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THE NÖRLUND ORLICZ SPACE OF DOUBLE GAI SEQUENCES

Abstract. Let χ^2 denotes the space of all double gai sequences. Let Λ^2 denotes the space of all double analytic sequences. This paper is devoted to a study of the general properties of Nörlund double Orlicz space of gai sequence space $\eta\left(\chi^2_M\right)$ and χ^2_M. and Nörlund double Orlicz space of analytic sequence space $\eta\left(\Lambda^2_M\right)$ and Λ^2_M.

Key words: analytic sequence, double sequence, Nörlund space, Orlicz space and gai space.

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1. Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^2 for the set of all complex sequences (x_{mn}), where $m, n \in \mathbb{N}$, the set of positive integers. Then, w^2 is a linear space under the coordinate wise addition and scalar multiplication.

Some initial works on double sequence spaces are due to Bromwich [4]. Later on, the double sequence spaces were studied by Hardy [5], Moricz [9], Moricz and Rhoades [10], Basarir and Solankan [2], Tripathy [17], Turkmenoglu [19], and many others.

Let us define the following sets of double sequences:

$$\mathcal{M}_u(t) := \left\{ (x_{mn}) \in w^2 : \sup_{m,n \in N} |x_{mn}|^{t_{mn}} < \infty \right\},$$

$$\mathcal{C}_p(t) := \left\{ (x_{mn}) \in w^2 : p - \lim_{m,n\to\infty} |x_{mn} - l|^{t_{mn}} = 1 \,\text{for some}\, l \in \mathbb{C} \right\},$$

$$\mathcal{C}_{0p}(t) := \left\{ (x_{mn}) \in w^2 : p - \lim_{m,n\to\infty} |x_{mn}|^{t_{mn}} = 1 \right\},$$

$$\mathcal{L}_u(t) := \left\{ (x_{mn}) \in w^2 : \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} |x_{mn}|^{t_{mn}} < \infty \right\},$$

$$\mathcal{C}_{bp}(t) := \mathcal{C}_p(t) \bigcap \mathcal{M}_u(t) \quad \text{and} \quad \mathcal{C}_{0bp}(t) = \mathcal{C}_{0p}(t) \bigcap \mathcal{M}_u(t);$$

where $t = (t_{mn})$ is the sequence of strictly positive reals t_{mn} for all $m, n \in \mathbb{N}$ and $p - \lim_{m,n\to\infty}$ denotes the limit in the Pringsheim's sense. In the case $t_{mn} = 1$ for all $m, n \in \mathbb{N}$; $\mathcal{M}_u(t)$, $\mathcal{C}_p(t)$, $\mathcal{C}_{0p}(t)$, $\mathcal{L}_u(t)$, $\mathcal{C}_{bp}(t)$ and $\mathcal{C}_{0bp}(t)$ reduce to the sets $\mathcal{M}_u$, $\mathcal{C}_p$, $\mathcal{C}_{0p}$, $\mathcal{L}_u$, $\mathcal{C}_{bp}$ and $\mathcal{C}_{0bp}$, respectively. Now, we may summarize the knowledge given in some document related to the double sequence spaces. Gökhan and Colak [21, 22] have proved that $\mathcal{M}_u(t)$ and $\mathcal{C}_p(t)$, $\mathcal{C}_{bp}(t)$ are complete paranormed spaces of double sequences and gave the $\alpha-, \beta-, \gamma-$ duals of the spaces $\mathcal{M}_u(t)$ and $\mathcal{C}_{bp}(t)$. Quite recently, in her PhD thesis, Zeltser [23] has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen and Edely [24] have recently introduced the statistical convergence and Cauchy for double sequences and given the relation between statistical convergent and strongly Cesàro summable double sequences. Nextly, Mursaleen [25] and Mursaleen and Edely [26] have defined the almost strong regularity of matrices for double sequences and applied these matrices to establish a core theorem and introduced the $M-$core for double sequences and determined those four dimensional matrices transforming every bounded double sequences $x = (x_{jk})$ into one whose core is a subset of the $M-$core of x. More recently, Altay and Basar [27] have defined the spaces $\mathcal{BS}, \mathcal{BS}(t), \mathcal{CS}_p, \mathcal{CS}_{bp}, \mathcal{CS}_r$ and $\mathcal{BV}$ of double sequences consisting of all double series whose sequence of partial sums are in the spaces $\mathcal{M}_u$, $\mathcal{M}_u(t)$, $\mathcal{C}_p$, $\mathcal{C}_{bp}$, $\mathcal{C}_r$ and $\mathcal{L}_u$, respectively, and also examined some properties of those sequence spaces and determined the $\alpha-$ duals of the spaces $\mathcal{BS}$, $\mathcal{BV}$, $\mathcal{CS}_{bp}$ and the $\beta(\vartheta)-$ duals of the spaces $\mathcal{CS}_{bp}$ and $\mathcal{CS}_r$ of double series. Quite recently Basar and Sever [28] have introduced the Banach space $\mathcal{L}_q$ of double sequences corresponding to the well-known space ℓ_q of single sequences and examined some properties of the space $\mathcal{L}_q$. Quite recently Subramanian and Misra [29] have studied the space $\chi^2_M(p, q, u)$ of double sequences and gave some inclusion relations.

