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ON $\hat{g}$-REGULAR AND $\hat{g}$-NORMAL SPACES

Abstract. The concept of $\hat{g}$-closed sets [10] (= ω-closed sets [8]) was introduced by Veerakumar [10] (Sheik John [8]). The aim of this paper is to introduce and characterize $\hat{g}$-regular spaces and $\hat{g}$-normal spaces via the concept of $\hat{g}$-closed sets.

Key words: $\hat{g}$-closed set, $\hat{g}$-open set, $\hat{g}$-regular space, $\hat{g}$-normal space, $\hat{g}$-continuous function, $\hat{g}$-irresolute function.

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1. Introduction and preliminaries

As a generalization of closed sets, in 1970, Levine [4] initiated the study of so called g-closed sets. As the strong forms of g-closed sets, the notion of $\tilde{g}$-closed sets and $\hat{g}$-closed sets (= ω-closed sets) were introduced and studied by Jafari et al [2] and Veerakumar [10] (Sheik John [8]) respectively. Using g-closed sets, Munshi [6] introduced g-regular and g-normal spaces in topological spaces. In a similar way, Rajesh and Ekici [7] introduced $\tilde{g}$-regular and $\tilde{g}$-normal spaces using $\tilde{g}$-closed sets in topological spaces.

In this paper, we introduce $\hat{g}$-regular spaces and $\hat{g}$-normal spaces in topological spaces. We obtain several characterizations of $\hat{g}$-regular and $\hat{g}$-normal spaces and some preservation theorems for $\hat{g}$-regular and $\hat{g}$-normal spaces.

Throughout this paper (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

We recall the following definitions, which are useful in the sequel.

Definition 1. *A subset A of a space (X, τ) is called semi-open [3] if $A \subseteq cl(int(A))$. The complement of semi-open set is called semi-closed.*

Definition 2. *A subset A of a space (X, τ) is called $\hat{g}$-closed [8, 10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of $\hat{g}$-closed set is said to be $\hat{g}$-open.*

Definition 3. *A function $f : (X, \tau) \to (Y, \sigma)$ is called:*

1. *a $\hat{g}$-continuous [8, 10] if $f^{-1}(V)$ is $\hat{g}$-closed in (X, τ) for every closed set V of (Y, σ).*
2. *a $\hat{g}$-irresolute [8, 10] if $f^{-1}(V)$ is $\hat{g}$-closed in (X, τ) for every $\hat{g}$-closed set V of (Y, σ).*
3. *a pre-semi-open [1] if $f(V)$ is semi-open in (Y, σ) for every semi-open set V of (X, τ).*
4. *an irresolute [1] if $f^{-1}(V)$ is semi-closed in (X, τ) for each semi-closed set V of (Y, σ).*
5. *a $\hat{g}$-closed [9] if the image of every closed set of (X, τ) is $\hat{g}$-closed of (Y, σ).*
6. *weakly continuous [5] if for each point $x \in X$ and each open set V in (Y, σ) containing $f(x)$, there exists an open set U containing x such that $f(U) \subseteq cl(V)$.*

Theorem 1 ([8])**.** *A set A is $\hat{g}$-open (X, τ) if and only if $F \subseteq int(A)$ whenever F is semi-closed in (X, τ) and $F \subseteq A$.*

Definition 4 ([8])**.** *Let x be a point of (X, τ) and V be a subset of X. Then V is called a $\hat{g}$-neighbourhood of x in (X, τ) if there exists a $\hat{g}$-open set U of (X, τ) such that $x \in U \subseteq V$.*

Definition 5 ([8])**.** *A space (X, τ) is called a $gT_{\hat{g}}$-space if every g-closed set in it is $\hat{g}$-closed.*

Theorem 2 ([8])**.** *Suppose that $B \subseteq A \subseteq X$, B is a $\hat{g}$-closed set relative to A and that A is open and $\hat{g}$-closed in (X, τ). Then B is $\hat{g}$-closed in (X, τ).*

2. $\hat{g}$-regular and $\hat{g}$-normal spaces

We introduce the following definition.

