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BETWEEN CLOSED SETS AND $^*g$-CLOSED SETS

Abstract. Veerakumar [16] introduced the notion of $^*g$-closed sets and further properties of $^*g$-closed sets are investigated. In this paper, we introduce the notion of m^*g-closed sets and obtain the unified characterizations for certain families of subsets between closed sets and $^*g$-closed sets.

Key words: $^*g$-closed set, m-structure, m-space and m^*g-closed set.

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1. Introduction

In 1970, Levine [4] introduced the notion of generalized closed(g-closed) sets in topological spaces. Veerakumar [15] introduced the notion of $\hat{g}$-closed sets in topological spaces. Recently, many variations of g-closed sets are introduced and investigated. One among them is $^*g$-closed sets which were introduced by Veerakumar[16].

In this paper, we introduce the notion of m^*g-closed sets and obtain the basic properties, characterizations and preservation properties. In the last section, we define several new subsets which lie between closed sets and $^*g$-closed sets.

2. Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. A subset A is said to be regular open [14] (resp. regular closed [14]) if $int(cl(A)) = A$ (resp. $cl(int(A)) = A$). The finite union of regular open sets is said to be π-open [18]. The complement of π-open set is said to be π-closed.

Definition 1. *A subset A of a topological space (X, τ) is said to be semi-open [3] (resp. α-open [7]) if $A \subset cl(int(A))$ (resp. $A \subset int(cl(int(A)))$).*

The complement of semi-open set (resp. α-open set) is said to be semi-closed (resp. α-closed).

The family of all semi-open (resp. α-open, regular open, π-open) sets in X is denoted by $SO(X)$ (resp. τ^{α}, $RO(X)$, $\pi O(X)$).

Definition 2. *A subset A of a topological space (X, τ) is said to be g-closed [4] (resp. $\hat{g}$-closed [15], πg-closed [2], rg-closed [10]) if $cl(A) \subset U$ whenever $A \subset U$ and U is open (resp. semi-open, π-open, regular open).*

The complements of the above closed sets are called their respective open sets.

The family of all $\hat{g}$-open sets in (X, τ) is a topology on X and it is denoted by $\hat{g}O(X)$.

The $\hat{g}$-closure (resp. α-closure) of a subset A of X is, denoted by $\hat{g}cl(A)$ (resp. $\alpha cl(A)$) defined to be the intersection of all $\hat{g}$-closed sets (resp. α-closed sets) containing A.

Definition 3. *A subset A of a topological space (X, τ) is said to be $^*g$-closed [16] if $cl(A) \subset U$ whenever $A \subset U$ and U is $\hat{g}$-open in (X, τ).*

Definition 4. *A subset A of a topological space (X, τ) is said to be αg-closed [5] (resp. αgs-closed [17], $\pi g\alpha$-closed [1], $r\alpha g$-closed [8]) if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is open (resp. semi-open, π-open, regular open) in (X, τ).*

Throughout the present paper, (X, τ) and (Y, σ) always denote topological spaces and $f : (X, \tau) \to (Y, \sigma)$ presents a function.

3. m-structures

Definition 5. *A subfamily m_x of the power set $\wp(X)$ of a nonempty set X is called a minimal structure (briefly m-structure) [11] on X if $\emptyset \in m_x$ and $X \in m_x$.*

By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space. Each member of m_x is said to be m_x-open (or briefly m-open) and the complement of an m_x-open set is said to be m_x-closed (or briefly m-closed).

Remark 1. Let (X, τ) be a topological space. Then the families τ, $SO(X)$, $RO(X)$, $\pi O(X)$ and $\hat{g}O(X)$ are all m-structures on X.

Definition 6. *Let (X, m_x) be an m-space. For a subset A of X, the m_x-closure of A and the m_x-interior of A are defined in [6] as follows:*

(a) $m_x - cl(A) = \cap\{F : A \subset F,\ X - F \in m_x\}$,

(b) $m_x - int(A) = \cup\{U : U \subset A,\ U \in m_x\}$.

Remark 2. Let (X, τ) be a topological space and A a subset of X. If $m_x = \tau$ (resp. τ^α, $\hat{g}O(X)$), then we have $m_x - cl(A) = cl(A)$ (resp. $\alpha cl(A)$, $\hat{g}cl(A)$).