We need the following inequality in the sequel of the paper. For $a, b \geq 0$ and $0 < p < 1$, we have

$$(a + b)^p \leq a^p + b^p. \tag{1}$$

The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence (s_{mn}) is convergent, where $s_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m, n \in \mathbb{N})$ (see[1]).

A sequence $x = (x_{mn})$ is said to be double analytic if $\sup_{mn} |x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences will be denoted by Λ^2. A sequence $x = (x_{mn})$ is called double gai sequence if $((m+n)! |x_{mn}|)^{1/m+n} \to 0$ as $m, n \to \infty$. The double gai sequences will be denoted by χ^2. By ϕ, we denote the set of all finite sequences.

Consider a double sequence $x = (x_{ij})$. The $(m, n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=1}^{m,n} x_{ij} \Im_{ij}$ for all $m, n \in \mathbb{N}$; where $\Im_{ij}$

denotes the double sequence whose only non zero term is $\frac{1}{(i+j)!}$ in the $(i,j)^{th}$ place for each $i,j \in \mathbb{N}$.

An FK-space(or a metric space)X is said to have AK property if $(\Im_{mn})$ is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$. An FDK-space is a double sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings $x = (x_k) \to (x_{mn})(m,n \in \mathbb{N})$ are also continuous.

Orlicz[13] used the idea of Orlicz function to construct the space $\left(L^M\right)$. Lindenstrauss and Tzafriri [7] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p\,(1 \leq p < \infty)$. subsequently, different classes of sequence spaces were defined by Parashar and Choudhary [14], Mursaleen et al. [11], Bektas and Altin [3], Tripathy et al. [18], Rao and Subramanian [15], and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in [6].

Recalling [13] and [6], an Orlicz function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing, and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by subadditivity of M, then this function is called modulus function, defined by Nakano [12] and further discussed by Ruckle [16] and Maddox [8], and many others.

An Orlicz function M is said to satisfy the Δ_2- condition for all values of u if there exists a constant $K > 0$ such that $M(2u) \leq KM(u)\,(u \geq 0)$. The Δ_2- condition is equivalent to $M(\ell u) \leq K\ell M(u)$, for all values of u and for $\ell > 1$.

Lindenstrauss and Tzafriri [7] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \quad \text{for some } \rho > 0 \right\}.$$

The space ℓ_M with the norm

$$\|x\| = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p\,(1 \leq p < \infty)$, the spaces ℓ_M coincide with the classical sequence space ℓ_p.

