F A S C I C U L I M A T H E M A T I C I

Nr 42 2009

Ishak Altun and Duran Turkoglu

SOME FIXED POINT THEOREMS FOR MAPPINGS SATISFYING CONTRACTIVE CONDITION OF INTEGRAL TYPE ON d-COMPLETE TOPOLOGICAL SPACES

Abstract. In this paper, we prove two fixed point theorems for mappings satisfying contractive condition of integral type on d-complete Hausdorff topological spaces.

Key words: fixed points, d-complete topological spaces contractive condition of integral type.

AMS Mathematics Subject Classification: 54H25, 47H10.

1. Introduction

Branciari [5] obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. The authors in [2], [3], [4], [13], [14] and [15] proved some fixed point theorems involving more general contractive conditions. Recently ([6]) some fixed point theorems have been proved in non-metric setting wherein the distance function used need not satisfying triangle inequality. The purpose of this paper is to investigate some new result of fixed points in non-metric settings. In the sequel, we use contractive condition of integral type on d-complete Hausdorff topological spaces.

Let (X, τ) be a topological space and $d : X \times X \to [0, \infty)$ be such that $d(x, y) = 0$ if and only if $x = y$. Then X is said to be d-complete if $\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty$ implies that the sequence $\{x_n\}$ is convergent in X. A mapping $T : X \to X$ is w-continuous at x if $x_n \to x$ implies $Tx_n \to Tx$. For details on d-complete topological spaces, we refer to Iseki [7] and Kasahara [9]-[11].

In the sequel, we shall use the following:

A symmetric function on a set X is a real valued d on $X \times X$ such that for all $x, y \in X$

(i) $d(x, y) \geq 0$, and $d(x, y) = 0$ if and only if $x = y$,

(ii) $d(x, y) = d(y, x)$.

Let d be a symmetric function on a set X, and for any $\varepsilon > 0$ and any $x \in X$, let $S(x,\varepsilon) = \{y \in X : d(x,y) < \varepsilon\}$. From [6], we can define a topology τ_d on X by $U \in \tau_d$ if and only if for each $x \in U$, some $S(x,\varepsilon) \subset U$. A symmetric function d is a semi-metric if for each $x \in X$ and for each $\varepsilon > 0$, $S(x,\varepsilon)$ is a neighborhood of x in the topology τ_d. A topological space X is said to be symmetrizable (resp. semi-metrizable) if its topology is induced by a symmetric function (resp. semi-metric) on X. The d-complete symmetrizable spaces form an important class of d-complete topological spaces. Other examples of d-complete topological spaces may be found in Hicks and Rhoades [6].

Hicks and Rhoades [6] proved the following theorem.

Theorem 1. *Let (X,τ) be a Hausdorff d-complete topological space and f, h be w-continuous self mappings on X satisfying*

$$d(hx,hy) \leq G(M^*(x,y))$$

for $x, y \in X$, where

$$M^*(x,y) = \max\{d(fx,fy), d(fx,hx), d(fy,hy)\}$$

and G is a real-valued function satisfying the following:
(a) $0 < G(y) < y$ for each $y > 0$; $G(0) = 0$,
(b) $g(y) = \frac{y}{y-G(y)}$ is a non-increasing function on $(0,\infty)$,
(c) $\int_0^{y_1} g(y)dy < \infty$ for each $y_1 > 0$,
(d) $G(y)$ is non-decreasing.
Suppose also that
(i) f and h commute,
(ii) $h(X) \subseteq f(X)$.
Then f and h have a unique common fixed point in X.

2. Main result

Now, we give our main theorems.