4. m^*g-closed sets

In this section, let (X, τ) be a topological space and m_x an m-structure on X. We obtain several basic properties of m^*g-closed sets.

Definition 7. *Let (X, τ) be a topological space and m_x an m-structure on X. A subset A of X is said to be*

(a) m-semiopen [13] if $A \subset m_x - cl(m_x - int(A))$,

(b) $m\hat{g}$-closed [12] if $cl(A) \subset U$ whenever $A \subset U$ and U is m-semiopen,

(c) $m\hat{g}$-open [12] if its complement is $m\hat{g}$-closed.

The family of all $m\hat{g}$-open sets in X is an m-structure on X and denoted by $m\hat{g}O(X)$.

Definition 8. *Let (X, τ) be a topological space and m_x an m-structure on X. A subset A of X is said to be*

(a) m^*g-closed if $cl(A) \subset U$ whenever $A \subset U$ and U is $m\hat{g}$-open,

(b) m^*g-open if its complement is m^*g-closed.

Definition 9. *Let $(X, m\hat{g}O(X))$ be an m-space. For a subset A of X, the $m\hat{g}$-closure of A and the $m\hat{g}$-interior of A are defined as follows:*

(a) $m\hat{g} - cl(A) = \cap\{F : A \subset F, X - F \in m\hat{g}O(X)\}$,

(b) $m\hat{g} - int(A) = \cup\{U : U \subset A, U \in m\hat{g}O(X)\}$.

Theorem 1. *Let $(X, m\hat{g}O(X))$ be an m-space and A a subset of X. Then $x \in m\hat{g} - cl(A)$ if and only if $U \cap A \neq \emptyset$ for every $m\hat{g}$-open set U containing x.*

Proof. Suppose that there exists $m\hat{g}$-open set U containing x such that $U \cap A = \emptyset$. Then $A \subset X - U$ and $X - (X - U) = U \in m\hat{g}O(X)$. Then by Definition 9, $m\hat{g} - cl(A) \subset X - U$. Since $x \in U$, we have $x \notin m\hat{g} - cl(A)$. Conversely, suppose that $x \notin m\hat{g} - cl(A)$. There exists a subset F of X such that $X - F \in m\hat{g}O(X)$, $A \subset F$ and $x \notin F$. Then there exists $m\hat{g}$-open set $X - F$ containing x such that $(X - F) \cap A = \emptyset$. ■

Definition 10. *An m-structure $m\hat{g}O(X)$ on a nonempty set X is said to have property $\mathcal{C}$ if the union of any family of subsets belonging to $m\hat{g}O(X)$ belongs to $m\hat{g}O(X)$.*

Example 1. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ and $m_x = \{\emptyset, X, \{a, b\}, \{a, c\}, \{b, d\}\}$. Then $m\hat{g}$-open sets are $\emptyset$, X, $\{a\}$,

$\{b\}$, $\{d\}$, $\{a,b\}$, $\{a,d\}$, $\{a,b,d\}$. It is shown that $m\hat{g}O(X)$ does not have property $\mathcal{C}$.

Remark 3. Let (X,τ) be a topological space. Then the families $SO(X)$, τ^{α} and $\hat{g}O(X)$ are all m-structures with property $\mathcal{C}$.

Lemma 1. *Let X be a nonempty set and $m\hat{g}O(X)$ an m-structure on X satisfying property $\mathcal{C}$. For a subset A of X, the following properties hold:*
(a) $A \in m\hat{g}O(X)$ if and only if $m\hat{g} - int(A) = A$,
(b) A is $m\hat{g}$-closed if and only if $m\hat{g} - cl(A) = A$,
(c) $m\hat{g} - int(A) \in m\hat{g}O(X)$ and $m\hat{g} - cl(A)$ is $m\hat{g}$-closed.

Remark 4. Let (X,τ) be a topological space and A a subset of X. If $m\hat{g}O(X) = \hat{g}O(X)$ (resp. $SO(X)$, τ, $\pi O(X)$, $RO(X)$) and A is m^*g-closed, then A is $^*g$-closed (resp. $\hat{g}$-closed, g-closed, πg-closed, rg-closed).

