F A S C I C U L I M A T H E M A T I C I

Nr 45 2010

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THE SEMI NORMED SPACE DEFINED BY A DOUBLE GAI SEQUENCE OF MODULUS FUNCTION

Abstract. In this paper we introduce the sequence spaces $\chi^2_M(p,q,u)$, using an modulus function M and defined over a semi normed space (X,q), semi normed by q. We study some properties of these sequence spaces and obtain some inclusion relations.

Key words: entire sequence, analytic sequence, modulus, invariant mean, semi norm.

AMS Mathematics Subject Classification: 40A05, 40C05, 40D05.

1. Introduction

Let (x_{mn}) be a double sequence of real or complex numbers. Then the series $\sum_{m,n=1}^{\infty} x_{mn}$is called a double series. The double series $\sum_{m,n=1}^{\infty} x_{mn}$is said to be convergent if and only if the double sequence $(S_{mn})$is convergent, where

$$S_{mn} = \sum_{i,j=1}^{m,n} x_{ij} \quad (m,n = 1,2,3,...) \quad \text{(see[1])}.$$

The class of all complex double sequences is denoted by w^2. A sequence $x = (x_{mn}) \in w^2$ is called as a double gai sequence if

$$((m+n)!\,|x_{mn}|)^{1/m+n} \to 0 \quad \text{as} \quad m,n \to \infty.$$

The vector space of all prime sense double gai sequences are usually denoted by χ^2. The space χ^2 is a metric space with the metric

$$(1) \qquad d(x,y) = \sup_{mn} \left\{ ((m+n)!\,|x_{mn} - y_{mn}|)^{1/m+n} : m, \ \ n : 1,2,3,\dots \right\},$$

forall $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in χ^2.

Modulus function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous non-decreasing and sub additive with $M(0), M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. The studies on sequence spaces defined by modulus was investigated at the initial stage by Ruckle[19], Maddox[20], Nakano[18] and many others.

Let us define the following sets of double sequences:

$$\mathcal{M}_u(t) := \left\{ (x_{mn}) \in w^2 : \sup_{m,n \in N} |x_{mn}|^{t_{mn}} < \infty \right\},$$

$$\mathcal{C}_p(t) := \left\{ (x_{mn}) \in w^2 : p - \lim_{m,n\to\infty} |x_{mn} - l|^{t_{mn}} = 1 \text{ for some } l \in C \right\},$$

$$\mathcal{C}_{0p}(t) := \left\{ (x_{mn}) \in w^2 : p - \lim_{m,n\to\infty} |x_{mn}|^{t_{mn}} = 1 \right\},$$

$$\mathcal{L}_u(t) := \left\{ (x_{mn}) \in w^2 : \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |x_{mn}|^{t_{mn}} < \infty \right\},$$

$$\mathcal{C}_{bp}(t) := \mathcal{C}_p(t) \bigcap \mathcal{M}_u(t) \quad \text{and} \quad \mathcal{C}_{0bp}(t) = \mathcal{C}_{0p}(t) \bigcap \mathcal{M}_u(t);$$