Theorem 2. *Let f be self-mapping of a Hausdorff d-complete topological space (X,τ) satisfying the following*

$$\int_0^{d(fx,fy)} \varphi(t)dt \leq G\left(\int_0^{M(x,y)} \varphi(t)dt\right) \tag{1}$$

for all $x, y \in X$, where $\varphi : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable on each compact subset of R^+, non-negative and such that

$$\varepsilon \leq \int_0^{\varepsilon} \varphi(t)dt \quad \text{for each} \quad \varepsilon > 0, \tag{2}$$

$$M(x,y) = \max\{d(x,y), d(x,fx), d(y,fy)\} \tag{3}$$

and G is real valued function satisfying the condition (a)-(d).

Then f has a unique fixed point in X.

Proof. Let $x \in X$ and, for brevity, define $x_n = f^n x$. For each integer $n \geq 1$, from (1)

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq G\left(\int_0^{M(x_{n-1},x_n)} \varphi(t)dt\right). \tag{4}$$

Using (3),

$$M(x_{n-1},x_n) = \max\{d(x_{n-1},x_n), d(x_n,x_{n+1})\}.$$

Substituting into (4), one obtains

$$\begin{aligned} \int_0^{d(x_n,x_{n+1})} \varphi(t)dt \ &\leq \ G\left(\int_0^{\max\{d(x_{n-1},x_n),d(x_n,x_{n+1})\}} \varphi(t)dt\right) \\ &= \ G\left(\max\left\{\int_0^{d(x_{n-1},x_n)} \varphi(t)dt, \int_0^{d(x_n,x_{n+1})} \varphi(t)dt\right\}\right). \end{aligned} \tag{5}$$

If $\int_0^{d(x_{n-1},x_n)} \varphi(t)dt \leq \int_0^{d(x_n,x_{n+1})} \varphi(t)dt$, then from (5) we have

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq G\left(\int_0^{d(x_n,x_{n+1})} \varphi(t)dt\right) < \int_0^{d(x_n,x_{n+1})} \varphi(t)dt,$$

which is a contradiction. Thus $\int_0^{d(x_{n-1},x_n)} \varphi(t)dt > \int_0^{d(x_n,x_{n+1})} \varphi(t)dt$ and so from (5)

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq G\left(\int_0^{d(x_{n-1},x_n)} \varphi(t)dt\right) \quad \text{for} \quad n \geq 1. \tag{6}$$

Next we define a sequence $\{S_n\}$ of real numbers by $S_{n+1} = G(S_n)$ with $S_1 = \int_0^{d(x,fx)} \varphi(t)dt > 0$. By (a), we then have $0 < S_{n+1} < S_n < S_1$, $n \geq 1$.

Moreover, by (b) and (c), the series $\sum_{n=1}^{\infty} S_n$ converges (see [1]). We shall show that $\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq S_{n+1}$, $n \geq 1$. From (6), we have $\int_0^{d(x_1,x_2)} \varphi(t)dt \leq G\left(\int_0^{d(x,fx)} \varphi(t)dt\right) = G(S_1) = S_2$ and the desired inequality is valid for $n = 1$. So, assume that it is true for some $n > 1$. From (6) again, we have $\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq G\left(\int_0^{d(x_{n-1},x_n)} \varphi(t)dt\right) \leq G(S_n) = S_{n+1}$.

Since $\sum_{n=1}^{\infty} S_n$ is convergent, it follows that $\sum_{n=1}^{\infty} \int_0^{d(x_n,x_{n+1})} \varphi(t)dt$ is convergent too. From (2) the series $\sum_{n=1}^{\infty} d(x_n, x_{n+1})$ converges.

Again, since X is d-complete $\{x_n\}$ converges to some $z \in X$.

From (1),

$$\begin{aligned}\int_0^{d(fz,x_{n+1})} \varphi(t)dt &\leq G\left(\int_0^{M(z,x_n)} \varphi(t)dt\right)\\ &= G\left(\max\left\{\int_0^{d(z,x_n)} \varphi(t)dt, \int_0^{d(z,fz)} \varphi(t)dt,\right.\right.\\ &\qquad\left.\left.\int_0^{d(x_n,x_{n+1})} \varphi(t)dt\right\}\right).\end{aligned}$$

Taking the limit as $n \to \infty$, one obtains

$$\int_0^{d(fz,z)} \varphi(t)dt \leq G\left(\int_0^{d(z,fz)} \varphi(t)dt\right),$$

which implies that $\int_0^{d(fz,z)} \varphi(t)dt = 0$ which from (2) implies that $d(z, fz) = 0$ or $z = fz$.

