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ON THE ASYMPTOTIC BEHAVIOR OF PEXIDERIZED ADDITIVE MAPPING ON SEMIGROUPS

Abstract. In this paper some asymptotic behaviors of the Pexiderized additive mappings can be proved for functions on commutative semigroup to a complex normed linear space under some suitable conditions. As a consequence of our result, we give some generalizations of Skof theorem and S.-M. Joung theorem. Furthermore, in this note we present a affirmative answer to problem 18, in the thirty-first ISFE.

Key words: asymptotic behavior, stability, additive mappings, Pexiderized additive mapping.

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1. Introduction

The starting point of the stability theory of functional equations was the problem formulated by S. M. Ulam in 1940 (see [33]), during a conference at Wisconsin University:

Let $(G,.)$ be a group $(B,.,d)$ be a metric group. Does for every $\varepsilon > 0$, there exists a $\delta > 0$ such that if a function $f : G \to B$ satisfies the inequality

$$d(f(xy), f(x)f(y)) \leq \delta, \quad x, y \in G,$$

there exists a homomorphism $g : G \to B$ such that

$$d(f(x), g(x)) \leq \varepsilon, \quad x \in G?$$

In 1941, Hyers [12] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G is a linear normed space and B is a Banach space. This is the reason for which today this type of stability is called Hyers-Ulam stability of functional equation. In 1950, Aoki [4] generalized Hyers' theorem for approximately additive functions. In 1978, Th. M. Rassias [28] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. Taking this fact into account, the additive functional equation $f(x + y) =$

$f(x)+f(y)$ is said to have the Hyers-Ulam-Rassias stability on (X,Y). This terminology is also applied to the case of other functional equations. For more detailed definitions of such terminology one can refer to [9] and [14]. Thereafter, the stability problem of functional equations has been extended in various directions and studied by several mathematicians [2, 3, 5, 6, 11, 25, 21, 30, 19, 23, 26, 29, 23].

The Hyers-Ulam stability of mappings is in development and several authors have remarked interesting applications of this theory to various mathematical problems. In fact the Hyers-Ulam stability has been mainly used to study problems concerning approximate isometries or quasi-isometries, the stability of Lorentz and conformal mappings, the stability of stationary points, the stability of convex mappings, or of homogeneous mappings, etc [15, 16, 7, 22, 32, 17].

Several authors have used asymptotic conditions in stating approximations to Cauchy's functional equation

$$f(x+y) = f(x) + f(y).$$

P. D. T. A. Elliott [8] showed that if the real function f belongs to the class $L^p(0,z)$ for every $z \geq 0$, where $p \geq 1$, and satisfies the asymptotic condition

$$\lim_{z\to\infty} \frac{\int_0^z \int_0^z |f(x+y) - f(x) - f(y)|^p dxdy}{z} = 0,$$

then there is a constant c such that $f(x) = cx$ almost everywhere on $\mathbb{R}^+$. One of the theorems of J. R. Alexander, C. E. Blair and L. A. Rubel [1] states that if $f \in L^1(0,b)$ for all $b > 0$, and if for almost all $x > 0$

$$\lim_{u\to\infty} \frac{\int_0^y [f(x+y) - f(x) - f(y)]dy}{u} = 0,$$

then for some real number c, $f(x) = cx$ for almost all $x \geq 0$.

F. Skof [31] proved the following theorem and applied the result to the study of an asymptotic behavior of additive functions.

Theorem 1. *Let E_1 and E_2 be a normed space and a Banach space, respectively. Given $a > 0$, suppose a function $f : E_1 \to E$ satisfies the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \delta$$

for some $\delta > 0$ and for all $x, y \in E_1$ with $\|x\| + \|y\| > a$. Then there exists a unique additive function $A : E_1 \to E_2$ such that

$$\|f(x) - A(x)\| \leq 9\delta$$

for all $x \in E_1$.

Using this theorem, F. Skof [31] has studied an interesting asymptotic behavior of additive functions as we see in the following theorem.

