F A S C I C U L I M A T H E M A T I C I

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ON THE SOLUTIONS OF A RATIONAL SYSTEM OF DIFFERENCE EQUATIONS

Abstract. In this paper we deal with the solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{y_{n-k}}, \qquad y_{n+1} = \frac{y_{n-k}}{x_n y_n},$$

with a nonzero real numbers initial conditions.

Key words: difference equations, boundedness, periodic solutions.

AMS Mathematics Subject Classification: 39A10.

1. Introduction

In this paper we deal with the solutions of the system of the difference equations

$$(1) \qquad x_{n+1} = \frac{1}{y_{n-k}}, \qquad y_{n+1} = \frac{y_{n-k}}{x_n y_n},$$

with a nonzero real numbers initial conditions.

Difference equations appear naturally as discrete analogues and as numerical solutions of differential and delay differential equations having applications in biology, ecology, economy, physics, and so on. So, recently there has been an increasing interest in the study of qualitative analysis of rational difference equations and systems of difference equations. Although difference equations are very simple in form, it is extremely difficult to understand thoroughly the behaviors of their solutions. See [1]–[29] and the references cited therein.

Cinar [1] has obtained the positive solution of the difference equation system

$$x_{n+1} = \frac{1}{y_n}, \qquad y_{n+1} = \frac{y_n}{x_{n-1} y_{n-1}}.$$

Also, Cinar et al. [4] has obtained the positive solution of the difference equation system

$$x_{n+1}=\frac{m}{y_n}, \qquad y_{n+1}=\frac{py_n}{x_{n-1}y_{n-1}}.$$

Elabbasy et al. [6] has obtained the solution of particular cases of the following general system of difference equations

$$x_{n+1}=\frac{a_1+a_2y_n}{a_3z_n+a_4x_{n-1}z_n}, \qquad y_{n+1}=\frac{b_1z_{n-1}+b_2z_n}{b_3x_ny_n+b_4x_ny_{n-1}},$$
$$z_{n+1}=\frac{c_1z_{n-1}+c_2z_n}{c_3x_{n-1}y_{n-1}+c_4x_{n-1}y_n+c_5x_ny_n}.$$

Özban [9] has investigated the solutions of the following system

$$x_{n+1}=\frac{a}{y_{n-3}}, \qquad y_{n+1}=\frac{by_{n-3}}{x_{n-q}y_{n-q}}.$$

Other related work see [1]-[14].

Definition 1 (Periodicity). *A sequence* $\{x_n\}_{n=-k}^{\infty}$ *is said to be periodic with period* p *if* $x_{n+p}=x_n$ *for all* $n\geq -k$.

2. Main results

2.1. When k-even. In this section we deal with the solutions of the system of the difference equations

$$x_{n+1}=\frac{1}{y_{n-2r}}, \qquad y_{n+1}=\frac{y_{n-2r}}{x_ny_n}, \tag{2}$$

with a nonzero real numbers initial conditions

Theorem 1. *Suppose that* $\{x_n,y_n\}$ *are solutions of system (2). Also, assume that* x_0, y_{-2r}, y_{-2r+1}, $\ldots$, y_0 *are arbitrary nonzero real numbers. Then all solutions of equation system (2) are periodic with period* $(4r+2)$.

Proof. From Eq.(2), we see that

$$\begin{aligned}
x_{n+1} &= \frac{1}{y_{n-2r}}, & y_{n+1} &= \frac{y_{n-2r}}{x_ny_n},\\
x_{n+2} &= \frac{1}{y_{n-2r+1}}, & y_{n+2} &= \frac{y_{n-2r+1}}{x_{n+1}y_{n+1}}=x_ny_ny_{n-2r+1},\\
x_{n+3} &= \frac{1}{y_{n-2r+2}}, & y_{n+3} &= \frac{y_{n-2r+2}}{x_{n+2}y_{n+2}}=\frac{y_{n-2r+2}}{x_ny_n},\\
x_{n+4} &= \frac{1}{y_{n-2r+3}}, & y_{n+1} &= \frac{y_{n-2r+3}}{x_{n+3}y_{n+3}}=x_ny_ny_{n-2r+3},
\end{aligned}$$

