F A S C I C U L I M A T H E M A T I C I

Nr 47 2011

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DECOMPOSITION OF BITOPOLOGICAL $(1,2)^*$-HOMEOMORPHISMS

Abstract. In this study, two new classes of generalized $(1,2)^*$-homeomorphisms are introduced. We investigate their relationship with other known generalized homeomorphisms. Moreover, some properties of these two $(1,2)^*$-homeomorphisms are obtained.

Key words: $(1,2)^*$-homeomorphism, $(1,2)^*$-sg-closed set, (1, 2)*- gsg-homeomorphism, $(1,2)^*$-gs-closed set, $(1,2)^*$-sgs-homeomorphism.

AMS Mathematics Subject Classification: 54E55.

1. Introduction

Levine [10] has generalized the concept of closed sets to generalized closed sets. Bhattacharyya and Lahiri [3] have generalized the concept of closed sets to semi-generalized closed sets with the help of semi-open sets and obtained various topological properties. Arya and Nour [2] have defined generalized semi-open sets with the help of semi-openness and used them to obtain some characterizations of s-normal spaces. Devi et al [9] defined two classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two classes of maps called sgc-homeomorphisms and gsc-homeomorphisms. In [1], sgs-homeomorphisms and gsg-homeomorphisms were recently introduced and investigated by Ozcelik and Narli.

In this paper, we introduce two classes of maps called $(1,2)^*$-sgs-homeomorphisms and $(1,2)^*$-gsg-homeomorphisms and study their properties. These bitopological notions are generalized from the topological notions in [1]. These generalizations are substantiated with suitable examples and investigated with utmost care.

2. Preliminaries

Throughout the present paper, (X,τ_1,τ_2) and (Y,σ_1,σ_2) denote bitopological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 1 ([13])**.** *Let A be a subset of X. Then A is said to be $\tau_{1,2}$-open if $A = M \cup N$ where $M \in \tau_1$ and $N \in \tau_2$.*

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Definition 2 ([13])**.** *Let A be a subset of X. Then*
(i) The $\tau_{1,2}$-interior of A, denoted by $\tau_{1,2} - int(A)$, is defined as $\cup\{F : F \subseteq A$ and F is $\tau_{1,2} - open\}$;
(ii) The $\tau_{1,2}$-closure of A, denoted by $\tau_{1,2} - cl(A)$, is defined as $\cap\{F : A \subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

Note 1 ([13])**.** Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 3. *Let A be a subset of X. Then A is said to be*
(i) $(1,2)^*$-semi-open [13] if $A \subseteq \tau_{1,2}cl(\tau_{1,2}int(A))$;
(ii) $(1,2)^*$-semi-closed [13] if $\tau_{1,2}int(\tau_{1,2}cl(A)) \subseteq A$.
The complement of $(1,2)^*$-semi-open set is called $(1,2)^*$-semi-closed.

Result 1 ([13])**.** (i) Every $\tau_{1,2}$-closed set is $(1,2)^*$-semi-closed but not conversely.
(ii) Every $\tau_{1,2}$-open set is $(1,2)^*$-semi-open but not conversely.

Definition 4 ([13])**.** *A map $f : X \to Y$ is called*
(i) $(1,2)^*$-closed if $f(F)$ is $\sigma_{1,2}$-closed in Y for each $\tau_{1,2}$-closed set $F \in X$;
(ii) $(1,2)^*$-open if $f(F)$ is $\sigma_{1,2}$-open in Y for each $\tau_{1,2}$-open set F in X;
(iii) $(1,2)^*$-semi-closed if $f(F)$ is $(1,2)^*$-semi-closed in Y for each $\tau_{1,2}$-closed set F in X.

Result 2. Every $(1,2)^*$-closed map is $(1,2)^*$-semi-closed but not conversely.