Theorem 2. *Let E_1 and E_2 be a normed space and a Banach space, respectively. Suppose z is a fixed point of E_1. For a function $f : E_1 \to E_2$ the following two conditions are equivalent:*

(a) $\|f(x+y) - f(x) - f(y)\| \to 0$ *as* $\|x\| + \|y\| \to \infty$;

(b) $f(x+y) - f(x) - f(y) = 0$

for all $x, y \in E_1$.

S.-M. Joung [20], proved that the Hyers-Ulam stability for Jensen's equation on a restricted domain and the result applied to the study of an interesting asymptotic behavior of the additive mappings-more precisely, he proved that a mapping $f : E_1 \to E_2$ satisfying $f(0) = 0$ is additive if and only if

(a) $\|2f(\frac{x+y}{2}) - f(x) - f(y)\| \to 0$ as $\|x\| + \|y\| \to \infty$.

As a consequence of our result in this paper, we give a simple proofs of Skof theorem (2) and S.-M. Joung theorem and show that Skof and S. M.-Joung theorem is true when E_2 be a complex normed linear space. Also we present some generalization of Skof and S.-M. Joung theorem. Furthermore, some asymptotic behaviors of Pexiderized additive mapping can be proved for functions on commutative semigroup to a complex normed linear space.

During the thirty-first International Symposium on Functional Equations (ISFE), Th. M. Rassias [27] introduced the term *mixed stability* of the function $f : E \to \mathbb{R}$ (or $\mathbb{C}$), where E is a Banach space, with respect to two operations 'addition' and 'multiplication' among any two elements of the set $\{x, y, f(x), f(y)\}$. Then the following question arises. Let $(S, \cdot)$ be an arbitrary semigroup or group and let a mapping $f : S \to R$ (the set of reals) be such that the set $\{f(x \cdot y) - f(x) - f(y) \mid x, y \in S\}$ is bounded. Is it true that there is a mapping $T : S \to R$ that satisfies

$$T(x \cdot y) - T(x) - T(y) = 0$$

for all $x, y \in S$ and that the set $\{T(x) - f(x) \mid x \in S\}$ is bounded?

G. L. Forti in [10] gave a negative answer to this problem (see also [13]). In this paper we give a affirmative answer to this problem under some suitable conditions.

2. Main results

Throughout this section, assume that $(S, +)$ is an arbitrary commutative semigroup, E_1 and E_2 be two complex normed space, $\mathbb{R}$ is real field, $\mathbb{N}$ is all positive integers and $\psi : S^2 \to [0, \infty)$ is a function.

2.1. Asymptotic behavior of additive mapping

The following Theorem is a affirmative answer to problem 18, in the thirty-first ISFE.

Theorem 3. *Let $f : S \to E_2$ be a function such that*

$$\|f(x+y) - f(x) - f(y)\| \leq \psi(x,y) \tag{1}$$

for all $x, y \in S$. Assume that
- $\lim\limits_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \psi(x+ix_0, x_0) = 0$;
- $\lim\limits_{n\to\infty} \frac{1}{n}\psi(x+nx_0, y+ny_0) = 0$

for any fixed $x_0, y_0, x, y \in S$. Then f is an additive function.

Proof. Let x_0 be any fixed element of S. From (1), its easy to show that the following inequality

$$\|f(x+nx_0) - nf(x_0) - f(x)\| \leq \sum_{i=0}^{n-1} \psi(x+ix_0, x_0)$$

for each fixed $x \in S$ and $n \in \mathbb{N}$. Now bye assumption $\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \psi(x+ix_0, x_0) = 0$, so

$$f(x_0) = \lim_{n\to\infty} \frac{f(x+nx_0)}{n}$$

for any fixed $x \in S$. Let x_0, y_0 be any two fixed element of S, then from (1), we obtain

$$\|f(x+y+n(x_0+y_0)) - f(x+nx_0) - f(y+ny_0)\| \leq \psi(x+nx_0, y+ny_0)$$

for any fixed $x, y \in S$. Now since $\lim_{n\to\infty} \frac{1}{n}\psi(x+nx_0, y+y_0) = 0$, thus

$$f(x_0+y_0) = f(x_0) + f(y_0),$$

which says that f is an additive mapping. ■

Corollary 1. *Let $f : E_1 \to E_2$ be a function such that*

$$\|f(x+y) - f(x) - f(y)\| \leq \|x\|^p + \|y\|^q \tag{2}$$

for all $x, y \in E_1$ and for some reals $p < 0$ and $q < 1$. Then f is an additive mapping.