$$\vdots$$

$$\begin{aligned}
x_{n+2r-1} &= \frac{1}{y_{n-2}}, \qquad y_{n+2r-1} = \frac{y_{n-2}}{x_{n+2r-2}y_{n+2r-2}} = \frac{y_{n-2}}{x_ny_n},\\
x_{n+2r} &= \frac{1}{y_{n-1}}, \qquad y_{n+2r} = \frac{y_{n-1}}{x_{n+2r-1}y_{n+2r-1}} = x_ny_ny_{n-1},\\
x_{n+2r+1} &= \frac{1}{y_n}, \qquad y_{n+2r+1} = \frac{y_n}{x_{n+2r}y_{n+2r}} = \frac{y_n}{x_ny_n} = \frac{1}{x_n},\\
x_{n+2r+2} &= \frac{1}{y_{n+1}} = \frac{x_ny_n}{y_{n-2r}},\\
y_{n+2r+2} &= \frac{y_{n+1}}{x_{n+2r+1}y_{n+2r+1}} = \frac{y_{n-2r}}{x_ny_n\frac{1}{y_n}\frac{1}{x_n}} = y_{n-2r},\\
x_{n+2r+3} &= \frac{1}{y_{n+2}} = \frac{1}{x_ny_ny_{n-2r+1}},\\
y_{n+2r+3} &= \frac{y_{n+2}}{x_{n+2r+2}y_{n+2r+2}} = \frac{x_ny_ny_{n-2r+1}}{\frac{x_ny_n}{y_{n-2r}}y_{n-2r}} = y_{n-2r+1},
\end{aligned}$$

$$\vdots$$

$$\begin{aligned}
x_{n+4r} &= \frac{1}{y_{n+2r-1}} = \frac{x_ny_n}{y_{n-2}},\\
y_{n+4r} &= \frac{y_{n+2r-1}}{x_{n+4r-1}y_{n+4r-1}} = \frac{y_{n-2}}{x_ny_n\frac{1}{x_ny_n}} = y_{n-2},\\
x_{n+4r+1} &= \frac{1}{y_{n+2r}} = \frac{1}{x_ny_ny_{n-1}},\\
y_{n+4r+1} &= \frac{y_{n+2r}}{x_{n+4r}y_{n+4r}} = \frac{x_ny_ny_{n-1}}{\frac{x_ny_n}{y_{n-2}}y_{n-2}} = y_{n-1},\\
x_{n+4r+2} &= \frac{1}{y_{n+2r+1}} = x_n,\\
y_{n+4r+2} &= \frac{y_{n+2r+1}}{x_{n+4r+1}y_{n+4r+1}} = \frac{1}{x_n\frac{1}{x_ny_ny_{n-1}}y_{n-1}} = y_n.
\end{aligned}$$

Hence, the proof is completed. ■

Proposition 1. *It is easy to see that the system*

$$x_{n+1} = \frac{1}{y_{n-2r}}, \qquad y_{n+1} = \frac{y_{n-2r}}{x_{n-p}y_{n-p}}$$

is periodic with period $(2p+2)(2r+1)$ *when* $2p \neq r$*, and periodic with period* $(4r+2)$ *when* $2p = r$*.*

2.2. When k-odd. In this section we deal with the solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{y_{n-2r-1}}, \qquad y_{n+1} = \frac{y_{n-2r-1}}{x_n y_n}, \tag{3}$$

with a nonzero real numbers initial conditions.