If X is a sequence space, we give the following definitions:

(i) $X^{'}$ = the continuous dual of X;

(ii) $X^{\alpha} = \left\{ a = (a_{mn}) : \sum_{m,n=1}^{\infty} |a_{mn}x_{mn}| < \infty, \text{ for each } x \in X \right\}$;

(*iii*) $X^{\beta} = \left\{ a = (a_{mn}) : \sum_{m,n=1}^{\infty} a_{mn}x_{mn} \text{ is convegent, for each } x \in X \right\};$

(*iv*) $X^{\gamma} = \left\{ a = (a_{mn}) : \sup_{MN} \geq 1 \left| \sum_{m,n=1}^{M,N} a_{mn}x_{mn} \right| < \infty, \text{ for each } x \in X \right\};$

(*v*) let X be an FK-space $\supset \phi$; then $X^{f} = \left\{ f(\Im_{mn}) : f \in X' \right\};$

(*vi*) $X^{\delta} = \left\{ a = (a_{mn}) : \sup_{mn} |a_{mn}x_{mn}|^{1/m+n} < \infty, \text{ for each } x \in X \right\};$

X^{α}, X^{β}, X^{γ} and X^{δ} are called $\alpha-$ (or Köthe-Toeplitz) dual of $X, \beta-$ (or generalized-Köthe-Toeplitz) dual of $X, \gamma-$ dual of $X, \delta-$ dual of X respectively. X^{α} is defined by Gupta and Kamptan [20]. It is clear that $X^{\alpha} \subset X^{\beta}$ and $X^{\alpha} \subset X^{\gamma}$, but $X^{\alpha} \subset X^{\gamma}$ does not hold, since the sequence of partial sums of a double convergent series need not to be bounded.

The notion of difference sequence spaces of single sequences was introduced by Kizmaz [30] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c$, c_0 and ℓ_{∞}, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$. Here c, c_0 and ℓ_{∞} denote the classes of convergent, null and bounded scalar valued single sequences respectively. The above difference spaces are Banach spaces normed by

$$\|x\| = |x_1| + sup_{k\geq 1} |\Delta x_k|\}.$$

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by

$$Z(\Delta) = \left\{ x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z \right\}$$

where $Z = \Lambda^2$, χ^2 and $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m, n \in \mathbb{N}$

Let $(P_{m,n})_{m,n=0}^{\infty}$ be a sequence of non-negative real numbers with $p_{00} > 0$. Consider the transformation

$$y_{mn} = \frac{1}{\sum\limits_{i=0}^{m}\sum\limits_{j=0}^{n} p_{ij}} \sum_{i=0}^{m}\sum_{j=0}^{n} p_{ij}x_{m-i,n-j}$$

for $m, n = 0, 1, 2, \cdots$. The set of all (x_{mn}) for which $(y_{mn}) \in \chi_M^2$ is called the Nörlund Orlicz space of double gai sequences. The Nörlund Orlicz space of double gai sequences and is denoted by $\eta\left(\chi_M^2\right)$. Similarly the set of all (x_{mn}) for which $(y_{mn}) \in \Lambda_M^2$ is called the Nörlund Orlicz space of double analytic sequences and is denoted by $\eta\left(\Lambda_M^2\right)$. We write $P_{mn} = p_{00} + \cdots + p_{mn}$, for $m, n = 0, 1, 2, \cdots$.

All absolutely convex absorbent closed subset of locally convex Topological Vector Space X is called barrel. X is called barreled space if each barrel is a neighbourhood of zero.

A locally convex Topological Vector Space X is said to be semi reflexive if each bounded closed set in X is $\sigma\left(X, X'\right)$ −compact.

Hardy [31] gave regularity conditions for a Nörlund matrix. Based on this Nörlund matrix transformation, in this paper the Nörlund Orlicz space $\eta\left(\chi^2_M\right)$ of double gai sequences is introduced. Similar results hold for the Orlicz space of analytic sequences. We have also examined as to whether the space χ^2_M is barrled space and semi reflexive space.

2. Definitions and preliminaries

χ^2_M and Λ^2_M denote the Pringscheim's sense of double Orlicz space of gai sequences and Pringscheim's sense of double Orlicz space of bounded sequences respecctively.

The notion of a modulus function was introduced by Nakano [12]. We recall that a modulus f is a function from $[0,\infty) \to [0,\infty)$, such that

(a) $f(x) = 0$ if and only if $x = 0$

(b) $f(x+y) \le f(x) + f(y)$, for all $x \ge 0$, $y \ge 0$,

(c) f is increasing,

(d) f is continuous from the right at 0. Since $|f(x) - f(y)| \le f(|x-y|)$, it follows from condition (d) that f is continuous on $[0,\infty)$.