Definition 6. *A space (X, τ) is said to be $\hat{g}$-regular if for every $\hat{g}$-closed set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subseteq U$ and $x \in V$.*

Remark 1. It is obvious that every $\hat{g}$-regular space is regular but not conversely. Consider $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then (X, τ) is a regular space but not $\hat{g}$-regular.

Theorem 3. *Let (X, τ) be a topological space. Then the following statements are equivalent:*

1. *(X, τ) is a $\hat{g}$-regular space.*
2. *For each $x \in X$ and each $\hat{g}$-open neighbourhood W of x there exists an open neighbourhood V of x such that $cl(V) \subseteq W$.*

Proof. 1. $\Rightarrow$ 2. Let W be any $\hat{g}$-open neighbourhood of x. Then there exists a $\hat{g}$-open set G such that $x \in G \subseteq W$. Since G^c is $\hat{g}$-closed and $x \notin G^c$, by hypothesis there exist open sets U and V such that $G^c \subseteq U$, $x \in V$ and $U \cap V = \emptyset$ and so $V \subseteq U^c$. Now, $cl(V) \subseteq cl(U^c) = U^c$ and $G^c \subseteq U$ implies $U^c \subseteq G \subseteq W$. Therefore $cl(V) \subseteq W$.

2. $\Rightarrow$ 1. Let F be any $\hat{g}$-closed set and $x \notin F$. Then $x \in F^c$ and F^c is $\hat{g}$-open and so F^c is a $\hat{g}$-neighbourhood of x. By hypothesis, there exists an open neighbourhood V of x such that $x \in V$ and $cl(V) \subseteq F^c$, which implies $F \subseteq (cl(V))^c$. Then $(cl(V))^c$ is an open set containing F and $V \cap (cl(V))^c = \emptyset$. Therefore, X is $\hat{g}$-regular. ■

Theorem 4. *For a space (X, τ) the following are equivalent:*

1. *(X, τ) is normal.*
2. *For every pair of disjoint closed sets A and B, there exist $\hat{g}$-open sets U and V such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$.*

Proof. 1. $\Rightarrow$ 2. Let A and B be disjoint closed subsets of (X, τ). By hypothesis, there exist disjoint open sets (and hence $\hat{g}$-open sets) U and V such that $A \subseteq U$ and $B \subseteq V$.

2. $\Rightarrow$ 1. Let A and B be closed subsets of (X, τ). Then by assumption, $A \subseteq G$, $B \subseteq H$ and $G \cap H = \emptyset$, where G and H are disjoint $\hat{g}$-open sets. Since A and B are semi-closed, by Theorem 1, $A \subseteq int(G)$ and $B \subseteq int(H)$. Further, $int(G) \cap int(H) = int(G \cap H) = \emptyset$. ■

Definition 7 ([4]). *A topological space (X, τ) will be termed symmetric if and only if for x and y in (X, τ), $x \in cl(y)$ implies that $y \in cl(x)$.*

Theorem 5 ([4]). *A space (X, τ) is symmetric if and only if $\{x\}$ is g-closed in (X, τ) for each point x of (X, τ).*

Theorem 6. *A $gT_{\hat{g}}$-space (X, τ) is symmetric if and only if $\{x\}$ is $\hat{g}$-closed in (X, τ) for each point x of (X, τ).*

Proof. Follows from Definition 7 and Theorem 5. ■

Theorem 7. *If (X, τ) is a $\hat{g}$-regular space and Y is an open and $\hat{g}$-closed subset of (X, τ), then the subspace Y is $\hat{g}$-regular.*

