^2 *is not rotund.*

Proof. Let us take $x = (x_{mn})$ and $y = (y_{mn})$ defined by

$$x_{mn} = \begin{pmatrix}1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots & & & & \end{pmatrix} \quad \text{and} \quad y_{mn} = \begin{pmatrix}1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \dots & & & & \end{pmatrix}$$

Then $x = (x_{mn})$ and $y = (y_{mn})$ are in Γ^2.

Also

$$d(x,y)$$
$$= \sup\begin{pmatrix}|x_{11}-y_{11}|^{\frac{1}{2}}, & |x_{12}-y_{12}|^{\frac{1}{3}}, & ..., & |x_{1n}-y_{1n}|^{\frac{1}{1+n}}, & 0, ... \\ \vdots & & & & \\ |x_{m1}-y_{m1}|^{\frac{1}{m+1}}, & |x_{m2}-y_{m2}|^{\frac{1}{m+2}}, & ..., & |x_{mn}-y_{mn}|^{\frac{1}{m+1}}, & 0, ... \\ 0, & ..., & ..., & & 0, ... \end{pmatrix}$$

Therefore

$$d(x,\theta) = \sup\begin{pmatrix}1, & 0, & 0, & 0, & \dots \\ 0, & 0, & 0, & 0, & \dots \\ \vdots & & & & \\ 0, & 0, & 0, & 0, & \dots \end{pmatrix}$$

$d(\theta,y) = 1$. Obviously $x = (x_{mn}) \neq y = (y_{mn})$.

But

$$(x_{mn}) + (y_{mn}) = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \dots \end{pmatrix}$$

$$d\left(\frac{x_{mn}+y_{mn}}{2}, \theta\right)$$
$$= \sup \begin{pmatrix} \left(\frac{|x_{11}+y_{11}|}{2}\right)^{\frac{1}{2}}, & \left(\frac{|x_{12}+y_{12}|}{2}\right)^{\frac{1}{3}}, & ..., & \left(\frac{|x_{1n}+y_{1n}|}{2}\right)^{\frac{1}{n+1}}, & 0, \; 0, \; ... \\ \vdots \\ \left(\frac{|x_{m1}+y_{m1}|}{2}\right)^{\frac{1}{m+1}}, & \left(\frac{|x_{m2}+y_{m2}|}{2}\right)^{\frac{1}{m+2}}, & ..., & \left(\frac{|x_{mn}+y_{mn}|}{2}\right)^{\frac{1}{m+1}}, & 0, \; 0, ... \\ ..., & ..., & ..., & ..., & ..., ..., ... \end{pmatrix}$$

$$d(\frac{x_{mn}+y_{mn}}{2}, \theta) = \sup \begin{pmatrix} 1, & 0, & 0, & 0, & \dots \\ 0, & 0, & 0, & 0, & \dots \\ \vdots \end{pmatrix} = 1.$$

Therefore Γ^2 is not rotund. This completes the proof. ∎

Proposition 8. *Weak convergence and strong convergence are equivalent in* Γ^2.

Proof. Step 1. Always strong convergence implies weak convergence.

Step 2. So it is enough to show that weakly convergence implies strongly convergence in Γ^2.

$y^{(\eta)}$ tends to y weakly in Γ^2, where $(y_{mn}^{(\eta)}) = y^{(\eta)}$ and $y = (y_{mn})$. Take any $x = (x_{mn}) \in \Gamma^2$ and

$$f(z) = \sum_{m,n=1}^{\infty} |z_{mn}x_{mn}|^{1/m+n}, \quad \text{for each } z = (z_{mn}) \in \Gamma^2. \tag{5}$$

Then $f \in (\Gamma^2)^*$ by Proposition 2. By hypothesis $f(y^{(\eta)}) \to f(y)$ as $\eta \to \infty$.

$$f(y^{(\eta)} - y) \to 0, \quad \text{as } \eta \to \infty. \tag{6}$$

$\Rightarrow \sum_{m,n=1}^{\infty} (|y_{mn}^{(\eta)} - y_{mn}|^{1/m+n} |x_{mn}|^{1/m+n}) \to 0$, as $\eta \to \infty$. By using (5) and (6) we get

Since $x = (x_{mn}) \in \Lambda^2$ we have $\sum_{m,n=1}^{\infty} |x_{mn}|^{1/m+n} < \infty$, for all $x \in \Lambda^2$.