Definition 5 ([13])**.** *Let A be a subset of X. Then*
(i) $(1,2)^*$-$sint(A) = \cup\{G_i : G_i$ is $(1,2)^*$-semi-open in X and $G_i \subset A\}$;
(ii) $(1,2)^*$-$scl(A) = \cap\{H_i : H_i$ is $(1,2)^*$-semi-closed in X and $H_i \supset A\}$.

Definition 6 ([13])**.** *Let A be a subset of X. Then A is said to be $(1,2)^*$-sg-closed if $(1,2)^*$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*$-semi -open.*
The complement of $(1,2)^*$-sg-closed set is called $(1,2)^*$-sg-open.
The family of all $(1,2)^*$-sg-closed sets of X is denoted by $(1,2)^*$-$sgc(X)$.

Result 3 ([13])**.** Every $(1,2)^*$-semi-closed set is $(1,2)^*$-sg-closed but not conversely.

Definition 7 ([12]). *Let A be a subset of X. Then A is said to be $(1,2)^*$-gs-closed if $(1,2)^*$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.*
The complement of $(1,2)^*$-gs-closed set is $(1,2)^*$-gs-open.
The family of all $(1,2)^*$-gs-closed sets of X is denoted by $(1,2)^*$-$gsc(X)$.

Result 4 ([12]). Every $(1,2)^*$-sg-closed set is $(1,2)^*$-gs-closed but not conversely.

Definition 8 ([12],[13],[14]). *A map $f : X \to Y$ is called*
(i) $(1,2)^*$-continuous if $f^{-1}(V)$ is $\tau_{1,2}$-closed in X for each $\sigma_{1,2}$-closed set V in Y;
(ii) $(1,2)^*$-sg-continuous if $f^{-1}(V)$ is $(1,2)^*$-sg-closed in X for each $\sigma_{1,2}$-closed set V of Y;
(iii) $(1,2)^*$-gs-continuous if $f^{-1}(V)$ is $(1,2)^*$-gs-closed in X for each $\sigma_{1,2}$-closed set V of Y;
(iv) $(1,2)^*$-sg-closed if $f(F)$ is $(1,2)^*$-sg-closed in Y for each $\tau_{1,2}$-closed set F of X;
(v) $(1,2)^*$-sg-open if $f(F)$ is $(1,2)^*$-sg-open in Y for each $\tau_{1,2}$-open set F of X.

Result 5 ([13]). Every $(1,2)^*$-semi-closed map is a $(1,2)^*$-sg-closed.

Definition 9 ([12]). *A map $f : X \to Y$ is called*
(i) $(1,2)^*$-gs-open if $f(F)$ is $(1,2)^*$-gs-open in Y for each $\tau_{1,2}$-open set F of X;
(ii) $(1,2)^*$-gs-closed if $f(F)$ is $(1,2)^*$-gs-closed in Y for each $\tau_{1,2}$-closed set F of X.

Result 6 ([12]). Every $(1,2)^*$-sg-closed map is $(1,2)^*$-gs-closed.

Definition 10 ([13],[14]). *A map $f : X \to Y$ is called*
(i) $(1,2)^*$-sg-irresolute if $f^{-1}(V)$ is $(1,2)^*$-sg-closed in X for each $(1,2)^*$ -sg-closed set V in Y;
(ii) $(1,2)^*$-gs-irresolute if $f^{-1}(V)$ is $(1,2)^*$-gs-closed in X for each $(1,2)^*$ -gs-closed set V in Y.

Definition 11 ([13],[14]). *A bijective map $f : X \to Y$ is called*
(i) $(1,2)^*$-homeomorphism if f is both $(1,2)^*$-continuous and $(1,2)^*$-open;
(ii) $(1,2)^*$-sg-homeomorphism if f is both $(1,2)^*$-sg-continuous and $(1,2)^*$ -sg-open;
(iii) $(1,2)^*$-sgc-homeomorphism if f is $(1,2)^*$-sg-irresolute and f^{-1} is $(1,2)^*$-sg-irresolute;
(iv) $(1,2)^*$-gs-homeomorphism if f is both $(1,2)^*$-gs-continuous and $(1,2)^*$-gs-open;
(v) $(1,2)^*$-gsc-homeomorphism if f is $(1,2)^*$-gs-irresolute and f^{-1} is $(1,2)^*$-gs-irresolute.