Proof. Let F be any $\hat{g}$-closed subset of Y and $y \in F^c$. By Theorem 2, F is $\hat{g}$-closed in (X, τ). Since (X, τ) is $\hat{g}$-regular, there exist disjoint open sets U and V of (X, τ) such that $y \in U$ and $F \subseteq V$. Therefore, $U \cap Y$ and $V \cap Y$ are disjoint open sets of the subspace Y such that $y \in U \cap Y$ and $F \subseteq V \cap Y$. Hence the subspace Y is $\hat{g}$-regular. ■

Theorem 8. *A topological space (X, τ) is $\hat{g}$-regular if and only if for each $\hat{g}$-closed set F of (X, τ) and each $x \in F^c$ there exist open sets U and V of (X, τ) such that $x \in U$, $F \subseteq V$ and $cl(U) \cap cl(V) = \emptyset$.*

Proof. Let F be a $\hat{g}$-closed set of (X, τ) and $x \notin F$. Then there exist open sets U_0 and V of (X, τ) such that $x \in U_0$, $F \subseteq V$ and $U_0 \cap V = \emptyset$, which implies $U_0 \cap cl(V) = \emptyset$. Since $cl(V)$ is closed, it is $\hat{g}$-closed and $x \notin cl(V)$. Since (X, τ) is $\hat{g}$-regular, there exist open sets G and H of (X, τ) such that $x \in G$, $cl(V) \subseteq H$ and $G \cap H = \emptyset$, which implies $cl(G) \cap H = \emptyset$. Let $U = U_0 \cap G$, then U and V are open sets of (X, τ) such that $x \in U$, $F \subseteq V$ and $cl(U) \cap cl(V) = \emptyset$.

Converse part is trivial. ∎

Corollary 1. *If a space (X, τ) is $\hat{g}$-regular, symmetric and $gT_{\hat{g}}$-space, then it is Urysohn.*

Proof. Let x and y be any two distinct points of (X, τ). Since (X, τ) is symmetric and $gT_{\hat{g}}$-space, $\{x\}$ is $\hat{g}$-closed by Theorem 6. Therefore, by Theorem 8, there exist open sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$. ∎

Theorem 9. *Let (X, τ) be topological space. Then the following statements are equivalent:*

1. *(X, τ) is $\hat{g}$-regular.*
2. *For each point $x \in X$ and for each $\hat{g}$-open neighbourhood W of x, there exists an open neighbourhood U of x such that $cl(U) \subseteq W$.*
3. *For each point $x \in X$ and for each $\hat{g}$-closed set F not containing x, there exists an open neighbourhood V of x such that $cl(V) \cap F = \emptyset$.*

Proof. 1. $\Rightarrow$ 2. It is obvious from Theorem 3.

2. $\Rightarrow$ 3. Let $x \in X$ and F be a $\hat{g}$-closed set such that $x \notin F$. Then F^c is a $\hat{g}$-open neighbourhood of x and by hypothesis, there exists an open neighbourhood V of x such that $cl(V) \subseteq F^c$ and hence $cl(V) \cap F = \emptyset$.

3. $\Rightarrow$ 2. Let $x \in X$ and W be a $\hat{g}$-open neighbourhood of x. Then there exists a $\hat{g}$-open set G such that $x \in G \subseteq W$. Since G^c is $\hat{g}$-closed and $x \notin G^c$, by hypothesis there exists an open neighbourhood U of x such that $cl(U) \cap G^c = \emptyset$. Therefore, $cl(U) \subseteq G \subseteq W$. ∎

Definition 8. *Let (X, τ) be a topological space and $E \subseteq X$. We define the $\hat{g}$-closure of E (briefly $\hat{g} - cl(E)$) [8] to be the intersection of all $\hat{g}$-closed sets containing E.*

Definition 9. *For a subset A of a topological space (X, τ), $cl_\theta(A) = \{x \in X : cl(U) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$ [11].*

Theorem 10. *The following are equivalent for a space* (X, τ).

1. (X, τ) *is* $\hat{g}$*-regular,*
2. $cl_\theta(A) = \hat{g} - cl(A)$ *for each subset* A *of* (X, τ),
3. $cl_\theta(A) = A$ *for each* $\hat{g}$*-closed set* A.

