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A BITOPOLOGICAL VIEW OF γ-OPEN SETS

Abstract. The aim of this paper is to introduce the γ-open sets in bitopological spaces and utilize them to define and characterize $(i,j)\gamma$-T_0 spaces and a class of continuous mappings.

Key words: $(i,j)\gamma$-open sets, $(i,j)\gamma$-$cl(A)$, $(i,j)\gamma d(A)$, $(i,j)\gamma-$ $ker(A)$, $(i,j)\gamma sl(A)$, $(i,j)\gamma - T_0$ and p.γ -continuity.

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1. Introduction

In 1982, Mashhour et al. [15] introduced the notion of preopen sets, also called locally dense sets by Corson and Michael [6]. The class of preopen sets properly contains the class of open sets. As the intersection of two preopen sets may fail to be preopen, the class of preopen sets does not always form a topology. In a submaximal space i.e. a topological space X in which every dense subset is open, collection of all preopen sets form a topology. Indeed, many notions in Topology can be defined in terms of preopen sets (see [4], [7], [9], [16] and [20]). In 1987, Andrijevic [3] offered a new class of open sets called γ-open sets by utilizing preopen sets. Recently, Abd El Monsef et al. [1] have applied preopen sets in connection with the topological applications of rough set measures in information systems. Moreover, it has been shown in [8] that the notion preopen sets is important with respect to the digital topology.

Andrijevic [3] proved that the family τ_γ of γ-sets in a topological space contains the family τ_α of α-sets [18]. In a bitopological space, the class of α-sets [18] (resp. preopen sets [15], semiopen sets [13] and semi-preopen sets [2] are generalized in the form of $(i,j)\alpha$-sets [14](resp.(i,j)preopen sets [11], (i,j)semi-open sets [5], and (i,j)semi-preopen sets [12].

In this paper, we introduce the $(i,j)\gamma$-open sets and observe that the family of $(i,j)\gamma$ -open sets is a topology. This concept of $(i,j)\gamma$ -open sets is used to define a space called $(i,j)\gamma$-T_0. A characterization of this space is offered and the pairwise γ-continuity is introduced and studied.

We recall some definitions and concepts which are useful in the following sections.

2. Preliminaries

Definition 1. *A subset A of a topological space* (X, τ) *is called a* γ*-set [3] if* $A \cap S \in PO(X)$ *for each* $S \in PO(X)$.

In the above definition, $PO(X)$ *is the family of all preopen sets in* X. *The family of all* γ*-sets in* X *is denoted by* $\gamma O(X)$.

Definition 2. *A subset A of a bitopological space* (X, τ_1, τ_2) *is called a*

(i) $(i,j)\alpha$*-open [14] if* $A \subset \tau_i int(\tau_j cl(\tau_i int(A)))$.
(ii) (i,j) *semiopen [5] if* $A \subset \tau_j cl(\tau_i int(A))$.
(iii) (i,j)*preopen [11] if* $A \subset \tau_i int(\tau_j cl(A))$.
(iv) (i,j) *semi-preopen [12] if* $A \subset \tau_j cl(\tau_i int(\tau_j cl(A)))$.

In all the above definitions $i, j = 1, 2$ and $i \neq j$. $\tau_i int(A)$ (resp. $\tau_j cl(A)$) represents the interior of A with respect to τ_i (resp. the closure of A with respect of τ_j).

Definition 3. *A mapping* $f:(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called*

(i) *Pairwise continuous [19] if the induced functions* $f:(X, \tau_1) \to (Y, \sigma_1)$ *and* $f:(X, \tau_2) \to (Y, \sigma_2)$ *are both continuous.*
(ii) (i,j)*-pre continuous [11] iff the inverse image of each* σ_i*-open set of* Y *is* (i,j)*preopen in* X, $i, j = 1, 2$; $i \neq j$.
(iii) (i,j)*-semi continuous [5] iff the inverse image of each* σ_i*-open set of* Y *is* (i,j)*semiopen in* $X, i, j = 1, 2$; $i \neq j$.
(iv) *Pairwise semi-pre continuous [12] if the inverse image of each* σ_i*-openset of* Y *is* (i,j) *semi-preopen in* $X, i, j = 1, 2$; $i \neq j$.

3. $(i,j)\gamma$-open sets

In the following sections by a space X, we mean a bitopological space (X, τ_1, τ_2).

Definition 4. *A subset A of X is called* $(i,j)\gamma$*-open, if* $A \cap B$ *is* (i,j) *preopen for every* (i,j) *preopen set* B *in* X.

We denote the family of $(i,j)\gamma$-open sets in X by $(i,j)\gamma O(X)$.