Result 7. (*i*) Every $(1,2)^*$-sgc-homeomorphism is $(1,2)^*$-sg-homeomorphism but not conversely [14];

(*ii*) Every $(1,2)^*$-sg-homeomorphism is $(1,2)^*$-gs-homeomorphism but not conversely [14];

(*iii*) Every $(1,2)^*$-gsc-homeomorphism is $(1,2)^*$-gs-homeomorphism but not conversely [12].

Definition 12 ([12]). *A space X is called*

(*i*) $(1,2)^*$-$T_{1/2}$ *if and only if every* $(1,2)^*$*-gs-closed set is* $(1,2)^*$*-semi-closed;*

(*ii*) $(1,2)^*$-T_b *if every* $(1,2)^*$*-gs-closed set is* $\tau_{1,2}$*-closed.*

We introduce the following definitions.

Definition 13. *A map $f : X \to Y$ is called* $(1,2)^*$*-gsg-irresolute if* $f^{-1}(F)$ *is* $(1,2)^*$*-sg-closed in X for each* $(1,2)^*$*-gs-closed in Y.*

Definition 14. *A bijective map $f : X \to Y$ is called* $(1,2)^*$*-gsg-homeomorphism if f and f^{-1} are both* $(1,2)^*$*-gsg-irresolute.*

If there exists a $(1,2)^*$*-gsg-homeomorphism from X to Y, then the spaces X and Y are said to be* $(1,2)^*$*-gsg-homeomorphic.*

The family of all $(1,2)^*$*-gsg-homeomorphisms of X is denoted by* $(1,2)^*$ *-gsgh*(X).

Definition 15. *A map $f : X \to Y$ is called a* $(1,2)^*$*-sgs-irresolute if* f^{-1} (A) *is* $(1,2)^*$*-gs-closed in X for each* $(1,2)^*$*-sg-closed set A of Y.*

Definition 16. *A bijective map $f : X \to Y$ is called a* $(1,2)^*$*-sgs-homeomorphism if f and f^{-1} are both* $(1,2)^*$*-sgs-irresolute.*

If there exists a $(1,2)^*$*-sgs-homeomorphism from X to Y, then the spaces X and Y are said to be* $(1,2)^*$*-sgs-homemorphic spaces.*

3. Properties of $(1,2)^*$-gsg-homeomorphism

Remark 1. The following two examples show that the concepts of $(1,2)^*$-homeomorphism and $(1,2)^*$-gsg-homeomorphism are independent of each other.

Example 1. Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X\}$ and $\tau_2 = \{\varphi, X, \{a\}\}$. Then the sets in $\{\varphi, X, \{a\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\varphi, X, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Let $I_x : (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ be the identity map. Clearly, I_x is a $(1,2)^*$-homeomorphism but it is not a $(1,2)^*$-gsg-homeomorphism.

Example 2. Let $X = \{a, b\}$, $\tau_1 = \{\varphi, X, \{b\}\}$, $\tau_2 = \{\varphi, X, \{a\}\}$, $\sigma_1 = \{\varphi, X\}$ and $\sigma_2 = \{\varphi, X\}$. Then the sets in $\{\varphi, X, \{a\}, \{b\}\}$ are called $\tau_{1,2}$-open and $\tau_{1,2}$-closed; and the sets in $\{\phi, X\}$ are $\sigma_{1,2}$ -open and

$\sigma_{1,2}$-closed. Let $I_x : (X, \tau_1, \tau_2) \to (X, \sigma_1, \sigma_2)$ be the identity map. Clearly, I_x is a $(1,2)^*$-gsg-homeomorphism but it is not a $(1,2)^*$-homeomorphism.