Define the sets:

$$\chi^2_M = \left\{ x \in w^2 : \left(M\left(\frac{((m+n)!\,|x_{mn}|)^{1/m+n}}{\rho} \right) \right) \to 0 \right.$$
$$\left. \text{as } m,n \to \infty \text{ for some } \rho > 0 \right\}$$

and

$$\Lambda^2_M = \left\{ x \in w^2 : \sup_{m,n \ge 1} \left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right) < \infty \text{ for some } \rho > 0 \right\}.$$

The space Λ^2_M is a metric space with the metric

$$d(x,y) = \inf\left\{ \rho > 0 : \sup_{m,n \ge 1} \left(M\left(\frac{|x_{mn} - y_{mn}|}{\rho} \right) \right)^{1/m+n} \le 1 \right\}.$$

The space χ^2_M is a metric space with the metric

$$\widetilde{d}(x,y) = \inf\left\{\rho > 0 : \sup_{m,n\geq 1}\left(M\left(\frac{(m+n)!\,|x_{mn}-y_{mn}|}{\rho}\right)\right)^{1/m+n} \leq 1\right\}.$$

3. Main results

Proposition 1. $\eta\left(\chi^2_M\right) = \chi^2_M$.

Proof. Let $x = (x_{mn}) \in \eta\left(\chi^2_M\right)$. Then $y \in \chi^2_M$ so that for every $\epsilon > 0$, we have a positive integer n_0 such that

$$\left(M\left(\left|\frac{p_{00}(m+n)!x_{mn} + \cdots + p_{mn}(0+0)!x_{00}}{\rho P_{mn}}\right|\right)\right) < \epsilon^{m+n}$$

for all $m,n \geq n_0$. Take $p_{00} = 1$; $p_{11} = \cdots = p_{mn} = 0$. We then have $\left(M\left(\frac{(m+n)!|x_{mn}|}{\rho}\right)\right) < \epsilon^{m+n}, \forall m,n \geq n_0$. Therefore $x = (x_{mn}) \in \chi^2_M$. Hence

$$\eta\left(\chi^2_M\right) \subset \chi^2_M. \tag{2}$$

On the other hand, let $x = (x_{mn}) \in \chi^2_M$. But for any given $\epsilon > 0$, there exists a positive integer n_0 such that $\left(M\left(\frac{(m+n)!|x_{mn}|}{\rho}\right)\right) < \epsilon^{m+n}, \forall m,n \geq n_0$. We have

$$\begin{aligned}
&\left(M\left(\frac{(m+n)!\,|y_{mn}|}{\rho}\right)\right) \\
&\quad\leq \left(M\left(\left|\frac{p_{00}(m+n)!x_{mn} + \cdots + p_{mn}(0+0)!x_{00}}{\rho P_{mn}}\right|\right)\right) \\
&\quad\leq \frac{1}{P_{mn}}\left[p_{00}\left(M\left(\frac{(m+n)!\,|x_{mn}|}{\rho}\right)\right) + \cdots \right. \\
&\qquad\qquad \left. + \, p_{mn}\left(M\left(\frac{(0+0)!\,|x_{00}|}{\rho}\right)\right)\right] \\
&\quad\leq \frac{1}{P_{mn}}\left[p_{00}\epsilon^{m+n} + \cdots + p_{mn}\epsilon^{0+0}\right] \\
&\quad\leq \frac{\epsilon^{m+n}}{P_{mn}}\left[p_{00} + \cdots + p_{mn}\right] \\
&\quad\leq \frac{\epsilon^{m+n}}{P_{mn}}P_{mn} = \epsilon^{m+n}, \ \ \forall m,n \geq n_0.
\end{aligned}$$

Therefore $(y_{mn}) \in \chi^2_M$. Consequently $x \in \eta\left(\chi^2_M\right)$. Hence

$$\chi^2_M \subset \eta\left(\chi^2_M\right). \tag{3}$$

From (2) and (3) we obtain $\eta\left(\chi^2_M\right) = \chi^2_M$. This completes the proof. ■

Proposition 2. $\eta\left(\Lambda_M^2\right) = \Lambda_M^2$.