Proof. 1. $\Rightarrow$ 2. For any subset A of (X, τ), we have always $A \subseteq \hat{g} - cl(A) \subseteq cl_\theta(A)$. Let $x \in (\hat{g} - cl(A))^c$. Then there exists a $\hat{g}$-closed set F such that $x \in F^c$ and $A \subseteq F$. By assumption, there exist disjoint open sets U and V such that $x \in U$ and $F \subseteq V$. Now, $x \in U \subseteq cl(U) \subseteq V^c \subseteq F^c \subseteq A^c$ and therefore $cl(U) \cap A = \emptyset$. Thus, $x \in (cl_\theta(A))^c$ and hence $cl_\theta(A) = \hat{g} - cl(A)$.

2. $\Rightarrow$ 3. It is trivial.

3. $\Rightarrow$ 1. Let F be any $\hat{g}$-closed set and $x \in F^c$. Since F is $\hat{g}$-closed, by assumption $x \in (cl_\theta(F))^c$ and so there exists an open set U such that $x \in U$ and $cl(U) \cap F = \emptyset$. Then $F \subseteq (cl(U))^c$. Let $V = (cl(U))^c$. Then V is an open such that $F \subseteq V$. Also, the sets U and V are disjoint and hence (X, τ) is $\hat{g}$-regular. ■

Theorem 11 ([8])**.** *If* $f : (X, \tau) \to (Y, \sigma)$ *is bijective, pre-semi-open and* $\hat{g}$*-continuous, then* f *is* $\hat{g}$*-irresolute.*

Theorem 12. *If* (X, τ) *is a* $\hat{g}$*-regular space and* $f : (X, \tau) \to (Y, \sigma)$ *is bijective, pre-semi-open,* $\hat{g}$*-continuous and open, then* (Y, σ) *is* $\hat{g}$*-regular.*

Proof. Let F be any $\hat{g}$-closed subset of (Y, σ) and $y \notin F$. Since the function f is $\hat{g}$-irresolute by Theorem 11, we have $f^{-1}(F)$ is $\hat{g}$-closed in (X, τ). Since f is bijective, let $f(x) = y$, then $x \notin f^{-1}(F)$. By hypothesis, there exist disjoint open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq V$. Since f is open and bijective, we have $y \in f(U)$, $F \subseteq f(V)$ and $f(U) \cap f(V) = \emptyset$. This shows that the space (Y, σ) is also $\hat{g}$-regular. ■

Theorem 13 ([8])**.** *If* $f : (X, \tau) \to (Y, \sigma)$ *is irresolute* $\hat{g}$*-closed and* A *is a* $\hat{g}$*-closed subset of* (X, τ)*, then* $f(A)$ *is* $\hat{g}$*-closed.*

Theorem 14. *If* $f : (X, \tau) \to (Y, \sigma)$ *is irresolute* $\hat{g}$*-closed continuous injection and* (Y, σ) *is* $\hat{g}$*-regular, then* (X, τ) *is* $\hat{g}$*-regular.*

Proof. Let F be any $\hat{g}$-closed set of (X, τ) and $x \notin F$. Since f is irresolute $\hat{g}$-closed, by Theorem 13, $f(F)$ is $\hat{g}$-closed in (Y, σ) and $f(x) \notin f(F)$. Since (Y, σ) is $\hat{g}$-regular and so there exist disjoint open sets U and V in (Y, σ) such that $f(x) \in U$ and $f(F) \subseteq V$. i.e., $x \in f^{-1}(U)$, $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Therefore, (X, τ) is $\hat{g}$-regular. ■

Theorem 15. *If* $f : (X, \tau) \to (Y, \sigma)$ *is weakly continuous* $\hat{g}$*-closed injection and* (Y, σ) *is* $\hat{g}$*-regular, then* (X, τ) *is regular.*