Example 1. Let $X = \{a, b, c\}, \tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. $(2,1)\gamma O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$.

Proposition 1. *Every $(i,j)\gamma$-open set is (i,j) preopen. But the converse is not true, in general. In Example 1, $\{b,c\}$ is $(2,1)$preopen but not $(2,1)\gamma$-open.*

Remark 1. In general, $(1,2)\gamma O(X)$ and $(2,1)\gamma O(X)$ are not equal. In Example 1, $(1,2)\gamma O(X) \neq (2,1)\gamma O(X)$.

Definition 5. *A subset A of X is called a pairwise γ-open set if A is $(1,2)\gamma$-open and $(2,1)\gamma$-open.*

Theorem 1. *If $A \subset X$ is $(i,j)\gamma$-open and $O \in \tau_1 \cap \tau_2$, then $O \cap A$ is a $(i,j)\gamma$-open set.*

Proof. Let $O \in \tau_1 \cap \tau_2$. If A is $(i,j)\gamma$-open, then for every $B \in (i,j)PO(X)$, $A \cap B$ is in $(i,j)PO(X)$. Therefore, there exists $U \in \tau_i$ such that $A \cap B \subset U \subset \tau_j\ cl(A \cap B)$. This implies that $(O \cap A) \cap B = O \cap (A \cap B) \subset O \cap U \subset O \cap \tau_j\ cl(A \cap B) \subset \tau_j cl(O \cap (A \cap B)) = \tau_j cl(O \cap A) \cap B)$. Since $O \cap U$ is τ_i-open, $(O \cap A) \cap B \in (i,j)PO(X))$, for every $B \in (i,j)PO(X)$. Hence $O \cap A$ is $(i,j)\gamma$-open. ■

Theorem 2. *The family of all $(i,j)\gamma$-open sets in X forms a topology on X.*

Proof. It is obvious that X and $\emptyset$ are in $(i,j)PO(X)$.If A_k where $k \in I$ is an arbitrary collection of $(i,j)\gamma$-open sets then for each $k \in I$, $A_k \cap B \in (i,j)PO(X)$ for every $B \in (i,j)PO(X)$. Therefore, $(\bigcup_{k\in I} A_k) \cap B = \bigcup_{k\in I}(A_k \cap B) \in (i,j)PO(X)$ [12]. If U and V are $(i,j)\gamma$-open sets then$(U \cap V) \cap B = U \cap (V \cap B) \in (i,j)PO(X)$ for every $B \in (i,j)PO(X)$. Hence $U \cap V \in (i,j)\gamma O(X)$. ■

Remark 2. Andrijevic [3] proved that $\tau_\alpha \subset \tau_\gamma$ in a topological space. But in a bitopological space the case is different. That is, $(i,j)\alpha$-open sets and $(i,j)\gamma$-open sets are independent to each other as shown in Example 2.

Example 2. Let $X = \{a,b,c,d\}$,$\tau_1 = \{\emptyset, \{a\}, \{b,c\}, \{a,b,c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, X\}$. Here, $\{b,c\}$∈(1,2)αO(X) but $\{b,c\}$ ∉ (1,2)γO(X) and $\{a,c,d\} \in (1,2)\gamma O(X)$ but $\{a,c,d\} \notin (1,2)\alpha O(X)$.

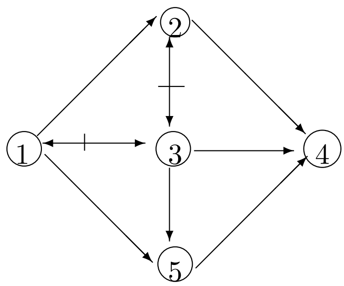
Remark 3. It is easy to verify that (i,j) semiopen sets are independent to $(i,j)\gamma$-open sets. In Example 2, $\{b,c,d\}$ is $(1,2)$semiopen but not $(1,2)\gamma$-open whereas $\{c\}$ is $(1,2)\gamma$-open but not $(1,2)$semiopen.

Remark 4. τ_i-open sets and $(i,j)\gamma$-open sets are independent. In Example 2, $\{b\} \in (2,1)\gamma O(X)$ but $\{b\} \notin \tau_2$ and $\{a,c,d\} \in \tau_2$ but $\{a,c,d\} \notin (2,1)\gamma O(X)$.

Remark 5. Every (i, j)preopen set is (i, j)semi-preopen [12]. Therefore, by Proposition 1, if a set $A \subset X$ is $(i, j)\gamma$-open then it is (i, j) semi-preopen. But it is not reversible. In Example 2, $\{a, d\}$ is $(2, 1)$ semi-preopen but not $(2, 1)\gamma$-open.