Proposition 1. *Let $\hat{g}O(X) \subset m\hat{g}O(X)$. Then the following implications hold:*

$$closed \longrightarrow m^*g\text{-}closed \longrightarrow^* g\text{-}closed$$

Proof. It is obvious that every closed set is m^*g-closed. Suppose that A is an m^*g-closed set. Let $A \subset U$ and $U \in \hat{g}O(X)$. Since $\hat{g}O(X) \subset m\hat{g}O(X)$, $cl(A) \subset U$ and hence A is $^*g$-closed. ■

Example 2. (a) Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{c\}, \{b,c\}\}$ and $m_x = \{\emptyset, X\}$. Then $m\hat{g}$-open sets are the power set of X; $\hat{g}$-open sets are $\emptyset$, X, $\{c\}$, $\{b,c\}$;m^*g-closed sets are $\emptyset$, X, $\{a\}$, $\{a,b\}$ and $^*g$-closed sets are $\emptyset$, X, $\{a\}$, $\{a,b\}$, $\{a,c\}$. It is clear that $\{a,c\}$ is $^*g$-closed set but it is not m^*g-closed.

(b) Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{c\}, \{b,c\}\}$ and $m_x = \{\emptyset, X, \{c\}\}$. Then $m\hat{g}$-open sets are $\emptyset$, X, $\{c\}$, $\{b,c\}$; $\hat{g}$-open sets are $\emptyset$, X, $\{c\}$, $\{b,c\}$ and m^*g-closed sets are $\emptyset$, X, $\{a\}$, $\{a,b\}$, $\{a,c\}$. It is clear that $\{a,c\}$ is m^*g-closed set but it is not closed.

Proposition 2. *If A and B are m^*g-closed, then $A \cup B$ is m^*g-closed.*

Proof. Let $A \cup B \subset U$ and $U \in m\hat{g}O(X)$. Then $A \subset U$ and $B \subset U$. Since A and B are m^*g-closed, we have $cl(A \cup B) = cl(A) \cup cl(B) \subset U$. Therefore, $A \cup B$ is m^*g-closed. ■

Proposition 3. *If A is m^*g-closed and $m\hat{g}$-open, then A is closed.*

Proposition 4. *If A is m^*g-closed and $A \subset B \subset cl(A)$, then B is m^*g-closed.*

Proof. Let $B \subset U$ and $U \in m\hat{g}O(X)$. Then $A \subset U$ and A is m^*g-closed. Hence $cl(B) \subset cl(A) \subset U$ and B is m^*g-closed. ■

Definition 11. *Let $(X, m\hat{g}O(X))$ be an m-space and A a subset of X. The $m\hat{g}$-frontier of A, $m\hat{g} - Fr(A)$, is defined as follows: $m\hat{g} - Fr(A) = m\hat{g} - cl(A) \cap m\hat{g} - cl(X - A)$.*

Proposition 5. *If A is m^*g-closed and $A \subset U \in m\hat{g}O(X)$, then $m\hat{g} - Fr(U) \subset int(X - A)$.*

Proof. Let A be m^*g-closed and $A \subset U \in m\hat{g}O(X)$. Then $cl(A) \subset U$. Suppose that $x \in m\hat{g} - Fr(U)$. Since $U \in m\hat{g}O(X)$, $m\hat{g} - Fr(U) = m\hat{g} - cl(U) \cap m\hat{g} - cl(X - U) = m\hat{g} - cl(U) \cap (X - U) = m\hat{g} - cl(U) - U$. Therefore, $x \notin U$ and $x \notin cl(A)$. This shows that $x \in int(X - A)$ and hence $m\hat{g} - Fr(U) \subset int(X - A)$. ■

Proposition 6. *A subset A of X is m^*g-open if and only if $F \subset int(A)$ whenever $F \subset A$ and F is $m\hat{g}$-closed.*

Proof. Suppose that A is m^*g-open. Let $F \subset A$ and F be $m\hat{g}$-closed. Then $X - A \subset X - F \in m\hat{g}O(X)$ and $X - A$ is m^*g-closed. Therefore, we have $X - int(A) = cl(X - A) \subset X - F$ and hence $F \subset int(A)$. Conversely, let $X - A \subset G$ and $G \in m\hat{g}O(X)$. Then $X - G \subset A$ and $X - G$ is $m\hat{g}$-closed. By the hypothesis, we have $X - G \subset int(A)$ and hence $cl(X - A) = X - int(A) \subset G$. Therefore, $X - A$ is m^*g-closed and A is m^*g-open. ■