Proof. Set $\psi(x,y) := \|x\|^p + \|y\|^q$ for all $x, y \in E_1$. Its easy to show that the followings relations
- $\lim\limits_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \psi(x+ix_0, x_0) = 0$;

- $\lim_{n\to\infty} \frac{1}{n}\psi(x+nx_0, y+ny_0) = 0$

for any fixed $x_0, y_0, x, y \in E_1$. Now Theorem 3 implies that f is an additive mapping. ■

Corollary 2. *Let $f : W \to V$ be a function such that*

$$\|f(x+y) - f(x) - f(y)\| \leq \frac{\|y\|^q}{\|x\|^p + \theta} \tag{3}$$

for all $x, y \in E_1$ and for some reals $p > 0$ and $q < 1$. Then f is an additive mapping.

Proof. Set $\psi(x,y) := \frac{\|y\|^q}{\|x\|^p+\theta}$ for all $x, y \in E_1$. Its easy to show that the followings relations

- $\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \psi(x+ix_0, x_0) = 0;$
- $\lim_{n\to\infty} \frac{1}{n}\psi(x+nx_0, y+ny_0) = 0$

for any fixed $x_0, y_0, x, y \in E_1$. Now Theorem 3 implies that f is an additive mapping. ■

In the following, by using Theorem 3, we give a simple proof of Skof Theorem 2 and also we show that Skof Theorem is true when E_2 be a complex normed space.

Theorem 4. *For a function $f : E_1 \to E_2$ the following two conditions are equivalent:*

(a) $\|f(x+y) - f(x) - f(y)\| \to 0$ *as* $\|x\| + \|y\| \to \infty$;

(b) $f(x+y) - f(x) - f(y) = 0$

for all $x, y \in E_1$.

Proof. Set $\psi(x,y) := \|f(x+y) - f(x) - f(y)\|$ for all $x, y \in E_1$. Now let $x_0, y_0 \in E_1$ be two arbirary fixed elements. Since $\|x+nx_0\| + \|y+ny_0\| \to \infty$ for each fixed $x, y \in E_1$, so

$$\lim_{n\to\infty} \psi(x+nx_0, y+ny_0) = 0,$$

for each fixed $x, y \in E_1$, hence its easy to show that the following relations

- $\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \psi(x+ix_0, x_0) = 0;$
- $\lim_{n\to\infty} \frac{1}{n}\psi(x+nx_0, y+ny_0) = 0$

for each fixed $x, y \in E_1$. Now by Theorem (3) implies that f is an additive mapping. The proof is complete. ■

Let $\mathfrak{S}$ be set all function $\rho : E_1^2 \to [0, \infty)$ such that

(a) $\rho(x+nx_0, y+ny_0) \to \infty$ as $n \to \infty$

for any fixed $x_0, y_0, x, y \in E_1$, where $\|x_0\| \neq 0$ or $\|y_0\| \neq 0$. Not that the functions $\rho_1, \rho_2, \rho_3 \in \mathfrak{S}$, in which $\rho_1(x,y) := \|x\| + \|y\|$, $\rho_2(x,y) := \|x+y\|$ and $\rho_3(x,y) := \max\{\|x\|, \|y\|\}$ for all $x, y \in E_1$. We now apply Theorem 3 to a generalization of Skof theorem.

Corollary 3. *For a function $f : E_1 \to E_2$ the following two conditions are equivalent:*
(a) $\|f(x+y) - f(x) - f(y)\| \to 0$ *as* $\rho(x,y) \to \infty$;
(b) $f(x+y) - f(x) - f(y) = 0$
for all $x, y \in E_1$, in which $\rho \in \mathfrak{S}$.