Theorem 2. *Suppose that* $\{x_n, y_n\}$ *are solutions of system (3). Also, assume that* x_0*,* y_{-2r-1}*,* y_{-2r}*,* $\dots$*,* y_0*, are arbitrary nonzero real numbers and* $A = x_0 y_0$*. Then*

(i) *If* $A = 1$*, the solutions of equation system (3) are periodic with period* $(2r+2)$*.*

(ii) *If* $A \neq 1$*, the solutions are unbounded and given by*

$$x_{(2r+2)n+s} = \begin{cases} \dfrac{A^n}{y_{-2r-2+s}}, & s - odd, \\ \dfrac{1}{A^n y_{-2r-2+s}}, & s - even, \end{cases} \qquad s = 1, 2, \dots, (2r+2),$$

and

$$y_{(2r+2)n+s} = \begin{cases} \dfrac{y_s}{A^n}, & s - odd, \\ A^n y_s, & s - even, \end{cases} \qquad s = -2r-1, -2r, -2r+1, \dots, 2, 1, 0,$$

where $n = 0, 1, 2, \dots$ *.*

Proof. (i) If $A = 1$, from Eq.(3), we see that

$$\begin{aligned} x_1 &= \frac{1}{y_{-2r-1}}, & y_1 &= \frac{y_{-2r-1}}{x_0 y_0} = y_{-2r-1}, \\ x_2 &= \frac{1}{y_{-2r}}, & y_2 &= \frac{y_{-2r}}{x_1 y_1} = y_{-2r}, \\ x_3 &= \frac{1}{y_{-2r+1}}, & y_3 &= \frac{y_{-2r+1}}{x_2 y_2} = y_{-2r+1}, \\ &\vdots \\ x_{2r+1} &= \frac{1}{y_{-1}}, & y_{2r+1} &= \frac{y_{-1}}{x_{2r} y_{2r}} = y_{-1}, \\ x_{2r+2} &= \frac{1}{y_0}, & y_{2r+2} &= \frac{y_0}{x_{2r+1} y_{2r+1}} = y_0, \\ x_{2r+3} &= \frac{1}{y_1} = \frac{1}{y_{-2r-1}} = x_1, & y_{2r+3} &= \frac{y_1}{x_{2r+2} y_{2r+2}} = y_1. \end{aligned}$$

(*ii*) If $A \neq 1$. For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n - 1$. That is;

$$x_{(2r+2)n-2r-1} = \frac{A^{n-1}}{y_{-2r-1}}, \qquad x_{(2r+2)n-2r} = \frac{1}{A^{n-1}y_{-2r}},$$
$$x_{(2r+2)n-2r+1} = \frac{A^{n-1}}{y_{-2r+1}},$$
$$\vdots$$
$$x_{(2r+2)n-2} = \frac{1}{A^{n-1}y_{-2}}, \qquad x_{(2r+2)n-1} = \frac{A^{n-1}}{y_{-1}},$$
$$x_{(2r+2)n} = \frac{1}{A^{n-1}y_0},$$

and

$$y_{(2r+2)n-4r-3} = \frac{y_{-2r-1}}{A^{n-1}}, \qquad y_{(2r+2)n-4r-2} = A^{n-1}y_{-2r},$$
$$y_{(2r+2)n-4r-1} = \frac{y_{-2r+1}}{A^{n-1}},$$
$$\vdots$$
$$y_{(2r+2)n-2r-4} = A^{n-1}y_{-2}, \qquad y_{(2r+2)n-2r-3} = \frac{y_{-1}}{A^{n-1}},$$
$$y_{(2r+2)n-2r-2} = A^{n-1}y_0,$$

it follows from Eq.(3) that

$$y_{(2r+2)n-2r-1} = \frac{y_{(2r+2)n-4r-3}}{x_{(2r+2)n-2r-2}y_{(2r+2)n-2r-2}} = \frac{y_{-2r-1}}{A^{n-1}\frac{1}{A^{n-2}y_0}A^{n-1}y_0}$$
$$= \frac{y_{-2r-1}}{A^n},$$
$$x_{(2r+2)n+1} = \frac{1}{y_{(2r+2)n-2r-1}} = \frac{A^n}{y_{-2r-1}},$$

$$y_{(2r+2)n-2r} = \frac{y_{(2r+2)n-4r-2}}{x_{(2r+2)n-2r-1}y_{(2r+2)n-2r-1}} = \frac{A^{n-1}y_{-2r}}{\frac{A^{n-1}}{y_{-2r-1}}\frac{y_{-2r-1}}{A^n}} = A^n y_{-2r},$$
$$x_{(2r+2)n+2} = \frac{1}{y_{(2r+2)n-2r}} = \frac{1}{A^n y_{-2r}},$$