Remark 6. We enlarge the diagram in [12].

1. τ_i-open 2. (i, j)-semi-open 3. (i, j)-γ-open
4. (i, j)-semi-preopen 5. (i, j)-preopen.



In the diagram, $a \to b$ (resp. $a \mapsto b$ represents that a implies b but b does not imply a resp. a and b are independent).

Theorem 3. *In a space X, the following statements are equivalent. For all $x \in X$,*
(a) $\{x\} \in (i, j)SPO(X)$.
(b) $\{x\} \in (i, j)PO(X)$.
(c) $\{x\} \in (i, j)\gamma O(X)$.

In the following section, we introduce the $(i, j)\gamma$-closed sets and the $(i, j)\gamma$-closure of A for $A \subset X$. Also we define $(i, j)\gamma$-derived set, $(i, j)\gamma$-kernal and $(i, j)\gamma$-shell for any A⊂X.

4. $(i, j)\gamma$-closed sets

Definition 6. *A subset $A \subset X$ is called $(i, j)\gamma$-closed if its complement, A^c in X is $(i, j)\gamma$-open.*

Proposition 2. *Every $(i, j)\gamma$-closed set is (i, j)pre closed but the converse is not true. In Example 3.9, the set $\{b\}$ is $(2, 1)$preclosed but not $(2, 1)\gamma$-closed.*

Definition 7. *For any $A \subset X$*

(i) $(i, j)\gamma$-closure of A is the intersection of all the $(i, j)\gamma$-closed sets containing A and is written as $(i, j)\gamma$-$cl(A)$.
(ii) $(i, j)\gamma$-kernal of A is the intersection of all the $(i, j)\gamma$-open sets containing A and is written as $(i, j)\gamma$-$ker(A)$.

(iii) $(i,j)\gamma$*-derived set of* A*,written as* $(i,j)\gamma d(A)$ *is equal to* $(i,j)\gamma$*-cl*(A) $\setminus A$.

(iv) $(i,j)\gamma$*-shell of* A*, written as* $(i,j)\gamma sl(A)$ *is equal to* $(i,j)\gamma ker(A)\setminus A$.

Theorem 4. *For any* $x \in X$*,*

(i) $(i,j)\gamma\text{-}cl(\{x\}) = \{y : x \in (i,j)\gamma\text{-}ker\{y\}\}$.

(ii) $(i,j)\gamma d(\{x\}) = \{y : x \in (i,j)\gamma\text{-}ker\{y\}\}$ *and* $y \neq x$.

(iii) $(i,j)\gamma\text{-}ker(\{x\}) = \{y : x \in (i,j)\gamma\text{-}cl\{y\}\}$.

(iv) $(i,j)\gamma sl(\{x\}) = \{y : x \in (i,j)\gamma\text{-}cl\{y\}\}$ *and* $y \neq x$.

Theorem 5. *For any two subsets* A, B *of* X *and* $x \in X$*,*

(i) A *is* $(i,j)\gamma$*-closed if any only if* $A = (i,j)\gamma\text{-}cl(A)$.

(ii) $x \in (i,j)\gamma\text{-}cl(A)$ *if and only if* $A \cap U = \emptyset$ *for every* $(i,j)\gamma$*-open set* U *containing* x.

(iii) *If* $A \subset B$*, then* $(i,j)\gamma\text{-}cl(A) \subset (i,j)\gamma\text{-}cl(B)$.

(iv) $(i,j)\gamma\text{-}cl((i,j)\gamma\text{-}cl(A)) = (i,j)\gamma\text{-}cl(A)$.

Throghout this study by a degenerate set, we mean the null set or a singleton set.

Theorem 6. *In a space* X*,* $(i,j)\gamma sl(\{x\}) \cap (i,j)\gamma sl(\{y\}) = \emptyset$ *for any pair of distinct points* x *and* y *if and only if* $(i,j)\gamma d(\{x\})$ *is degenerate.*

Proof. Necessity. If $y, z \in (i,j)\gamma d(\{x\})$ for some $x \in X$, then $(i,j)\gamma sl(\{y\})$ and $(i,j)\gamma sl(\{z\})$ are not disjoint for $y, z \in X$. Therefore, $(i,j)\gamma d(\{x\})$ is degenerate.