Proof. Set $\psi(x,y) := \|f(x+y) - f(x) - f(y)\|$ for all $x, y \in E_1$. Now let $x_0, y_0 \in E_1$ be two arbirary fixed elements. Since $\rho \in \mathfrak{S}$, so
(a) $\rho(x + nx_0, y + ny_0) \to \infty$ as $n \to \infty$
for any fixed $x_0, y_0, x, y \in E_1$, where $\|x_0\| \neq 0$ or $\|y_0\| \neq 0$. Thus

$$\lim_{n\to\infty} \psi(x + nx_0, y + ny_0) = 0,$$

for each fixed $x_0, y_0, x, y \in E_1$, where $\|x_0\| \neq 0$ or $\|y_0\| \neq 0$. Hence, its easy to show that the following relations
- $\lim\limits_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \psi(x + ix_0, x_0) = 0$;
- $\lim\limits_{n\to\infty} \frac{1}{n} \psi(x + nx_0, y + ny_0) = 0$

for any fixed $x_0, y_0, x, y \in E_1$. Now by Theorem 3 implies that f is an additive mapping. The proof is complete. ■

2.2. Asymptotic behavior of Pexiderized additive mapping

Theorem 5. *Let S be with identity e and $f, g, h : S \to V$ be three functions such that $g(e) = h(e) = 0$ and*

$$\|f(x+y) - g(x) - h(y)\| \leq \psi(x,y) \tag{4}$$

for all $x, y \in S$. Assume that
- $\lim\limits_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \psi(x + ix_0, x_0) = 0$;
- $\lim\limits_{n\to\infty} \frac{1}{n} \psi(x + nx_0, y + ny_0) = 0$

for any fixed $x_0, y_0, x, y \in S$. Then f, g and h are additive function and $f(x+y) - g(x) - h(y) = 0$ for all $x, y \in S$.

Proof. Set $\widetilde{\psi}(x,y) := \psi(x,y) + \psi(x,e) + \psi(e,y)$ and $\widehat{\psi}(x,y) := \psi(x+y, e) + \psi(x,e) + \psi(e,y)$ for all $x, y \in S$. From inequality (4) and assumptions, we obtain the following inequalities

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\leq \psi(x,y) + \|f(x) - g(x)\| + \|f(y) - h(y)\| \\ &\leq \psi(x,y) + \psi(x,e) + \psi(e,y) = \widetilde{\psi}(x,y) \end{aligned}$$

and

$$\begin{aligned}\|g(x+y)-g(x)-g(y)\| &\leq \psi(x+y,e)+\|f(x+y)-g(x)-g(y)\|\\ &\leq \psi(x+y,e)+\|f(x+y)-f(x)-f(y)\|\\ &\quad +\|f(x)-g(x)\|+\|f(y)-g(y)\|\\ &\leq \psi(x+y,e)+\psi(x,y)+2\psi(x,e)+2\psi(e,y)\\ &= \widetilde{\psi}(x,y)+\widehat{\psi}(x,y)\end{aligned}$$

and also

$$\begin{aligned}\|h(x+y)-h(x)-h(y)\| &\leq \psi(x+y,e)+\|f(x+y)-h(x)-h(y)\|\\ &\leq \psi(x+y,e)+\|f(x+y)-f(x)-f(y)\|\\ &\quad +\|f(x)-h(x)\|+\|h(y)-h(y)\|\\ &\leq \psi(x+y,e)+\psi(x,y)+2\psi(x,e)+2\psi(e,y)\\ &= \widetilde{\psi}(x,y)+\widehat{\psi}(x,y)\end{aligned}$$

for all $x, y \in S$. With assumptions its easy to show that

- $\lim\limits_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1}\phi(x+ix_0,x_0)=0$;
- $\lim\limits_{n\to\infty} \frac{1}{n}\phi(x+nx_0,y+ny_0)=0$

for any fixed $x_0, y_0, x, y \in S$, in which the function ϕ is $\widetilde{\psi}$ or $\widetilde{\psi}+\widehat{\psi}$. Now by Theorem 3 f, g and h is additive mapping and also