Suppose that z and w are fixed points of f. Then from (1),

$$\int_0^{d(z,w)} \varphi(t)dt = \int_0^{d(fz,fw)} \varphi(t)dt \leq G\left(\int_0^{d(z,w)} \varphi(t)dt\right)$$

which implies that $\int_0^{d(z,w)} \varphi(t)dt = 0$, which from (2), implies $d(z, w) = 0$ or $z = w$ and the fixed point is unique. ■

Theorem 3. *Let (X, τ) be Hausdorff d-complete topological space, f, h w-continuous self-mappings of X satisfying*

$$\int_0^{d(hx,hy)} \varphi(t)dt \leq G\left(\int_0^{M^*(x,y)} \varphi(t)dt\right) \tag{7}$$

for all $x, y \in X$, where φ and G are as in Theorem 2 and

$$M^*(x, y) = \max\{d(fx, fy), d(fx, hx), d(fy, hy)\}.$$

Suppose also that
(i) f and h commute,
(ii) $h(X) \subseteq f(X)$.
Then f and h have a unique common fixed point in X.

Proof. Let $x \in X$ and define $T_1 = \int_0^{d(fx_0,hx_0)} \varphi(t)dt$. If $T_1 = 0$, then

$$\int_0^{d(hhx_0,hx_0)} \varphi(t)dt \leq G\left(\int_0^{M^*(hx_0,x_0)} \varphi(t)dt\right),$$

where

$$M^*(hx_0,x_0) = \max\{d(fhx_0,fx_0), d(fhx_0,hhx_0), d(fx_0,hx_0)\}.$$

Since f and h commute and $fx_0 = hx_0$, $d(fhx_0,fx_0) = 0$. Therefore $M^*(hx_0,x_0) = d(hhx_0,hx_0)$ and $M^*(hx_0,x_0)$ must be zero. For, otherwise we have

$$\begin{aligned}\int_0^{d(hhx_0,hx_0)} \varphi(t)dt &\leq G\left(\int_0^{M^*(hx_0,x_0)} \varphi(t)dt\right)\\ &= G\left(\int_0^{d(hhx_0,hx_0)} \varphi(t)dt\right) < \int_0^{d(hhx_0,hx_0)} \varphi(t)dt\end{aligned}$$

a contradiction. Thus $M^*(hx_0,x_0) = 0$ and hx_0 is a fixed point of h. But then $fhx_0 = hfx_0 = hhx_0 = hx_0$ and hx_0 is also a fixed point of f.

Suppose that $T_1 > 0$. By (ii) there exists an $x_1 \in X$ such that $fx_1 = hx_0$. In general define $\{x_n\} \subset X$ so that $fx_n = hx_{n-1}$ for $n \geq 1$.

Without loss of generality we may assume that $fx_n \neq hx_n$ for each n. For, if $fx_n = hx_n$ for some n, the above argument, with x_0 replaced with x_n, yields fx_n as a common fixed point of f and h.

Define $\{T_n\}$ by $T_{n+1} = G(T_n)$, with $T_1 = \int_0^{d(fx_0,hx_0)} \varphi(t)dt > 0$. By (a), $0 < T_{n+1} < T_n < T_1$, $n \geq 1$.

Moreover, by (b) and (c) the series $\sum_{n=1}^{\infty} T_n$ converges. We shall show that $\int_0^{d(hx_{n-1},hx_n)} \varphi(t)dt \leq T_n, n \geq 1$.