Corollary 1. *Let $\hat{g}O(X) \subset m\hat{g}O(X)$. Then the following properties hold:*

(*a*) *Every open set is m^*g-open and every m^*g-open set is $^*g$-open,*
(*b*) *If A and B are m^*g-open, then $A \cap B$ is m^*g-open,*
(*c*) *If A is m^*g-open and $m\hat{g}$-closed, then A is open,*
(*d*) *If A is m^*g-open and $int(A) \subset B \subset A$, then B is m^*g-open.*

Proof. This follows from Propositions 1, 2, 3 and 4. ■

5. Characterizations of m^*g-closed sets

In this section, let (X, τ) be a topological space and m_x an m-structure on X. We obtain some characterizations of m^*g-closed sets.

Theorem 2. *A subset A of X is m^*g-closed if and only if $cl(A) \cap F = \emptyset$ whenever $A \cap F = \emptyset$ and F is $m\hat{g}$-closed.*

Proof. Suppose that A is m^*g-closed. Let $A \cap F = \emptyset$ and F be $m\hat{g}$-closed. Then $A \subset X - F \in m\hat{g}O(X)$ and $cl(A) \subset X - F$. Therefore, we have $cl(A) \cap F = \emptyset$. Conversely, let $A \subset U$ and $U \in m\hat{g}O(X)$. Then $A \cap (X - U) = \emptyset$ and $X - U$ is $m\hat{g}$-closed. By the hypothesis, $cl(A) \cap (X - U) = \emptyset$ and hence $cl(A) \subset U$. Therefore, A is m^*g-closed. ■

Theorem 3. *Let $\hat{g}O(X) \subset m\hat{g}O(X)$ and $m\hat{g}O(X)$ have property $\mathcal{C}$. A sub-set A of X is m^*g-closed if and only if $cl(A) - A$ contains no nonempty $m\hat{g}$-closed set.*

Proof. Suppose that A is m^*g-closed. Let $F \subset cl(A) - A$ and F be $m\hat{g}$-closed. Then $F \subset cl(A)$ and $F \not\subseteq A$ and so $A \subset X - F \in m\hat{g}O(X)$ and hence $cl(A) \subset X - F$. Therefore, we have $F \subset X - cl(A)$. Hence $F = \emptyset$. Conversely, suppose that A is not m^*g-closed. Then by Theorem 2, $\emptyset \neq cl(A) - U$ for some $U \in m\hat{g}O(X)$ containing A. Since $\tau \subset \hat{g}O(X) \subset m\hat{g}O(X)$ and $m\hat{g}O(X)$ has property $\mathcal{C}$, $cl(A) - U$ is $m\hat{g}$-closed. Moreover, we have $cl(A) - U \subset cl(A) - A$, a contradiction. Hence A is m^*g-closed. ■

Theorem 4. *Let $\hat{g}O(X) \subset m\hat{g}O(X)$ and $m\hat{g}O(X)$ have property $\mathcal{C}$. A sub-set A of X is m^*g-closed if and only if $cl(A) - A$ is m^*g-open.*

Proof. Suppose that A is m^*g-closed. Let $F \subset cl(A) - A$ and F be $m\hat{g}$-closed. By Theorem 3, we have $F = \emptyset$ and $F \subset int(cl(A) - A)$. It follows from Proposition 6, $cl(A) - A$ is m^*g-open. Conversely, let $A \subset U$ and $U \in m\hat{g}O(X)$. Then $cl(A) \cap (X - U) \subset cl(A) - A$ and $cl(A) - A$ is m^*g-open. Since $\tau \subset \hat{g}O(X) \subset m\hat{g}O(X)$ and $m\hat{g}O(X)$ has property $\mathcal{C}$, $cl(A) \cap (X - U)$ is $m\hat{g}$-closed and by Proposition 6, $cl(A) \cap (X - U) \subset int(cl(A) - A)$. Now, $int(cl(A) - A) = int(cl(A)) \cap int(X - A) \subset cl(A) \cap int(X - A) = cl(A) \cap (X - cl(A)) = \emptyset$. Therefore, we have $cl(A) \cap (X - U) = \emptyset$ and hence $cl(A) \subset U$. This shows that A is m^*g-closed. ■