$$y_{(2r+2)n-2r+1} = \frac{y_{(2r+2)n-4r-1}}{x_{(2r+2)n-2r}y_{(2r+2)n-2r}} = \frac{y_{-2r+1}}{A^{n-1}\frac{1}{A^{n-1}y_{-2r}}A^n y_{-2r}}$$

$$= \frac{y_{-2r+1}}{A^n},$$

$$x_{(2r+2)n+3} = \frac{1}{y_{(2r+2)n-2r+1}} = \frac{A^n}{y_{-2r+1}},$$

$$\vdots$$

$$x_{(2r+2)n+2r} = \frac{1}{y_{(2r+2)n-2}} = \frac{1}{A^n y_{-2}},$$

$$y_{(2r+2)n-2} = \frac{y_{(2r+2)n-2r-4}}{x_{(2r+2)n-3}y_{(2r+2)n-3}} = \frac{A^{n-1}y_{-2}}{\frac{A^{n-1}}{y_{-3}}\frac{y_{-3}}{A^n}} = A^n y_{-2},$$

$$x_{(2r+2)n+2r+1} = \frac{1}{y_{(2r+2)n-1}} = \frac{A^n}{y_{-1}},$$

$$y_{(2r+2)n-1} = \frac{y_{(2r+2)n-2r-3}}{x_{(2r+2)n-2}y_{(2r+2)n-2}} = \frac{y_{-1}}{A^{n-1}\frac{1}{A^{n-1}y_{-2}}A^n y_{-2}} = \frac{y_{-1}}{A^n},$$

$$x_{(2r+2)n+2r+2} = \frac{1}{y_{(2r+2)n}} = \frac{1}{A^n y_0},$$

$$y_{(2r+2)n} = \frac{y_{(2r+2)n-2r-2}}{x_{(2r+2)n-1}y_{(2r+2)n-1}} = \frac{A^{n-1}y_0}{\frac{A^{n-1}}{y_{-1}}\frac{y_{-1}}{A^n}} = A^n y_0.$$

Hence, the proof is completed. ∎

Proposition 2. *It is easy to see that for the following system*

$$x_{n+1} = \frac{1}{y_{n-2r-1}}, \qquad y_{n+1} = \frac{y_{n-2r-1}}{x_{n-1}y_{n-1}}.$$

(i) If $A = B = 1$, the solutions are periodic with period $(2r+2)$.
(ii) If A or $B \neq 1$, the solutions are unbounded and given by

$$x_{(2r+2)n+s} = \begin{cases} \dfrac{B^{n\sin(\frac{s\pi}{2})}}{y_{-2r-2+s}}, & s-odd, \\ \dfrac{A^{n\cos(\frac{(s+2)\pi}{2})}}{y_{-2r-2+s}}, & s-even, \end{cases} \qquad s = 1, 2, ..., (2r+2),$$

and

$$y_{(2r+2)n+s} = \begin{cases} \dfrac{y_s}{B^{n\sin(\frac{s\pi}{2})}}, & s-odd, \\ \dfrac{y_s}{A^{n\cos(\frac{(s+2)\pi}{2})}}, & s-even, \end{cases} \quad s=-2r-1,-2r,-2r+1,...,2,1,0,$$

where $n = 0, 1, 2, ...$ *and* $A = x_0y_0$, $B = x_{-1}y_{-1}$.

3. Numerical examples

In order to illustrate the results of the previous sections and to support our theoretical discussions, we consider several interesting numerical examples in this section. These examples represent different types of qualitative behavior of solutions to nonlinear difference equations.