For $n = 1$, we have

$$\int_0^{d(hx_0,hx_1)} \varphi(t)dt \leq G\left(\int_0^{M^*(x_0,x_1)} \varphi(t)dt\right),$$

where

$$\begin{aligned}M^*(x_0,x_1) &= \max\{d(fx_0,fx_1), d(fx_0,hx_0), d(fx_1,hx_1)\}\\ &= \max\{d(fx_0,hx_0), d(hx_0,hx_1)\}.\end{aligned}$$

If $M^*(x_0,x_1) = d(hx_0,hx_1)$, then

$$\begin{aligned}\int_0^{d(hx_0,hx_1)} \varphi(t)dt &\leq G\left(\int_0^{M^*(x_0,x_1)} \varphi(t)dt\right)\\ &< \int_0^{d(hx_0,hx_1)} \varphi(t)dt,\end{aligned}$$

a contradiction. Thus $M^*(x_0, x_1) = d(fx_0, hx_0)$, and the desired inequality is valid for $n = 1$, in fact

$$\int_0^{d(hx_0,hx_1)} \varphi(t)dt \leq G\left(\int_0^{d(fx_0,hx_0)} \varphi(t)dt\right) = G(T_1) < T_1.$$

Assume that it is true for some $n > 1$. Then

$$\int_0^{d(hx_n,hx_{n+1})} \varphi(t)dt \leq G\left(\int_0^{M^*(x_n,x_{n+1})} \varphi(t)dt\right),$$

where

$$M^*(x_n, x_{n+1}) = \max\{d(hx_{n-1}, hx_n), d(hx_n, hx_{n+1})\}.$$

By assumption, $M^*(x_n, x_{n+1}) \neq 0$ for each n. If $M^*(x_n, x_{n+1}) = d(hx_n, hx_{n+1})$, then we get

$$\begin{aligned}\int_0^{d(hx_n,hx_{n+1})} \varphi(t)dt &\leq G\left(\int_0^{M^*(x_n,x_{n+1})} \varphi(t)dt\right)\\ &< \int_0^{d(hx_n,hx_{n+1})} \varphi(t)dt,\end{aligned}$$

a contradiction. Therefore, $M^*(x_n, x_{n+1}) = d(hx_{n-1}, hx_n)$ and

$$\begin{aligned}\int_0^{d(hx_n,hx_{n+1})} \varphi(t)dt &\leq G\left(\int_0^{M^*(x_n,x_{n+1})} \varphi(t)dt\right)\\ &= G\left(\int_0^{d(hx_{n-1},hx_n)} \varphi(t)dt\right)\\ &\leq G(T_n) = T_{n+1}.\end{aligned}$$

Since $\sum_{n=1}^{\infty} T_n$ is convergent, it follows that $\sum_{n=1}^{\infty} \int_0^{d(hx_n,hx_{n+1})} \varphi(t)dt$ is convergent too. Therefore the series $\sum_{n=1}^{\infty} d(hx_n, hx_{n+1})$ converges.

Now X is d-complete so $\{hx_n\}$ converges to some $z \in X$. Then w-continuity of f implies that $fhx_n \to fz$. Since f and h commute, and h is w-continuous, $fhx_n = hfx_n = hhx_{n-1} \to hz$. Since X is Hausdorff, $hz = fz$. Again using (7),

$$\int_0^{d(hhz,hz)} \varphi(t)dt \leq G\left(\int_0^{M^*(hz,z)} \varphi(t)dt\right)$$

and

$$M^*(hz, z) = d(fhz, hz) = d(hfz, hz) = d(hhz, hz),$$

since $hz = fz$ and h and f commute. If $hz \neq hhz$, then we obtain the contradiction

$$\int_0^{d(hhz,hz)} \varphi(t)dt \;\le\; G\left(\int_0^{M^*(hz,z)} \varphi(t)dt\right) \\ < \int_0^{d(hhz,hz)} \varphi(t)dt.$$

Thus hz is a fixed point of h. Since $fhz = hfz = hhz = hz$, hz is also fixed point of f. The uniqueness of the common fixed point can be easily shown using (7). ■

Remark 1. If $\varphi(t) = 1$ in Theorem 3, we have Theorem 1.