Theorem 5. *Let $(X, m\hat{g}O(X))$ be an m-structure with property $\mathcal{C}$. A sub-set A of X is m^*g-closed if and only if $m\hat{g} - cl(\{x\}) \cap A \neq \emptyset$ for each $x \in cl(A)$.*

Proof. Suppose that A is m^*g-closed and $m\hat{g} - cl(\{x\}) \cap A = \emptyset$ for some $x \in cl(A)$. By Lemma 1, $m\hat{g} - cl(\{x\})$ is $m\hat{g}$-closed and $A \subset X - (m\hat{g} - cl(\{x\})) \in m\hat{g}O(X)$. Since A is m^*g-closed, $cl(A) \subset X - (m\hat{g} - cl(\{x\})) \subset X - \{x\}$, a contradiction, since $x \in cl(A)$. Conversely, suppose that A is not m^*g-closed. Then by Theorem 2, $\emptyset \neq cl(A) - U$ for some $U \in m\hat{g}O(X)$ containing A. There exists $x \in cl(A) - U$. Since $x \notin U$, by Theorem 1, $m\hat{g} - cl(\{x\}) \cap U = \emptyset$ and hence $m\hat{g} - cl(\{x\}) \cap A \subset m\hat{g} - cl(\{x\}) \cap U = \emptyset$. This shows that $m\hat{g} - cl(\{x\}) \cap A = \emptyset$ for some $x \in cl(A)$. Hence A is m^*g-closed. ■

Corollary 2. *Let $\hat{g}O(X) \subset m\hat{g}O(X)$ and $m\hat{g}O(X)$ have property $\mathcal{C}$. For a subset A of X, the following properties are equivalent:*
(a) *A is m^*g-open,*
(b) *$A - int(A)$ contains no nonempty $m\hat{g}$-closed set,*
(c) *$A - int(A)$ is m^*g-open,*
(d) *$m\hat{g} - cl(\{x\}) \cap (X - A) \neq \emptyset$ for each $x \in A - int(A)$.*

Proof. This follows from Theorems 3, 4 and 5. ■

6. Preservation theorems

Definition 12. *A function $f : (X, m_x) \to (Y, m_y)$ is said to be*
(a) *$M\hat{g}$-continuous if $f^{-1}(V)$ is $m\hat{g}$-closed in (X, m_x) for every $m\hat{g}$-closed V in (Y, m_y),*
(b) *$M\hat{g}$-closed if for each $m\hat{g}$-closed set F of (X, m_x), $f(F)$ is $m\hat{g}$-closed in (Y, m_y).*

Theorem 6. *Let $m\hat{g}O(X)$ be an m-structure with property $\mathcal{C}$. Let $f : (X, m_x) \to (Y, m_y)$ be a function from a minimal space (X, m_x) into a minimal space (Y, m_y). Then the following are equivalent:*
(a) *f is $M\hat{g}$-continuous,*
(b) *$f^{-1}(V) \in m\hat{g}O(X)$ for every $V \in m\hat{g}O(Y)$.*

Proof. Assume that $f : (X, m_x) \to (Y, m_y)$ is $M\hat{g}$-continuous. Let $V \in m\hat{g}O(Y)$. Then V^c is $m\hat{g}$-closed in (Y, m_y). Since f is $M\hat{g}$-continuous, $f^{-1}(V^c)$ is $m\hat{g}$-closed in (X, m_x). But $f^{-1}(V^c) = X - f^{-1}(V)$. Thus $X - f^{-1}(V)$ is $m\hat{g}$-closed in (X, m_x) and so $f^{-1}(V)$ is $m\hat{g}$-open in (X, m_x). Conversely, let for each $V \in m\hat{g}O(Y)$, $f^{-1}(V) \in m\hat{g}O(X)$. Let F be any $m\hat{g}$-closed set in (Y, m_y). By assumption, $f^{-1}(F^c)$ is $m\hat{g}$-open in (X, m_x). But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is $m\hat{g}$-open in (X, m_x) and so $f^{-1}(F)$ is $m\hat{g}$-closed in (X, m_x). Hence f is $M\hat{g}$-continuous. ■