Sufficiency. If there exists $z \in X$ such that $z \in (i,j)\gamma sl(\{x\}) \cap (i,j)\gamma sl(\{y\})$ for x, $y \in X$, $x \neq y$ then $x, y \in (i,j)\gamma\text{-}cl(\{z\})$ and this implies that $(i,j)\gamma d(\{x\})$ is not degenerate, a contradiction. ■

5. $(i,j)\gamma$-T_0 spaces

Definition 8. *A space* X *is called* $(i,j)\gamma$-T_0 *if for* $x, y \in X$*,* $x \neq y$*, there exists* $U \in (i,j)\gamma O(X)$ *such that* U *contains only one of* x *and* y *but not the other where* $i, j = 1, 2$*,* $i \neq j$.

Example 3. Let X be the space in Example 1. This space X is $(1,2)\gamma$-T_0 and $(2,1)\gamma$-T_0.

Definition 9. *A space* X *is called pairwise* γ-T_0 *if it is* $(1,2)\gamma$-T_0 *and* $(2,1)\gamma$-T_0.

Recall that a space X is pairwise-T_0 if for each pair of distinct point of X, there is either a τ_1-open set or a τ_2-open set containing one of the points but not the other.

Remark 7. Pairwise γ-T_0ness and pairwise-T_0 ness are independent. In the Example 4, given below, X is pairwise γ-T_0 but not pairwise-T_0. In Example 5, X is pairwise-T_0 but not pairwise γ-T_0.

Example 4. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, X\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Here X is pairwise γ-T_0 but not pairwise-T_0.

Example 5. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{d\}, X\}$. Here X is pairwise-T_0 but not pairwise γ-T_0 since for the pair a, c there does not exist a $(2, 1)\gamma$ -open set containing one of them but not the other.

Theorem 7. *A space X is $(i, j)\gamma$-T_0 if and only if the $(i, j)\gamma$-closures of distinct points are distinct.*

Proof. Let X be$(i, j)\gamma$-T_0 and $x, y \in X$, $x \neq y$. Then there exists a $(i, j)\gamma$-open set U containing x but not y, say. The set $X \setminus U$ is $(i, j)\gamma$-closed in X and $x \notin X \setminus V$ but $y \in X \setminus U$. Therefore, $x \notin (i, j)\gamma\text{-}cl(\{y\})$. Since $y \in (i, j)\gamma\text{-}cl(\{y\}), (i, j)\gamma\text{-}cl(\{x\}) \neq (i, j)\ \gamma\text{-}cl(\{y\})$.
Conversely, if for $x, y \in X$, $x \neq y$, $(i, j)\gamma\text{-}cl(\{x\})$ and $(i, j)\gamma\text{-}cl(\{y\})$ are not equal, then there exists $z \in X$ such that $z \in (i, j)\gamma\text{-}cl(\{x\})$ but $z \notin (i, j)\gamma\text{-}cl(\{y\})$. If $x \in (i, j)\gamma\text{-}cl(\{y\})$ then $\{x\} \subset (i, j)\gamma\text{-}cl(\{y\})$ which implies that $(i, j)\gamma\text{-}cl(\{x\}) \subset (i, j)\gamma\text{-}cl(\{y\})$ and so $z \in (i, j)\gamma\text{-}cl(\{y\})$. Hence $x \notin (i, j)\gamma\text{-}cl(\{y\})$ or $x \in ((i, j)\gamma\text{-}cl(\{y\}))^c$ which is a $(i, j)\gamma$-open set not containing y. ■

Theorem 8. *In a space X, if for all $x, y, x \neq y$, $(i, j)\gamma\text{-}ker(\{x\}) \cap (i, j)\gamma\text{-}ker(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$ then X is $(i, j)\gamma$-T_0.*

Proof. Let X be a space in which for all $x \neq y$, $(i, j)\gamma\text{-}ker(\{x\}) \cap (i, j)\gamma\text{-}ker(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$. Consequently, $(i, j)\gamma\text{-}ker(\{x\}) \neq (i, j)\gamma\text{-}ker(\{y\})$ and hence the space X is $(i, j)\gamma$-T_0. ■

6. Pairwise γ-continuous mappings

Definition 10. *A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called pairwise γ-continuous (briefly p.γ-continuous) if the inverse image of each σ_i-open set of Y is $(i, j)\gamma$-open in X for $i, j = 1, 2$ and $i \neq j$.*

Example 6. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, \{a, b, d\}, X\}$ and $Y = \{p, q, r\}$, $\sigma_1 = \{\emptyset, \{p\}, \{q\}, \{p, q\}, Y\}$, and $\sigma_2 = \{\emptyset, \{p, q, r\}, Y\}$. Define a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ as follows. $f(a) = p$, $f(b) = q$, $f(c) = r$ and $f(d) = s$, then f is p.γ-continuous.