Remark 2. If we take a complete metric space instead of Hausdorff d-complete topological space in Theorems 2 and 3, we have the following theorems. Note that the condition (2) has been weakened in these theorems, but we have changed the conditions of the function G.

We need the following lemma for the proofs of these theorems.

Lemma ([12])**.** *Let $G : R^+ \to R^+$ be right continuous function such that $G(t) < t$ for every $t > 0$, then $\lim_{n\to\infty} G^n(t) = 0$.*

Theorem 4. *Let f be self-mapping of a complete metric space (X, d) satisfying the following*

$$\int_0^{d(fx,fy)} \varphi(t)dt \le G\left(\int_0^{M(x,y)} \varphi(t)dt\right)$$

for all $x, y \in X$, where $\varphi : R^+ \to R^+$ is a Lebesgue integrable mapping which is summable on each compact subset of R^+, non-negative and such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \ \text{ for each } \ \varepsilon > 0, \tag{8}$$

$$M(x, y) = \max\{d(x, y), d(x, fx), d(y, fy)\}$$

and $G : R^+ \to R^+$ is a right continuous and nondecreasing function such that $G(0) = 0$, and $G(t) < t$ for each $t > 0$.

Then f has a unique fixed point in X.

Proof. Let $x \in X$ and define $x_n = f^n x$. As in the proof of Theorem 2, we can obtain

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \le G\left(\int_0^{d(x_{n-1},x_n)} \varphi(t)dt\right) \quad \text{for } \ n \ge 1. \tag{9}$$

Now, from (9), we have

$$\begin{aligned}
\int_0^{d(x_n,x_{n+1})}\varphi(t)dt &\leq G\left(\int_0^{d(x_{n-1},x_n)}\varphi(t)dt\right)\\
&\leq G^2\left(\int_0^{d(x_{n-2},x_{n-1})}\varphi(t)dt\right)\\
&\vdots\\
&\leq G^n\left(\int_0^{d(x_0,x_1)}\varphi(t)dt\right),
\end{aligned}$$

and, taking the limit as $n \to \infty$ and using Lemma, we have

$$\lim_{n\to\infty}\int_0^{d(x_n,x_{n+1})}\varphi(t)dt \leq \lim_{n\to\infty} G^n\left(\int_0^{d(x_0,x_1)}\varphi(t)dt\right) = 0,$$

which from (8), implies that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{10}$$

We now show that $\{x_n\}$ is a Cauchy sequence. Suppose that it is not. Then there exists an $\varepsilon > 0$ and subsequences $\{m(k)\}$ and $\{n(k)\}$ such that $m(k) < n(k) < m(k+1)$ with

$$d(x_{m(k)}, x_{n(k)}) \geq \varepsilon,\ \ d(x_{m(k)}, x_{n(k)-1}) < \varepsilon. \tag{11}$$

Now from (10), we have

$$\lim_{k\to\infty}\int_0^{d(x_{m(k)-1},x_{m(k)})}\varphi(t)dt = \lim_{k\to\infty}\int_0^{d(x_{n(k)-1},x_{n(k)})}\varphi(t)dt = 0. \tag{12}$$

On the other hand, using the triangular inequality and (11), we have

$$\begin{aligned}
d(x_{m(k)-1}, x_{n(k)-1}) &\leq d(x_{m(k)-1}, x_{m(k)}) + d(x_{m(k)}, x_{n(k)-1})\\
&< d(x_{m(k)-1}, x_{m(k)}) + \varepsilon.
\end{aligned} \tag{13}$$