Lemma 2. *A function $f : (X, m_x) \to (Y, m_y)$ is $M\hat{g}$-closed if and only if for each subset B of Y and each $U \in m\hat{g}O(X)$ containing $f^{-1}(B)$, there exists $V \in m\hat{g}O(Y)$ such that $B \subset V$ and $f^{-1}(V) \subset U$.*

Proof. Suppose that f is $M\hat{g}$-closed. Let $B \subset Y$ and $U \in m\hat{g}O(X)$ containing $f^{-1}(B)$. Put $V = Y - f(X - U)$. Then V is $m\hat{g}$-open in (Y, m_y) and $f^{-1}(V) \subset f^{-1}(Y) - (X - U) = X - (X - U) = U$. Also, since $f^{-1}(B) \subset U$, then $X - U \subset f^{-1}(Y - B)$ which implies $f(X - U) \subset Y - B$ and hence $B \subset V$. Hence we obtain $V \in m\hat{g}O(Y)$, $B \subset V$ and $f^{-1}(V) \subset U$. Conversely, let F be any $m\hat{g}$-closed set of (X, m_x). Set $f(F) = B$, then $F \subset f^{-1}(B)$ and $f^{-1}(Y - B) \subset X - F \in m\hat{g}O(X)$. By the hypothesis, there exists $V \in m\hat{g}O(Y)$ such that $Y - B \subset V$ and $f^{-1}(V) \subset X - F$ and so $F \subset f^{-1}(Y - V)$. Therefore $f(F) \subset Y - V$. Hence, we obtain $Y - V \subset B = f(F) \subset Y - V$. Therefore $f(F) = Y - V$ is $m\hat{g}$-closed in (Y, m_y). Hence f is $M\hat{g}$-closed. ■

Theorem 7. *If $f : (X, \tau) \to (Y, \sigma)$ is closed and $f : (X, m_x) \to (Y, m_y)$ is $M\hat{g}$-continuous, where $m\hat{g}O(X)$ has property $\mathcal{C}$, then $f(A)$ is m^*g-closed in (Y, m_y) for each m^*g-closed set A of (X, m_x).*

Proof. Let A be any m^*g-closed set of (X, m_x) and $f(A) \subset V \in m\hat{g}O(Y)$. Then, by Theorem 6, $A \subset f^{-1}(V) \in m\hat{g}O(X)$. Since A is m^*g-closed, $cl(A) \subset f^{-1}(V)$ and $f(cl(A)) \subset V$. Since f is closed, $cl(f(A)) \subset f(cl(A)) \subset V$. Hence $f(A)$ is m^*g-closed in (Y, m_y). ■

Theorem 8. *If $f : (X, \tau) \to (Y, \sigma)$ is continuous and $f : (X, m_x) \to (Y, m_y)$ is $M\hat{g}$-closed, then $f^{-1}(B)$ is m^*g-closed in (X, m_x) for each m^*g-closed set B of (Y, m_y).*

Proof. Let B be any m^*g-closed set of (Y, m_y) and $f^{-1}(B) \subset U \in m\hat{g}O(X)$. Since f is $M\hat{g}$-closed, by Lemma 2, there exists $V \in m\hat{g}O(Y)$ such that $B \subset V$ and $f^{-1}(V) \subset U$. Since B is m^*g-closed, $cl(B) \subset V$ and since f is continuous, $cl(f^{-1}(B)) \subset f^{-1}(cl(B)) \subset f^{-1}(V) \subset U$. Hence $f^{-1}(B)$ is m^*g-closed in (X, m_x). ■

7. New forms of closed sets in topological spaces

By $\hat{g}O(X)$ (resp. $GO(X)$, $\pi GO(X)$, $RGO(X)$, $\alpha GSO(X)$, $\alpha GO(X)$, $\pi G\alpha O(X)$, $R\alpha GO(X)$), we denote the collection of all $\hat{g}$-open (resp. g-open, πg-open, rg-open, αgs-open, αg-open, $\pi g\alpha$-open, $r\alpha g$-open) sets of a topological space (X, τ). These collections are m-structure on X.