Remark 8. Every p.γ-continuous mapping is (i, j)- pre-continuous but the converse is not true, in general. It is shown in Example 7.

Example 7. *Let X be the space given in Example 1, $Y = \{p, q, r\}$, $\sigma_1 = \{\emptyset, \{p, q\}, Y\}$ and $\sigma_2 = \{\emptyset, \{q, r\}, Y\}$. Define a mapping $f : X \to Y$ as follows. $f(a) = p$, $f(b) = q$ and $f(c) = r$. Then f is (i, j)- pre-continuous but not p.γ-continuous since $f^{-1}(\{p, q\}) = \{a, b\} \notin (1, 2)\gamma O(X)$.*

Remark 9. Pairwise continuity and pairwise γ-continuity are independent. In Example 7, f is pairwise continuous but not p.γ-continuous. In Example 8, given below, the mapping f is p.γ-continuous but not pairwise continuous.

Example 8. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{c\}, X\}$ and Y $= \{p, q, r, s\}$, $\sigma_1 = \{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, \{p, q, r\}, Y\}$ and $\sigma_2 = \{\emptyset, \{p\}, \{q\}, \{p, q\}, \{p, q, r\}, \{p, q, s\}, Y\}$. Define a mapping $f : X \to Y$ as $f(a) = p$, $f(b) = q$, $f(c) = r$ and $f(d) = s$. Then f is p.γ-continuous but not pairwise continuous because $f\mathbf{:}(X, \tau_1) \to (Y, \sigma_1)$ is not continuous.

Remark 10. Pairwise γ-continuity and (i, j)- semi-continuity are independent. Let X be the space as is Example 2. Let Y $= \{p, q, r\}$ with $\sigma_1 = \{\emptyset, \{p, q\}, Y\}$ and $\sigma_2 = \{\emptyset, \{q, r\}, Y\}$. Define a mapping $f : X \to Y$ as $f(a) = r$, $f(b) = p$, $f(c) = q$ and $f(d) = r$. Then f is (i, j)-semi-continuous but not p.γ-continuous since $f^{-1}(\{p, q\}) = \{b, c\} \notin (1, 2)\gamma O(X)$. With the same space (X, τ_1, τ_2) and $Y = \{p, q, r\}$ with $\sigma_1 = \{\emptyset, \{q, r\}, Y\}$ and $\sigma_2 = \{\emptyset, \{p\}, Y\}$, define a mapping $f : X \to Y$ as $f(a) = q$, $f(b) = p$ and $f(c) = r = f(d)$. Then f is p.γ-continuous but not (i, j)-semi-continuous since $f^{-1}(\{q, r\}) = \{a, c\} \notin (1, 2)SO(X)$.

Proposition 3. *Every p.γ-continuous mapping is pairwise semi-pre continuous.*

Proof. Follows from Remark 6.3 and Remark 5.1 [12]. ■

The above proposition is not reversible as shown in the following Example.

Example 9. Let X be the space in Example 1, $Y = \{p, q, r\}$, $\sigma_1 = \{\emptyset, \{p\}, \{q, r\}, Y\}$ and $\sigma_2 = \{\emptyset, \{p, r\}, Y\}$. Define a mapping $f : X \to Y$ as $f(a) = q$, $f(b) = r$ and $f(c) = p$. Then f is pairwise semi-pre continuous but not p.γ-continuous since $f^{-1}(\{q, r\}) = \{a, b\} \notin (1, 2)\gamma O(X)$.

Theorem 9. *For a mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.*

(a) f is p.γ-continuous.

(b) *The inverse image of each σ_j-closed set of Y is $(i,j)\gamma$-closed in X.*
(c) *For each $x \in X$ and each $V \in \sigma_i$ containing $f(x)$, there exists a $(i,j)\gamma$-open set U of X containing x such that $f(U) \subset V$.*
(d) *$(i,j)\gamma$-$cl(f^{-1}(B)) \subset f^{-1}(\tau_j cl(B))$ for every $B \subset Y$.*
(e) *$f((i,j)\gamma$-$cl(A)) \subset \tau_j cl(f(A))$ for every $A \subset X$.*
On each statement above $i \neq j$ and $i,j = 1,2$.

Proof. Omitted. ■

Theorem 10. *If $f : (X,\tau_1,\tau 2) \to (Y,\sigma_1,\sigma_2)$ is p.γ-continuous and $g : (Y,\sigma_1,\sigma_2) \to (Z,\rho_1,\rho_2)$ is pariwise continuous then $g \circ f$ is p.γ-continuous.*

Proof. The proof is evident. ■

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