$\Rightarrow \sum_{m,n=1}^{\infty} (|y_{mn}^{(\eta)} - y_{mn}|^{1/m+n}) \to 0$ as $\eta \to \infty$.

$\Rightarrow \sup_{mn}(|(y_{mn}^{(\eta)} - y_{mn}), 0|^{1/m+n}) \to 0$ as $\eta \to \infty$.

$\Rightarrow \sup_{mn}(|(y_{mn}^{(\eta)} - y_{mn})|^{1/m+n}) \to 0$, as $\eta \to \infty$.

$\Rightarrow d((y^{(\eta)} - y), 0) \to 0, \quad \text{as} \quad \eta \to \infty.$

$\Rightarrow d((y^{(\eta)} - y) \to 0$, as $\eta \to \infty$.

This completes the proof. ■

Proposition 9. *We shall construct an infinite matrix A for which $\Gamma_A^2 = \Gamma^2$.*

Example. Consider the matrix

$$\begin{pmatrix} y_{11} & y_{12} & \dots & y_{1n} & 0 & 0 & \dots \\ y_{21} & y_{22} & \dots & y_{2n} & 0 & 0 & \dots \\ \vdots & & & & & & \\ y_{m1} & y_{m2} & \dots & y_{mn} & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots \\ \vdots & & & & & & \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & \dots \\ 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & & & \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} & 0 & 0 & \dots \\ x_{21} & x_{22} & \dots & x_{2n} & 0 & 0 & \dots \\ \vdots & & & & & & \\ x_{m1} & x_{m2} & \dots & x_{mn} & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots \\ \vdots & & & & & & \end{pmatrix}$$

$$\begin{pmatrix} y_{11} & y_{12} & \dots & y_{1n} & 0 & 0 & \dots \\ y_{21} & y_{22} & \dots & y_{2n} & 0 & 0 & \dots \\ \vdots \\ y_{m1} & y_{m2} & \dots & y_{mn} & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots \\ \vdots \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} & 0 & 0 & \dots \\ x_{11} & x_{12} & \dots & x_{1n} & 0 & 0 & \dots \\ x_{21} & x_{22} & \dots & x_{2n} & 0 & 0 & \dots \\ x_{21} & x_{22} & \dots & x_{2n} & 0 & 0 & \dots \\ x_{21} & x_{22} & \dots & x_{2n} & 0 & 0 & \dots \\ x_{21} & x_{22} & \dots & x_{2n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ x_{31} & x_{32} & \dots & x_{3n} & 0 & 0 & \dots \\ \vdots \end{pmatrix}$$

$$y_{11}, y_{12}, \dots, y_{1n} = x_{11}, x_{12}, \dots, x_{1n}$$

$$y_{21}, y_{22}, \dots, y_{2n} = x_{11}, x_{12}, \dots, x_{1n}$$

$$y_{31}, y_{32}, \dots, y_{3n} = x_{21}, x_{22}, \dots, x_{2n}$$

$$y_{41}, y_{42}, \dots, y_{4n} = x_{21}, x_{22}, \dots, x_{2n}$$

$$y_{51}, y_{52}, \dots, y_{5n} = x_{21}, x_{22}, \dots, x_{2n}$$

$$y_{61}, y_{62}, \dots, y_{6n} = x_{21}, x_{22}, \dots, x_{2n}$$

$$\vdots$$

and so on. For any $x = (x_{mn}) \in \Gamma^2$. $|(Ax)_{mn}| = \lim_{m,n\to\infty} |\sum x_{mn}|^{1/m+n} \leq d(x, 0)$, where metric defined on Γ^2 is given by

$$[d(x,\theta)]_{\Gamma^2_A} \leq [d(x,\theta)]_{\Gamma^2} \tag{7}$$

Conversely. Given $x \in [d(x,\theta)]_{\Gamma^2_A}$ fix any m, n then, $\lim_{m,n\to\infty} |x_{mn}|^{1/m+n} \leq (Ax)_{mn} \Rightarrow \lim_{m,n\to\infty} |x_{mn}|^{1/m+n} \leq [d(x,\theta)]_{\Gamma^2_A} \Rightarrow$