where $t = (t_{mn})$ is the sequence of strictly positive reals t_{mn} for all $m, n \in \aleph$ and $p - lim_{m,n\to\infty}$ denotes the limit in the Pringsheim's sense. In the case $t_{mn} = 1$ for all $m, n \in \aleph$; $\mathcal{M}_u(t)$, $\mathcal{C}_p(t)$, $\mathcal{C}_{0p}(t)$, $\mathcal{L}_u(t)$, $\mathcal{C}_{bp}(t)$ and $\mathcal{C}_{0bp}(t)$ reduce to the sets $\mathcal{M}_u$, $\mathcal{C}_p$, $\mathcal{C}_{0p}$, $\mathcal{L}_u$, $\mathcal{C}_{bp}$ and $\mathcal{C}_{0bp}$, respectively. Now, we may summarize the knowledge given in some document related to the double sequence spaces. Gökhan and Colak [22, 23] have proved that $\mathcal{M}_u(t)$ and $\mathcal{C}_p(t), \mathcal{C}_{bp}(t)$ are complete paranormed spaces of double sequences and gave the $\alpha-, \beta-, \gamma-$ duals of the spaces $\mathcal{M}_u(t)$ and $\mathcal{C}_{bp}(t)$. Quite recently, in her PhD thesis, Zelter [24] has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen and Edely [25] have recently introduced the statistical convergence and Cauchy for double sequences and given the relation between statistical convergent and strongly Cesàro summable double sequences. Nextly, Mursaleen [26] and Mursaleen and Edely [27] have defined the almost strong regularity of matrices for double sequences and applied these matrices to establish a core theorem and introduced the $M-$core for double sequences and determined those four dimensional matrices transforming every bounded double sequences $x = (x_{jk})$ into one whose core is a subset of the $M-$core of x. More recently, Altay and Basar [28] have defined the spaces $\mathcal{BS}, \mathcal{BS}(t), \mathcal{CS}_p, \mathcal{CS}_{bp}, \mathcal{CS}_r$ and $\mathcal{BV}$ of double sequences consisting of all double series whose sequence of partial sums are in the spaces $\mathcal{M}_u$, $\mathcal{M}_u(t)$, $\mathcal{C}_p$, $\mathcal{C}_{bp}$, $\mathcal{C}_r$ and $\mathcal{L}_u$, respectively, and also examined some properties of those sequence spaces and determined the $\alpha-$ duals of the spaces $\mathcal{BS}$, $\mathcal{BV}$, $\mathcal{CS}_{bp}$ and the $\beta(\vartheta)-$ duals of the spaces $\mathcal{CS}_{bp}$ and $\mathcal{CS}_r$ of double series. Quite recently Basar and Sever [29] have introduced the Banach space $\mathcal{L}_q$

of double sequences corresponding to the well-known space ℓ_q of single se quences and examined some properties of the space $\mathcal{L}_q$.

Consider a double sequence $x = (x_{ij})$. The $(m, n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij} \Im_{ij}$ for all $m, n \in \aleph$,

$$\Im_{mn} = \begin{pmatrix} 0, & 0, & ...0, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \\ . \\ . \\ . \\ 0, & 0, & ...\frac{1}{(m+n)!}, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \end{pmatrix}$$

with $\frac{1}{(m+n)!}$ in the $(m, n)^{th}$ position and zero other wise. An FK-space (or a metric space) X is said to have AK property if $(\Im_{mn})$ is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$. We need the following inequality in the sequel of the paper:

Lemma 1. *For $a, b \geq 0$ and $0 < p < 1$, we have*

$$(a + b)^p \leq a^p + b^p$$

2. Preliminaries

Some initial works on double sequence spaces is found in Bromwich [3]. Later on it was investigated by Hardy [5], Moricz [6], Moricz and Rhoades [7], Basarir and Solankan [2], Tripathy [8], Colak and Turkmenoglu [4], Turkmenoglu [9], and many others. Orlicz[10] used the idea of Orlicz function to construct the space $\left(L^M\right)$. Lindenstrauss and Tzafriri [11] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p\left(1 \leq p < \infty\right)$. subsequently, different classes of sequence spaces were defined by Parashar and Choudhary [12], Mursaleen et al. [13], Bektas and Altin [14], Tripathy et al. [15], Rao and Subramanian [16], and many others.

3. Definitions

Definition 1. *The space consisting of all those sequences x in w^2 such that $\left(M\left(\frac{((m+n)!|x_{mn}|)^{1/m+n}}{\rho}\right)\right) \to 0$ as $m, n \to \infty$ for some arbitrarily fixed $\rho > 0$ is denoted by χ^2_M, M being a modulus function. In other words $\left(M\left(\frac{((m+n)!|x_{mn}|)^{1/m+n}}{\rho}\right)\right)$ is a modulus space of double gai sequences.* χ^2_M

is called the modulus space of double gai sequences. The space χ^2_M *is a metric space with the metric*

$$d(x, y) = inf \left\{ \rho > 0 : \sup_{mn} \left(M \left(\frac{((m+n)! \left| x_{mn} - y_{mn} \right|)^{1/m+n}}{\rho} \right) \right) \leq 1 \right\}$$

for all $x = \{x_{mn}\}$ *and* $y = \{y_{mn}\}$ *in* χ^2_M.