- $f(x_0)=\lim\limits_{n\to\infty}\frac{f(x+nx_0)}{n}$
- $g(x_0)=\lim\limits_{n\to\infty}\frac{g(x+nx_0)}{n}$
- $h(x_0)=\lim\limits_{n\to\infty}\frac{h(x+nx_0)}{n}$

for each fixed $x_0, x \in S$. Let x_0, y_0 be any two fixed element of S, then from (4), we obtain

$$\|f(x+y+n(x_0+y_0))-g(x+nx_0)-h(y+ny_0)\| \leq \psi(x+nx_0,y+ny_0)$$

for any fixed $x, y \in S$. Now since $\lim_{n\to\infty}\frac{1}{n}\psi(x+nx_0,y+y_0)=0$, thus

$$f(x_0+y_0)=g(x_0)+h(y_0),$$

which says that $f(x+y)-g(x)-h(y)=0$ for all $x, y \in S$. The proof is complete. ■

In the following, by using Theorem 5, we give a generalization of Skof theorem for Pexiderized additive mapping.

Theorem 6. *Assume that $f, g, h : E_1 \to E_2$ are three functions such that $g(0)=h(0)=0$, then the following two conditions are equivalent:*

(a) $\|f(x+y)-g(x)-h(y)\| \to 0$ *as* $\rho(x,y) \to \infty$;

(b) $f(x+y)-g(x)-h(y)=0$

for all $x, y \in E_1$, *in which* $\rho \in \mathfrak{S}$.

Proof. Set $\psi(x,y) := \|f(x+y)-g(x)-h(y)\|$ for all $x, y \in E_1$. Now let $x_0, y_0 \in E_1$ be two arbirary fixed elements. Since $\rho \in \mathfrak{S}$, so

(a) $\rho(x+nx_0, y+ny_0) \to \infty$ as $n \to \infty$

for any fixed $x_0, y_0, x, y \in E_1$, where $\|x_0\| \neq 0$ or $\|y_0\| \neq 0$. Thus

$$\lim_{n\to\infty} \psi(x+nx_0, y+ny_0) = 0,$$

for each fixed $x_0, y_0, x, y \in E_1$, where $\|x_0\| \neq 0$ or $\|y_0\| \neq 0$. Hence, its easy to show that the following relations

- $\lim\limits_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \psi(x+ix_0, x_0) = 0$;
- $\lim\limits_{n\to\infty} \frac{1}{n}\psi(x+nx_0, y+ny_0) = 0$

for any fixed $x_0, y_0, x, y \in E_1$. Now by Theorem 5 implies that $f(x+y) - g(x) - h(y) = 0$ for all $x, y \in S$. The proof is complete. ■

In the following, by using Theorem 7, we give a simple proof of S.-M. Joung theorem (see [20]) and also we show that Skof theorem is true when E_2 be a complex normed space.

Theorem 7. *Assume that* $J : E_1 \to E_2$ *is a function such that* $J(0) = 0$, *then the following two conditions are equivalent:*

(a) $\|2J(\frac{x+y}{2}) - J(x) - J(y) \to 0$ *as* $\|x\| + \|y\| \to \infty$;

(b) $2J(\frac{x+y}{2}) - J(x) - J(y) = 0$

for all $x, y \in E_1$.

Proof. Sets $f(x) := 2J(\frac{x}{2})$ and $g(x) := J(x)$ for all $x \in E_1$. Now apply Theorem 7. ■

References

[1] Alexander R., Blair C.-E., Rubel L.-A., Approximate version of Cauchy's functional equation, *Illinois J. Math.,* 39(1995), 278-287.

[2] Alimohammady M., Sadeghi A., On the superstability of the Pexider type of exponential equation in Banach algebra, *Int. J. Nonlinear Anal. Appl.,* (2011)(*in press*).

[3] Alimohammady M., Sadeghi A., Some new results on the superstablity of the Cauchy equation on semigroup, *Results Math.,* (2012)(*in press*).

[4] Aoki T., On the stability of the linear transformation in Banach spaces, *J. Math. Soc. Japan,* 2(1950), 64–66.

[5] Baker J.A., A general functional equation and its stability, *Proc. Amer. Math. Soc.,* 133(2005), 1657-1664.