$$[d(x,\theta)]_{\Gamma^2} \leq [d(x,\theta)]_{\Gamma^2_A} \tag{8}$$

Therefore the matrix $A = (x_{mnlk})$ for which the summability field $[d(x,\theta)]_{\Gamma^2} = [d(x,\theta)]_{\Gamma^2_A}$ is given by

$$A = \begin{pmatrix} 1 & 0 & 0 & ... \\ 1 & 0 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ \vdots & & & \end{pmatrix}$$

```
//Program for generalization:

#include ⟨iostream.h⟩
#include ⟨conio.h⟩
#include ⟨math.h⟩
#include ⟨fstream.h⟩

void main()
{
clrscr() ;
int m,n,i,nn=0,j,count=1,k,1pp,abc;
ofstream fout,fout 1;
fout.open("aa1.txt");
fout1.open("aa2.txt");
cout << "enter the value of m:";
cin>> m;
for(i=1;i<=m;i++)
{
nn=nn+pow(2,i);
}
while(count<=nn)
{
cout<< " – ";
fout<< " – ";
for(abc=1;abc<=m+3;abc++)
```

```
{
cout<< " ";
fout<< " ";
}
cout<< " − \n";
fout<< " − "\n;
for(j = 1; j <= m; j + +)
{
for(k=1;k<=pow(2,j);k++)
{
for(pp=1;pp<=2;pp++)
{
fout1<< "Y" << count << "," << pp << " ";
}
fout1<< ".....Y" << count << ", n = ";
cout<< " | ";
fout<< " | ";
for(int q=1;q<=m+1;q++)
{
if(q==j)
{
cout<< "1";
fout<< "1";
}
else
{
cout<< "0";
fout<< "0";
}
}
for(l=1;l<=2;l++)
{
foutl<< "X" << "j" << "," <<l<< "";
}
fout1<< ".....X" << j << "n";
cout<< "... | \n";
fout<< "... | \n";
fout1<< "... | \n";
count++;
}
}
}
```

```
cout<< " · \n · \n · \n";
fout<< " · \n · \n · \n";
cout<< " | –";
fout<< " | –";
for(abc=1;abc<<=m+1;abc++)
{
cout<< " ";
fout<< " ";
}
cout<< "– | ";
fout<< "– | ";
fout1<< ".\n.\n";
fout.close();
fout1.close();
getch();
}
```

SAMPLE INPUT/OUTPUT

enter the value of m : 3

$$\begin{pmatrix} 1 & 0 & 0 & ... \\ 1 & 0 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ \vdots & & & \end{pmatrix}$$

$Y_{1,1}, Y_{1,2}, ..., Y_{1,n} = X_{1,1}, X_{1,2}, ..., X_{1,n}$

$X_{2,1}, X_{2,2}, ..., X_{2,n} = X_{1,1}, X_{1,2}, ..., X_{1,n}$

$Y_{3,1}, Y_{3,2}, ..., Y_{3,n} = X_{2,1}, X_{2,2}, ..., X_{2,n}$

$$Y_{4,1}, Y_{4,2}, ..., Y_{4,n} = X_{2,1}, X_{2,2}, ..., X_{2,n}$$

$$Y_{5,1}, Y_{5,2}, ..., Y_{5,n} = X_{2,1}, X_{2,2}, ..., X_{2,n}$$

$$Y_{6,1}, Y_{6,2}, ..., Y_{6,n} = X_{2,1}, X_{2,2}, ..., X_{2,n}$$

$$Y_{7,1}, Y_{7,2}, ..., Y_{7,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{8,1}, Y_{8,2}, ..., Y_{8,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{9,1}, Y_{9,2}, ..., Y_{9,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{10,1}, Y_{10,2}, ..., Y_{10,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{11,1}, Y_{11,2}, ..., Y_{11,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{12,1}, Y_{12,2}, ..., Y_{12,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{13,1}, Y_{13,2}, ..., Y_{13,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

$$Y_{14,1}, Y_{14,2}, ..., Y_{14,n} = X_{3,1}, X_{3,2}, ..., X_{3,n}$$

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