Definition 2. *Let* p, q *be semi norms on a vector space* X. *Then* p *is said to be stronger that* q *if whenever* (x_{mn}) *is a sequence such that* $p\,(x_{mn}) \to 0$, *then also* $q\,(x_{mn}) \to 0$. *If each is stronger than the others, the* p *and* q *are said to be equivalent.*

Lemma 2. *Let* p *and* q *be semi norms on a linear space* X. *Then* p *is stronger than* q *if and only if there exists a constant* M *such that* $q(x) \leq Mp(x)$ *for all* $x \in X$.

Definition 3. *A sequence space* E *is said to be solid or normal if* $(\alpha_{mn} x_{mn}) \in E$ *whenever* $(x_{mn}) \in E$ *and for all sequences of scalars* (α_{mn}) *with* $|\alpha_{mn}| \leq 1$, *for all* $m, n \in N$.

Definition 4. *A sequence space* E *is said to be monotone if it contains the canonical pre-images of all its step spaces.*

Remark 1. From the two above definitions it is clear that a sequence space E is solid implies that E is monotone.

Definition 5. *A sequence* E *is said to be convergence free if* $(y_{mn}) \in E$ *whenever* $(x_{mn}) \in E$ *and* $x_{mn} = 0$ *implies that* $y_{mn} = 0$.

Let $p = (p_{mn})$ *be a sequence of positive real numbers with* $0 < p_{mn} < \sup p_{mn} = G$ *and Let* $D = Max(1, 2^{G-1})$ *Then for* $a_{mn}, b_{mn} \in C$, *the set of complex numbers for all* $m, n \in N$ *we have*

$$|a_{mn} + b_{mn}|^{1/m+n} \leq D \left\{ |a_{mn}|^{1/m+n} + |b_{mn}|^{1/m+n} \right\} \tag{2}$$

By $S\,(X)$ we denote the linear space of all sequences $x = (x_{mn})$ with $(x_{mn}) \in X$ and the usual coordinate wise operations: $\alpha x = (\alpha x_{mn})$ and $x + y = (x_{mn} + y_{mn})$, for each $\alpha \in C$. If $\lambda = (\lambda_{mn})$ is a scalar sequence and $x \in S\,(X)$ then we shall write $\lambda x = (\lambda_{mn} x_{mn})$

Let U be the set of all sequences $u = (u_{mn})$ such that $u_{mn} \neq 0$ and complex for all $m, n = 1, 2, 3, \cdots$.

Let $M = (M_{mn})$ be a sequence of modulus functions, $p = (p_{mn})$ be a sequence of positive real numbers and α be a seminormed space with seminorm q. Given $u \in U$.

Let (X, q) be a semi normed space over the field C of complex numbers with the semi norm q. The symbol $\chi^2_M(X)$ denotes the spaces of all double gai sequences defined over X. We define the following sequence space:

$$\chi^2_M(p,q,u) = \left\{ x \in S(X) : u_{mn} \left[M_{mn} \left(q \left(\frac{((m+n)! \, |x_{mn}|)^{1/m+n}}{\rho} \right) \right) \right]^{p_{mn}} \right. \left. \to 0 \quad \text{as} \quad m,n \to \infty, \quad \rho > 0 \right\}.$$

We get the following sequence spaces from $\chi^2_M(p,q,u)$ on giving particular values to p and u. Taking $p_{mn} = 1$ for all $m, n \in N$ we have

$$\chi^2_M(q,u) = \left\{ x \in S(X) : u_{mn} \left[M_{mn} \left(q \left(\frac{((m+n)! \, |x_{mn}|)^{1/m+n}}{\rho} \right) \right) \right] \right. \left. \to 0 \quad \text{as} \quad m,n \to \infty, \quad \rho > 0 \right\}.$$

If we take $u_{mn} = 1$, then we have

$$\chi^2_M(p,q) = \left\{ x \in S(X) : \left[M_{mn} \left(q \left(\frac{((m+n)! \, |x_{mn}|)^{1/m+n}}{\rho} \right) \right) \right]^{p_{mn}} \right. \left. \to 0 \quad \text{as} \quad m,n \to \infty, \quad \rho > 0 \right\}.$$