Example 3. Every $(1,2)^*$-gsg-homeomorphism implies both a $(1,2)^*$-gsc-homeomorphism and a $(1,2)^*$-sgc-homeomorphism.

However the converse is not true as shown by the following example.

Example 4. Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{b\}\}$ and $\tau_2 = \{\varphi, X\}$. Then the sets in $\{\varphi, X, \{b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\varphi, X, \{a, c\}\}$ are called $\tau_{1,2}$-closed. We have $(1,2)^*$-sgc$(X) = \{\varphi, X, \{a\}, \{c\}, \{a, c\}\}$ and $(1,2)^*$-gsc$(X) = \{\varphi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Let $I_x : (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ be the identity map. Clearly I_x is both $(1,2)^*$-gsc-homeomorphism and $(1,2)^*$-sgc-homeomorphism. Since the set $\{b, c\}$ is $(1,2)^*$-gs-closed but the set $I_x^{-1}(\{b, c\}) = \{b, c\}$ is not $(1,2)^*$-sg-closed, the identity map I_x is not a $(1,2)^*$-gsg-homeomorphism on X.

Remark 2. Every $(1,2)^*$-gsg-homeomorphism implies both a $(1,2)^*$-gs-homeomorphism and a $(1,2)^*$-sg-homeomorphism.

However the converse is not true as shown by the following example.

Example 5. In Example 4, Clearly I_x is both $(1,2)^*$-gs-homeomorphism and $(1,2)^*$-sg-homeomorphism. However, I_x is not $(1,2)^*$-gsg-homeomorphism.

4. Properties of $(1,2)^*$-sgs-homeomorphism

Remark 3. Every $(1,2)^*$-sgc-homeomorphism and $(1,2)^*$-gsc-homeomorphism implies a $(1,2)^*$-sgs-homeomorphism.

However the converse is not true as shown by the following examples.

Example 6. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\varphi, X, \{b\}, \{b, c\}\}$, $\sigma_1 = \{\varphi, Y, \{b\}\}$ and $\sigma_2 = \{\varphi, Y, \{a, b\}\}$. Then the sets in $\{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\varphi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Moreover the sets in $\{\varphi, Y, \{b\}, \{a, b\}\}$ are called $\sigma_{1,2}$-open and the sets in $\{\varphi, Y, \{c\}, \{a, c\}\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^*$-sgc$(X) = (1,2)^*$-gsc$(X) = P(X) \backslash \{\{b\}, \{a, b\}\}$ where $P(X)$ is the power set of X and $(1,2)^*$-sgc$(Y) = \{\varphi, X, \{a\}, \{c\}, \{a, c\}\}$ and $(1,2)^*$-gsc$(Y) = P(Y) \backslash \{\{b\}, \{a, b\}\}$. Clearly the identity map $I_X : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a $(1,2)^*$-sgs-homeomorphism but it is not a $(1,2)^*$-sgc-homeomorphism.

Example 7. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}\}$, $\tau_2 = \{\varphi, X\}$, $\sigma_1 = \{\varphi, Y, \{b\}\}$ and $\sigma_2 = \{\varphi, Y, \{a, b\}\}$. We have $(1,2)^*$-sgc$(X) = \{\varphi, X, \{b\}, \{c\}, \{b, c\}\}$, $(1,2)^*$-gsc$(X) = \{\varphi, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $(1,2)^*$-sgc(Y)

$= \{\varphi, Y, \{a\}, \{c\}, \{a, c\}\}$ and $(1,2)^*$-gsc$(Y) = \{\varphi, Y, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Define $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = b$; $f(b) = a$; $f(c) = c$. Clearly f is a $(1,2)^*$-sgs-homeomorphism but it is not a $(1,2)^*$-gsc-homeomorphism.

Result 8. Every $(1,2)^*$-homeomorphism is a $(1,2)^*$-sgs-homeomorphism.