Hence,

$$\begin{aligned}
\int_0^{\varepsilon}\varphi(t)dt &\leq \int_0^{d(x_{m(k)},x_{n(k)})}\varphi(t)dt\\
&= \int_0^{d(fx_{m(k)-1},fx_{n(k)-1})}\varphi(t)dt
\end{aligned} \tag{14}$$

$$\leq G\left(\int_0^{M(x_{m(k)-1},x_{n(k)-1})}\varphi(t)dt\right)$$

$$\leq G\left(\int_0^{\max\{d(x_{m(k)-1},x_{n(k)-1}),d(x_{m(k)-1},x_{m(k)}),d(x_{n(k)-1},x_{n(k)})\}}\varphi(t)dt\right)$$

$$\leq G\left(\int_0^{\max\{d(x_{m(k)-1},x_{m(k)})+\varepsilon,d(x_{m(k)-1},x_{m(k)}),d(x_{n(k)-1},x_{n(k)})\}}\varphi(t)dt\right).$$

Using (11), (12), (13) and (14), we have

$$\int_0^{\varepsilon}\varphi(t)dt \leq \int_0^{d(x_{m(k)},x_{n(k)})}\varphi(t)dt \leq G\left(\int_0^{\varepsilon}\varphi(t)dt\right),$$

which is a contradiction. Therefore $\{x_n\}$ is Cauchy. Since X is complete $\{x_n\}$ converges to some $z \in X$. Therefore we can complete the proof as in the proof of Theorem 2. ■

We can prove the following theorem using the proofs of Theorem 3 and Theorem 4.

Theorem 5. *Let (X,d) be complete metric space, f,h continuous self-mappings of X satisfying*

$$\int_0^{d(hx,hy)}\varphi(t)dt \leq G\left(\int_0^{M^*(x,y)}\varphi(t)dt\right) \tag{15}$$

for all $x,y \in X$, where φ and G are as in Theorem 4 and

$$M^*(x,y) = \max\{d(fx,fy), d(fx,hx), d(fy,hy)\}.$$

Suppose also that
(i) f and h commute,
(ii) $h(X) \subseteq f(X)$.
Then f and h have a unique fixed point in X.

Remark 3. If $\varphi(t) = 1$ in Theorem 5, we have a generalization of main theorem of [8].

Example. Let $X = \{\frac{1}{n} : n = 2,3,...\} \cup \{0\}$ with the metric induced by $d(x,y) = |x-y|$, thus since X is a closed subset of it is a complete metric space. We consider now two mappings $h, f : X \to X$ defined by

$$hx = \begin{cases} \frac{1}{n+1}, & x = \frac{1}{n} \\ 0, & x = 0 \end{cases} \quad \text{and} \quad fx = x.$$

It is obvious that f and h commute and $h(X) \subseteq f(X)$. Then h and f satisfies (7) with $\varphi : [0,\infty) \to [0,\infty)$

$$\varphi(t) = \begin{cases} \frac{1+\ln 2}{4}, & t > \frac{1}{2} \\ t^{\frac{1}{t}-2}[1-\ln t], & 0 < t \leq \frac{1}{2} \\ 0, & t = 0 \end{cases}$$

and $G(s) = \frac{s}{2}$. In this context one has, if $0 < t \leq \frac{1}{2}$, $\int_0^t \varphi(s)ds = t^{\frac{1}{t}}$ so that, since $\sup\{d(x,y) : x,y \in X\} = \dfrac{1}{2}$, (15) for $x \neq y$ is equivalent to:

$$d(hx,hy)^{\frac{1}{d(hx,hy)}} \leq G\left(M^*(x,y)^{\frac{1}{M^*(x,y)}}\right) = \frac{1}{2}M^*(x,y)^{\frac{1}{M^*(x,y)}}. \tag{16}$$

Since $d(x,y) \leq M^*(x,y)$ and $\int_0^t \varphi(s)ds = t^{\frac{1}{t}}$ is non-decreasing, we show sufficiently that

$$d(hx,hy)^{\frac{1}{d(hx,hy)}} \leq G\left(d(x,y)^{\frac{1}{d(x,y)}}\right) = \frac{1}{2}d(x,y)^{\frac{1}{d(x,y)}} \tag{17}$$

instead of (16). Using [5, Example 3.6] we can show the condition (17) is satisfied. Thus h and f satisfies (15). Therefore the Theorem 5 is applicable in this example.