By the definitions, we obtain the following diagram:

$$\begin{array}{ccccccc} \hat{g}\text{-}open & \longrightarrow & g\text{-}open & \longrightarrow & \pi g\text{-}open & \longrightarrow & rg\text{-}open \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \alpha gs\text{-}open & \longrightarrow & \alpha g\text{-}open & \longrightarrow & \pi g\alpha\text{-}open & \longrightarrow & r\alpha g\text{-}open \end{array}$$

Diagram 1.

For subsets of a topological space (X, τ), we can define many new variations of closed sets.

Definition 13. *A subset A of a topological space (X, τ) is said to be $^*g$-closed (resp. g^*-closed, πgg-closed, rgg-closed, αgsg-closed, αgg-closed, $\pi g\alpha g$-closed, $r\alpha gg$-closed) if $cl(A) \subset U$ whenever $A \subset U$ and U is $\hat{g}$-open* (*resp. g-open, πg-open, rg-open, αgs-open, αg-open, $\pi g\alpha$-open, $r\alpha g$-open*) *in* (X, τ).

By Diagram 1 and Definition 13, we have the following diagram:

$$\begin{array}{ccccccccc}
{}^*g\text{-closed} & \longleftarrow & g^*\text{-closed} & \longleftarrow & \pi gg\text{-closed} & \longleftarrow & rgg\text{-closed} & & \\
\uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
\alpha gsg\text{-closed} & \longleftarrow & \alpha gg\text{-closed} & \longleftarrow & \pi g\alpha g\text{-closed} & \longleftarrow & r\alpha gg\text{-closed} & \longleftarrow & \text{closed}
\end{array}$$

Diagram 2.

References

[1] Arockiarani I., Balachandran K., Janaki C., On contra-$\pi g\alpha$-continuous functions, *Kochi J. Math.,* 3(2008), 201-209.

[2] Dontchev J., Noiri T., Quasi-normal spaces and πg-closed sets, *Acta Math. Hungar.,* 89(3)(2000), 211-219.

[3] Levine N., Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly,* 70(1963), 36-41.

[4] Levine N., Generalized closed sets in topology, *Rend. Circ. Mat. Palermo* (2)*,* 19(1970),89−96.

[5] Maki H., Devi R., Balachandran K., Associated topologies of generalized α-closed sets and α-generalized closed sets, *Mem. Sci. Kochi Univ. Ser. A. Math.,* 15(1994), 51-63.

[6] Maki H., Rao K.C., Nagoor Gani A., On generalizing semi-open and preopen sets, *Pure Appl. Math. Sci.,* 49(1999), 17-29.

[7] Njastad O., On some classes of nearly open sets, *Pacific J. Math.,* 15(1965), 961-970.

[8] Noiri T., Almost αg-closed functions and separation axioms, *Acta Math. Hungar.,* 82(3)(1999), 193-205.

[9] Noiri T., Popa V., Between closed sets and g-closed sets, *Rend. Circ. Mat. Palermo* (2)*,* 55(2006), 175-184.

[10] Palaniappan N., Rao K.C., Regular generalized closed sets, *Kyungpook Math. J.,* 33(1993), 211-219.

[11] Popa V., Noiri T., On M-continuous functions, *Anal. Univ. "Dunărea de Jos" Galati. Ser. Mat. Fiz. Mec. Teor.* (2)*,* 18(23)(2000), 31-41.

[12] Ravi O., Tharmar S., $m\hat{g}$-closed sets in minimal structures, (*submitted*).

[13] Roses E., Rajesh N., Carpintero C., Some new types of open and closed sets in minimal structures-II^1, *International Mathematical Forum,* 44(4)(2009), 2185-2198.

[14] Stone M.,, Application of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.,* 41(1937), 374-481.

[15] Veerakumar M.K.R.S., On $\hat{g}$-closed sets in topological spaces, *Bull. Allah. Math. Soc.,* 18(2003), 99-112.

[16] Veerakumar M.K.R.S., Between g^*-closed sets and g-closed sets, *Antarctica J. Math.,* (3)(1)(2006), 43-65.

[17] Viswanathan K., *Studies on α Generalized Semi-Closed Sets in Topological and Bitopological Spaces,* Ph. D Thesis, Bharathiar University, Coimbatore 2006.

[18] Zaitsev V., On certain classes of topological spaces and their bicompactifications, *Dokl. Akad. Nauk SSSR,* 178(1968), 778-779.

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