If we take $p_{mn} = 1$ and $u_{mn} = 1$ for all $m, n \in N$, then we have

$$\chi^2_M(q) = \left\{ x \in S(X) : \left[M_{mn} \left(q \left(\frac{((m+n)! \, |x_{mn}|)^{1/m+n}}{\rho} \right) \right) \right] \right. \left. \to 0 \quad \text{as} \quad m,n \to \infty, \quad \rho > 0 \right\}.$$

In addition to the above sequence spaces, we have $\chi^2_M(p,q,u) = \chi^2_M(p)$, on taking $u_{mn} = 1$ for all $m, n \in N$, $q(x) = |x|$, $(M_{mn}) = M$ for all $m, n \in N$ and $X = C$.

4. Main results

Theorem 1. *If M is a modulus function, then $\chi^2_M(p,q,u)$ are linear spaces over the set of complex numbers.*

Proof. The proof is easy, so omitted. ■

Theorem 2. *$\chi^2_M(p,q,u)$ are paranormed spaces with*

$$g(x) = \inf\left\{ \rho^{p_v/H} : \sup_{m,n\geq 1} u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho} \right)\right)\right] \leq 1, \quad v \in N, \ \ \rho > 0 \right\}$$

where $H = \max(1, \sup_{mn} p_{mn})$

Proof. Clearly $g(x) = g(-x)$ and $g(\theta) = 0$, where θ is the zero sequence. It can be easily verified that $g(x+y) \leq g(x) + g(y)$. Next $x \to \theta, \lambda$ fixed implies $g(\lambda x) \to 0$. Also $x \to \theta$ and $\lambda \to 0$ implies $g(\lambda x) \to 0$. The case $\lambda \to 0$ and x fixed implies that $g(\lambda x) \to 0$ follows from the following expressions.

$$g(\lambda x) = \inf\left\{ \rho^{p_v/H} : \sup_{m,n\geq 1} u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho} \right)\right)\right] \leq 1, \quad v \in N, \ \ \rho > 0 \right\}$$

$$g(\lambda x) = \inf\left\{ (|\lambda|\, r)^{p_v/H} : \sup_{m,n\geq 1} u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho} \right)\right)\right] \leq 1, \quad v \in N, \ \ \rho > 0 \right\}$$

where $r = \frac{\rho}{|\lambda|}$. Hence $\chi^2_M(p,q,u)$ is a paranormed space. This completes the proof. ■

Theorem 3. *Let $M = (M_{mn})$ and $T = (T_{mn})$ be two modulus function. Then*

$$\chi^2_M(p,q,u) \bigcap \chi^2_T(p,q,u) \subseteq \chi^2_{M+T}(p,q,u).$$

Proof. The proof is easy, so omitted. ■

Remark 2. Let $M = (M_{mn})$ be a modulus function q_1 and q_2 be two semi norms on X, we have

(*i*) $\chi^2_M(p, q_1, u) \bigcap \chi^2_M(p, q_2, u) \subseteq \chi^2_M(p, q_1 + q_2, u)$.

(*ii*) If q_1 is stronger than q_2 then $\chi^2_M(p, q_1, u) \subseteq \chi^2_M(p, q_2, u)$.

(*iii*) If q_1 is equivalent to q_2 then $\chi^2_M(p, q_1, u) = \Gamma^2_M(p, q_2, u)$.

Theorem 4. *(i) Let $0 \le p_{mn} \le r_{mn}$ and $\left\{\frac{r_{mn}}{p_{mn}}\right\}$ be bounded. Then $\chi^2_M(r, q, u) \subset \chi^2_M(p, q, u)$.*

(ii) $u_1 \le u_2$ implies $\chi^2_M(p, q, u_1) \subset \chi^2_M(p, q, u_2)$.