But, since

$$\sup_{\{x,y\in X: x\neq y\}} \frac{d(hx,hy)}{M^*(x,y)} \geq 1,$$

then there is not any constant $k \in (0,1)$ such that $d(hx,hy) \leq kM^*(x,y)$. Thus the main theorem of [8] is not applicable in this example.

Acknowledgement. The authors thank the referees for their valuable comments and suggestions.

References

[1] ALTMAN M., An integral test for series and generalized contractions, *Amer. Math. Montly,* 82(1975), 827-829.

[2] ALTUN I., TURKOGLU D., A fixed point theorem on general topological spaces with a τ-distance, *Indian J. Math.,* 50(1)(2008), 139-148.

[3] ALTUN I., TURKOGLU D., Some fixed point theorems for weakly compatible mappings satisfying an implicit relation, *Taiwanese J. Math.,* 13(4)(2009), 1291-1304.

[4] ALTUN I., TURKOGLU D., RHOADES B.E., Fixed points of weakly compatible maps satisfying a general contractive condition of integral type, *Fixed Point Theory Appl.,* 2007), Article ID 17301, 9 pages, doi:10.1155/2007/17301.

[5] Branciari A., A fixed point theorem for mappings satisfying a general contractive condition of integral type, *Int. J. Math. Math. Sci.,* 29(9)(2002), 531-536.
[6] Hicks T.L., Rhoades B.E., Fixed point theorems for d-complete topological spaces II, *Math. Japonica,* 37(5)(1992), 847-853.
[7] Iseki K., An approach to fixed point theorems, *Math. Seminar Notes, Kobe Univ.,* 3(1975), 193-202.
[8] Jungck G., Commuting mappings and fixed points, *Amer. Math. Monthly,* 83(4)(1976), 261–263.
[9] Kasahara S., Fixed point iterations in L-space, *Math. Seminar Notes, Kobe Univ.,* 4(1976), 205-210.
[10] Kasahara S., On some generalizations of Banach contraction theorem, *Math. Seminar Notes, Kobe Univ.,* 39(1975), 161-169.
[11] Kasahara S., Some fixed point and coincidence theorems in L-space, *Math. Seminar Notes, Kobe Univ.,* 3(1976), 161-169.
[12] Matkowski J., Fixed point theorems for mappings with a contractive iterate at a point, *Proceedings of the American Mathematical Society,* 62(2)(1977), 344-348.
[13] Rhoades B.E., Two fixed point theorems for mappings satisfying a general contractive condition of integral type, *Int. J. Math. Math. Sci.,* 63(2003), 4007-4013.
[14] Turkoglu D., Altun I., A common fixed point theorem for weakly compatible mappings in symetric spaces satisfying an implicit relation, *Bol. Soc. Mat. Mexicana,* 13(1)(2007), 195-205.
[15] Vijayaraju P., Rhoades B.E., Mohanraj R., A fixed point theorem for a pair of maps satisfying a general contractive condition of integral type, *Int. J. Math. Math. Sci.,* 15(2005), 2359-2364.

Ishak Altun
Department of Mathematics
Faculty of Science and Arts
Kirikkale University
71450-Yahsihan, Kirikkale, Turkey
e-mail: ialtun@kku.edu.tr *or* ishakaltun@yahoo.com

Duran Turkoglu
Department of Mathematics
Faculty of Science and Arts
Gazi University
06500-Teknikokullar, Ankara, Turkey
e-mail: dturkoglu@gazi.edu.tr

Received on 15.02.2008 and, in revised form, on 12.02.2009.