However the converse is not true as seen from the following example.

Example 8. In Example 7, clearly f is $(1,2)^*$-sgs-homeomorphism but it is not a $(1,2)^*$-homeomorphism.

Remark 4. Every $(1,2)^*$-sgs-homeomorphism is a $(1,2)^*$-gs-homeomorphism.

However the converse is not true as seen from the following example.

Example 9. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a, b\}\}$, $\tau_2 = \{\varphi, X\}$, $\sigma_1 = \{\varphi, Y, \{b\}\}$ and $\sigma_2 = \{\varphi, Y, \{a, b\}\}$. We have $(1,2)^*$-sgc$(X) = (1,2)^*$-gsc$(X) = \{\varphi, X, \{c\}, \{a, c\}, \{b, c\}\}$, $(1,2)^*$-sgc$(Y) = \{\varphi, Y, \{a\}, \{c\}, \{a, c\}\}$ and $(1,2)^*$-gsc$(Y) = \{\varphi, Y, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Then, the identity map $I : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a $(1,2)^*$-gs-homeomorphism but it is not $(1,2)^*$-sgs-homeomorphism.

Example 10. The map $I : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is given by Example 9 is a $(1,2)^*$-sg-homeomorphism but it is not a $(1,2)^*$-sgs-homeomor- phism.

Result 9. (i) From the Example 10, we can see that any $(1,2)^*$-sg-homeomorphism is not a $(1,2)^*$-sgs-homeomorphism.

(ii) Every $(1,2)^*$-gsg-homeomorphism is a $(1,2)^*$-sgs-homeomorphism and the converse is not true as seen from the following example.

Example 11. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}\}$, $\tau_2 = \{\varphi, X, \{a, b\}\}$, $\sigma_1 = \{\varphi, Y, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma_2 = \{\varphi, Y, \{b, c\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Clearly f is a $(1,2)^*$-sgs-homeomorphism but it is not a $(1,2)^*$-gsg-homeomorphism.

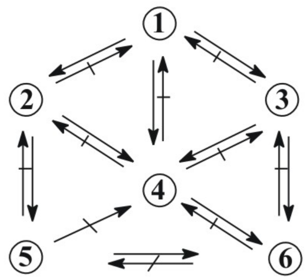
Theorem 1. (i) *Every $(1,2)^*$-sgs-homeomorphism from a $(1,2)^*$-$T_{1/2}$ space onto itself is a $(1,2)^*$-gsg-homeomorphism. This implies that $(1,2)^*$-sgs-homeomorphism is both a $(1,2)^*$-sgc-homeomorphism and $(1,2)^*$-gsc-homeomorphism.*

(ii) *Every $(1,2)^*$-sgs-homeomorphism from a $(1,2)^*$- T_b space onto itself is a $(1,2)^*$-homeomorphism. This implies that $(1,2)^*$-sgs-homeomorphism is a $(1,2)^*$-gs-homeomorphism, a $(1,2)^*$-sg-homeomorphism, a $(1,2)^*$-sgc-homeomorphism, a $(1,2)^*$-gsc-homeomorphism and a $(1,2)^*$-gsg-homeomorphism.*

Proof. *(i)* In a $(1,2)^*$-$T_{1/2}$ space, every $(1,2)^*$-gs-closed set is $(1,2)^*$-semiclosed.

(ii) In a $(1,2)^*$-T_b space, every $(1,2)^*$-gs-closed set is $\tau_{1,2}$-closed. ■

5. Conclusion



where

(1)	$(1,2)^*$-gsg-homeomorphism
(2)	$(1,2)^*$-sgc-homeomorphism
(3)	$(1,2)^*$-gsc-homeomorphism
(4)	$(1,2)^*$-sgs-homeomorphism
(5)	$(1,2)^*$-sg-homeomorphism
(6)	$(1,2)^*$-gs-homeomorphism

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Received on 19.08.2010 and, in revised form, on 15.01.2011.