Proof. Let

$$x \in \chi^2_M(r, q, u) \tag{3}$$

$$u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{r_{mn}} \to 0 \quad \text{as} \quad m, n \to \infty. \tag{4}$$

Let $t_{mn} = u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{r_{mn}}$ and $\lambda_{mn} = \frac{p_{mn}}{r_{mn}}$. Since $p_{mn} \le r_{mn}$, we have $0 \le \lambda_{mn} \le 1$. Take $0 < \lambda < \lambda_{mn}$.

Define $u_{mn} = t_{mn}(t_{mn} \ge 1)$; $u_{mn} = 0(t_{mn} < 1)$ and $v_{mn} = 0(t_{mn} \ge 1)$; $v_{mn} = t_{mn}(t_{mn} < 1)$; $t_{mn} = u_{mn} + v_{mn}$; $t^{\lambda_{mn}}_{mn} + v^{\lambda_{mn}}_{mn}$. Now it follows that

$$u^{\lambda_{mn}}_{mn} \le t_{mn} \qquad \text{and} \qquad v^{\lambda_{mn}}_{mn} \le v^{\lambda}_{mn} \tag{5}$$

i.e $t^{\lambda_{mn}}_{mn} = u^{\lambda_{mn}}_{mn} + v^{\lambda_{mn}}_{mn}$; $t^{\lambda_{mn}}_{mn} \le t_{mn} + v^{\lambda}_{mn}$ by (5)

$$u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)^{r_{mn}}\right]^{\lambda_{mn}} \le u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{r_{mn}}$$

$$u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)^{r_{mn}}\right]^{p_{mn}/r_{mn}} \le u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{r_{mn}}$$

$$u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{p_{mn}} \le u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{r_{mn}}.$$

But $u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{r_{mn}} \to 0$ as $m,n\to\infty$. By (4), we have

$$u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{p_{mn}} \to 0 \quad \text{as} \quad m,n\to\infty.$$

Hence

$$\chi^2_M(p,q,u) \tag{6}$$

From (3) and (6) we get $\chi^2_M(r,q,u) \subset \chi^2_M(p,q,u)$. This completes the proof. ∎

Proof. (ii): The proof is easy, so omitted. ∎

Theorem 5. *The space $\chi^2_M(p,q,u)$ is solid, hence is monotone.*

Proof. Let $(x_{mn}) \in \chi^2_{M_{mn}}(p,q,u)$ and (α_{mn}) be a sequence of scalars such that $|\alpha_{mn}|^{1/m+n} \le 1$ for all $m,n\in N$. Then

$$u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|\alpha_{mn}x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{p_{mn}} \le u_{mn}\left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/mn}}{\rho}\right)\right)\right]^{p_{mn}}$$

for all $m,n\in N$

$$\left[M_{mn}\left(q\left(\frac{((m+n)!\,|\alpha_{mn}x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{p_{mn}} \le \left[M_{mn}\left(q\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho}\right)\right)\right]^{p_{mn}}$$

for all $m,n\in N$ This completes the proof. ∎

5. Result

The space $\chi^2_M(p,q,u)$ are not convergence free in general.

Proof. The proof follows from the following example.

Example. Consider the sequences $(x_{mn}),(y_{mn}) \in \chi^2_M(p,q,u)$. Defined as $(x_{mn}) = \frac{1}{(m+n)!}\left(\frac{1}{m+n}\right)^{m+n}$ and $(y_{mn}) = \frac{1}{(m+n)!}\left(\frac{m-n}{m+n}\right)^{m+n}$. Hence $u_{mn}\left[M_{mn}\left(q\left(\frac{1}{\rho(m+n)}\right)\right)\right]^{p_{mn}} \to 0$ as $m,n\to\infty$. Which implies $(x_{mn})=0$.

Also $u_{mn}\left[M_{mn}\left(q\left(\frac{m-n}{\rho(m+n)}\right)\right)\right]^{p_{mn}} \to 0$ as $m,n\to\infty$. But $(y_{mn})\not\to 0$. Hence the space $\chi^2_M(p,q,u)$ are not convergence free in general. This completes the proof. ■

Acknowledgement: I wish to thank the referees for their several remarks and valuable suggestions that improved the presentation of the paper.

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Received on 16.04.2009 and, in revised form, on 18.06.2009.