Proof. Let $(x_{mn}) \in \Lambda_M^2$. Then there exists a positive constant T such that

$$\left(M\left(\frac{(m+n)!\,|x_{mn}|}{\rho}\right)\right) \leq T^{m+n} \quad \text{for} \quad m,n = 0,1,2,\cdots.$$

$$\begin{aligned}\left(M\left(\frac{(m+n)!\,|y_{mn}|}{\rho}\right)\right) &\leq \frac{p_{00}T^{m+n} + \cdots + p_{mn}T^{0+0}}{P_{mn}} \\ &\leq \frac{T^{m+n}}{P_{mn}}\left[p_{00} + \cdots + \frac{p_{mn}}{T^{m+n}}\right] \\ &\leq \frac{T^{m+n}}{P_{mn}}\left[p_{00} + \cdots + p_{mn}\right] \\ &\leq \frac{T^{m+n}}{P_{mn}}P_{mn} = T^{m+n}, \quad \text{for} \ \ m,n = 0,1,2,\cdots.\end{aligned}$$

Hence $(y_{mn}) \in \Lambda_M^2$. But then $x = (x_{mn}) \in \eta\left(\chi_M^2\right)$. Consequently

$$\Lambda_M^2 \subset \eta\left(\Lambda_M^2\right). \tag{4}$$

On the other hand let $(x_{mn}) \in \eta\left(\Lambda_M^2\right)$. Then $(y_{mn}) \in \Lambda_M^2$. Hence there exists a positive constant T such that $\left(M\left(\frac{|y_{mn}|}{\rho}\right)\right) < T^{m+n}$ for $m,n = 0,1,2,\cdots$. This in turn implies that

$$\left(M\left(\left|\frac{p_{00}(m+n)!x_{mn} + \cdots + P_{mn}(0+0)!x_{00}}{\rho P_{mn}}\right|\right)\right) < T^{m+n}.$$

Hence

$$\frac{1}{P_{mn}}\left(M\left(\frac{|p_{00}(m+n)!x_{mn} + \cdots + p_{mn}(0+0)!x_{00}|}{\rho}\right)\right) < T^{m+n}$$

and thus

$$\left(M\left(\frac{|p_{00}(m+n)!x_{mn} + \cdots + p_{mn}(0+0)!x_{00}|}{\rho}\right)\right) < P_{mn}T^{m+n}.$$

Take $p_{00} = 1$; $p_{11} = \cdots = p_{mn} = 0$. Then it follows that $P_{mn} = 1$ and so $\left(M\left(\frac{(m+n)!|x_{mn}|}{\rho}\right)\right) < T^{m+n}$ for all m,n. Consequently $x = (x_{mn}) \in \Lambda_M^2$. Hence

$$\eta\left(\Lambda_M^2\right) \subset \Lambda_M^2. \tag{5}$$

From (4) and (5) we get $\eta\left(\Lambda_M^2\right) = \Lambda_M^2$. This completes the proof. ∎

Proposition 3. *χ^2_M is not a barreled space.*

Proof. Let

$$A = \left\{ x \in \chi^2_M : \left(M \left(\frac{((m+n)! \, |x_{mn}|)^{\frac{1}{m+n}}}{\rho} \right) \right) \leq \frac{1}{m+n}, \ \forall m, n \right\}.$$

Then A is an absolutely convex, closed absorbent in χ^2_M. But A is not a neighbour hood of zero. Hence χ^2_M is not barreled. ■

Proposition 4. *χ^2_M is not semi reflexive.*

Proof. Let $\left\{ \Im^{(mn)} \right\} \in U$ be the closed unit ball in χ^2_M. Clearly no subsequence of $\left\{ \Im^{(mn)} \right\}$ which weakly converges to any $y \in \chi^2_M$. Hence χ^2_M is not semi reflexive. ■

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