Example 1. Consider the following difference system equation:

$$x_{n+1} = \frac{1}{y_{n-2}}, \qquad y_{n+1} = \frac{y_{n-2}}{x_n y_n},$$

with the initial conditions $y_{-2} = 0.8$, $y_{-1} = 4.2$, $y_0 = 9$, $x_0 = 0.7$. This solution is a period six solution and will be $\{x_n\} = \{0.7, 1.25, 0.238, 0.111, 7.875, 0.037, 0.7, 1.25, ...\}$, $\{y_n\} = \{0.8, 4.2, 9, 0.127, 26.46, 1.428, 0.8, 4.2, ...\}$. (See Fig. 1).

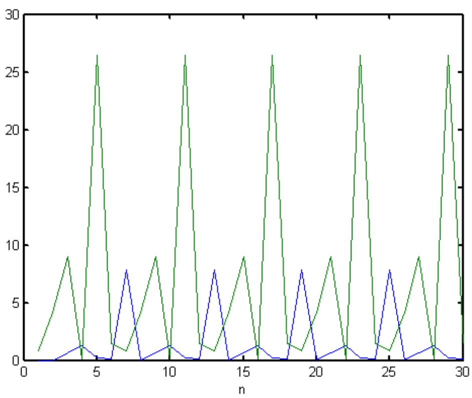

Figure 1.

Example 2. Consider the following difference system equation:

$$x_{n+1} = \frac{1}{y_{n-2}}, \qquad y_{n+1} = \frac{y_{n-2}}{x_{n-1} y_{n-1}},$$

with the initial conditions $y_{-2} = 8$, $y_{-1} = 0.19$, $y_0 = 5$, $x_{-1} = 3$, $x_0 = 6$. Also, this solution is periodic with period twelve and takes the form $\{x_n\} =$

$\{3, 6, 0.125, 5.26, 0.2, 0.07, 157.9, 0.35, 0.0023, 90, 10.5, 4.16E{-}3, 3, 6, ...\}$, $\{y_n\}$ $= \{8, 0.19, 5, 14, 6.33E-3, 2.85, 421, 1.11E-2,\ 9.5E-02, 240, 0.33, 0.16, 8,$ $0.19, 5, ...\}$. (See Fig. 2).

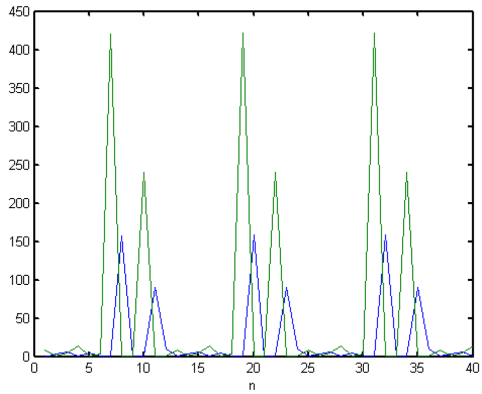

Figure 2.

Example 3. Consider the difference system equation

$$x_{n+1} = \frac{1}{y_{n-4}}, \qquad y_{n+1} = \frac{y_{n-4}}{x_{n-1}y_{n-1}},$$

with the initial conditions $y_{-4} = 2.7$, $y_{-3} = 0.3$, $y_{-2} = 0.5$, $y_{-1} = 6$, $y_0 = 5$, $x_{-1} = 0.8$, $x_0 = 0.2$. The solution is periodic with period ten. (See Fig. 3).

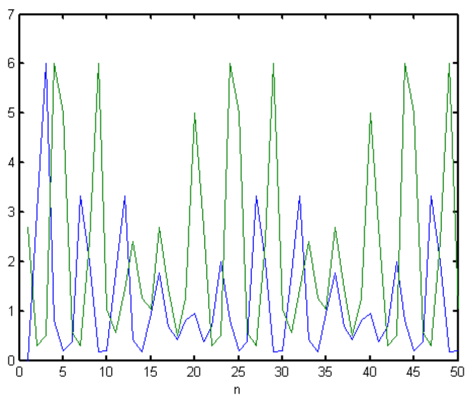

Figure 3.

Example 4. Consider the following difference system equation:

$$x_{n+1} = \frac{1}{y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-3}}{x_n y_n},$$

with the initial conditions $y_{-3} = 0.6$, $y_{-2} = 0.7$, $y_{-1} = 4.3$, $y_0 = 2.5$, $x_0 = 0.4$. We see that this solution is periodic with period four. (See Fig. 4).