[6] Baker J.A., The stability of the cosine equation, *Proc. Amer. Math. Soc.,* 80(1980), 411-416.

[7] Czerwik S., On the stability of the homogeneous mapping, *G. R. Math. Rep. Acad. Sci. Canada XIV,* 6(1992), 268-272.

[8] Elliott P.-D.-T.-A., Cauchy's functional equation in the mean, *Advances in Math.,* 51(1984), 253-257.

[9] Forti G.-L., Hyers-Ulam stability of functional equations in several variables, *Aeq. Math.,* 50(1995), 143-190.

[10] Forti G.-L., Remark 11 in: Report of the 22nd Internat. Symposium on Functional Equations, *Aequationes Math.,* 29(1980), (1985), 90-91.

[11] Ger R., Šemrl P., The stability of the exponential equation, *Proc. Amer. Math. Soc.,* 124(1996), 779-787.

[12] Hyers D.-H., On the stability of the linear functional equation, *Proc. Nat. Acad. Sci. U.S.A.,* 27(1941), 222-224.

[13] Hyers D.-H., Isac G., Rassias Th.-M., On the asymptoticity aspect of Hyers-Ulam stability of mappings, *Proc. Amer. Math. Soc.,* 126(2)(1998), 425-430.

[14] Hyers D.-H., Rassias Th.-M., Approximate homomorphisms, *Aeq. Math.,* 44(1992), 125-153.

[15] Hyers D.-H., The stability of homomorphisms and related topics, *in Global Analysis-Analysis on manifolds (ed. Th. M Rassias), Teubner-Texte zur Math., Leipzig,* 57(1983), 140-153.

[16] Hyers D.-H., Ulam S.-M., Approximately convex functions, *Proc. Amer. Math. Soc.,* 3(1952), 821-828.

[17] Hyers D.-H., Isac G., Rassias Th.-M., *Stability of Functional Equations in Several Variables,* Birkhauser, Boston,Basel, Berlin (1998).

[18] Isac G.-Th.-M., Rassias Th.-M., Stability of Ψ-additive mappings: Applications to nonlinear analysis, *Internat. J. Math. & Math. Sci.,* 19(2)(1996), 219-228.

[19] Jarosz K., Almost multiplicative functionals, *Studia Math.,* 124(1997), 37-58.

[20] Joung S.M., Hyers-Ulam-Rassias stability of Jensen's equation and its aplications, *Proc. Amer. Math. Soc.,* 126(1998), 3137-3143.

[21] Kannappan Pl., *Functional Equations and Inequalities with Applications,* Springer, New York, 2009.

[22] Nikodem K., Approximately quasiconvex functions, *C. R. Math. Rep. Acad. Sci. Canada,* 10(1988), 291-294.

[23] Johnson B.-E., Approximately multiplicative functionals, *J. London Math. Soc.,* 34(2)(1986), 489-510.

[24] Jung S.-M., Superstability of homogeneous functional equation, *Kyungpook Math. J.,* 38(1998), 251-257.

[25] Jung S.-M., *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis,* Springer, New York, 2011.

[26] Rassias Th.-M., On the stability of functional equations and a problem of Ulam, *Acta Applicandae Mathematicae,* 62(2000), 23-130.

[27] Rassias Th.-M., Problem 18, In: Report on the 31st ISFE, *Aequationes Math.,* 47(1994), 312-13.
[28] Rassias Th.-M., On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.,* 72(1978), 297-300.
[29] Rassias Th.-M., The problem of S. M. Ulam for approximately multiplicative mappings, *J. Math. Anal. Appl.,* 246(2000), 352-378.
[30] Rassias Th.-M., Brzdek J., (eds.), *Functional Equations in Mathematical Analysis,* Springer, New York, 2012.
[31] Skof F., Proprietá locali e approssimazione di operatori, *Rend. Sem. Mat. Fis. Milano,* 53(1983), 113-129.
[32] Tabor J., Tabor J., Homogenity is superstable, *Publ. Math. Debrecen,* 45 (1994), 123-130.
[33] Ulam S.M., *Problems in Modern Mathematics,* Science Editions, Wiley, New York, 1960.

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