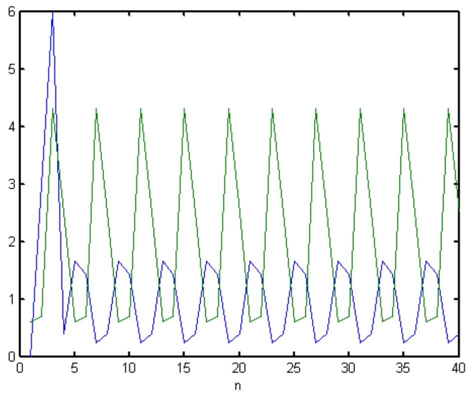

Figure 4.

Example 5. Consider the following difference system equation:

$$x_{n+1} = \frac{1}{y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-3}}{x_n y_n},$$

with the initial conditions $y_{-3} = 7$, $y_{-2} = 1.1$, $y_{-1} = 0.2$, $y_0 = 4$, $x_0 = 0.4$. Unbounded solution in this case. (See Fig. 5).

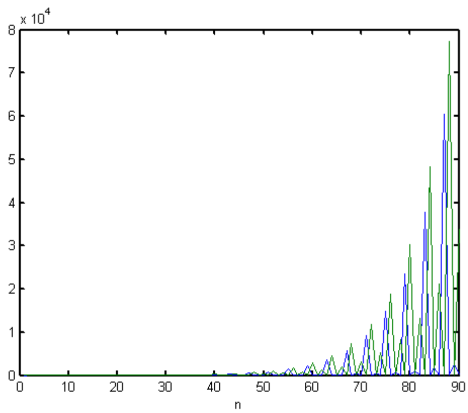

Figure 5.

Example 6. Consider the following difference system equation:

$$x_{n+1} = \frac{1}{y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-3}}{x_{n-1} y_{n-1}},$$

with the initial conditions $y_{-3} = 7$, $y_{-2} = 1.1$, $y_{-1} = 0.2$, $y_0 = 4$, $x_{-1} = 8$, $x_0 = 0.4$. Also, this is unbounded solution. (See Fig. 6).

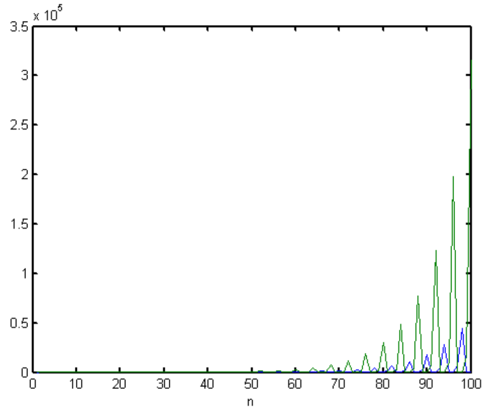

Figure 6.

Example 7. Consider the following difference system equation:

$$x_{n+1} = \frac{1}{y_{n-3}}, \qquad y_{n+1} = \frac{y_{n-3}}{x_{n-1}y_{n-1}},$$

with the initial conditions $y_{-3} = 7,\ y_{-2} = 1.1,\ y_{-1} = 0.2,\ y_0 = 4,\ x_{-1} = 5,\ x_0 = 0.25$. This solution is periodic with period four. (See Fig. 7).

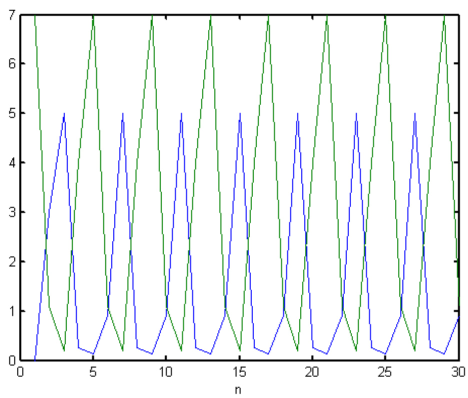

Figure 7.

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