Proof. Let F be any closed set of (X,τ) and $x \notin F$. Since f is $\hat{g}$-closed, $f(F)$ is $\hat{g}$-closed in (Y,σ) and $f(x) \notin f(F)$. Since (Y,σ) is $\hat{g}$-regular by Theorem 8 there exist open sets U and V such that $f(x) \in U$, $f(F) \subseteq V$ and $cl(U) \cap cl(V) = \emptyset$. Since f is weakly continuous it follows that [5, Theorem 1], $x \in f^{-1}(U) \subseteq int(f^{-1}(cl(U)))$, $F \subseteq f^{-1}(V) \subseteq int(f^{-1}(cl(V)))$ and $int(f^{-1}(cl(U))) \cap int(f^{-1}(cl(V))) = \emptyset$. Therefore, (X,τ) is regular. ■

We conclude this section with the introduction of $\hat{g}$-normal spaces in topological spaces.

Definition 10. *A topological space (X,τ) is said to be $\hat{g}$-normal if for any pair of disjoint $\hat{g}$-closed sets A and B, there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.*

Remark 2. It is obvious that every $\hat{g}$-normal space is normal but not conversely. Consider $X = \{a,b,c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b,c\}\}$. Then X is normal space but not $\hat{g}$-normal.

Theorem 16. *If (X,τ) is a $\hat{g}$-normal space and Y is an open and $\hat{g}$-closed subset of (X,τ), then the subspace Y is $\hat{g}$-normal.*

Proof. Let A and B be any disjoint $\hat{g}$-closed sets of Y. By Theorem 2, A and B are $\hat{g}$-closed in (X,τ). Since (X,τ) is $\hat{g}$-normal, there exist disjoint open sets U and V of (X,τ) such that $A \subseteq U$ and $B \subseteq V$. Then $A \subseteq U \cap Y$ and $B \subseteq V \cap Y$ and so the subspace Y is normal. ■

In the next theorem we characterize $\hat{g}$-normal space.

Theorem 17. *Let (X,τ) be a topological space. Then the following statements are equivalent:*
1. *(X,τ) is $\hat{g}$-normal.*
2. *For each $\hat{g}$-closed set F and for each $\hat{g}$-open set U containing F, there exists an open set V containing F such that $cl(V) \subseteq U$.*
3. *For each pair of disjoint $\hat{g}$-closed sets A and B in (X,τ), there exists an open set U containing A such that $cl(U) \cap B = \emptyset$.*
4. *For each pair of disjoint $\hat{g}$-closed sets A and B in (X,τ), there exist open sets U containing A and V containing B such that $cl(U) \cap cl(V) = \emptyset$.*

Proof. 1. $\Rightarrow$ 2. Let F be a $\hat{g}$-closed set and U be a $\hat{g}$-open set such that $F \subseteq U$. Then $F \cap U^c = \emptyset$. By assumption, there exist open sets V and W such that $F \subseteq V$, $U^c \subseteq W$ and $V \cap W = \emptyset$, which implies $cl(V) \cap W = \emptyset$. Now, $cl(V) \cap U^c \subseteq cl(V) \cap W = \emptyset$ and so $cl(V) \subseteq U$.

2. $\Rightarrow$ 3. Let A and B be disjoint $\hat{g}$-closed sets of (X,τ). Since $A \cap B = \emptyset$, $A \subseteq B^c$ and B^c is $\hat{g}$-open. By assumption, there exists an open set U containing A such that $cl(U) \subseteq B^c$ and so $cl(U) \cap B = \emptyset$.

3. $\Rightarrow$ 4. Let A and B be any two disjoint $\hat{g}$-closed sets of (X,τ). Then by assumption, there exists an open set U containing A such that $cl(U)\cap B=\emptyset$. Since $cl(U)$ is closed, it is $\hat{g}$-closed and so B and $cl(U)$ are disjoint $\hat{g}$-closed sets in (X,τ). Therefore again by assumption, there exists an open set V containing B such that $cl(V)\cap cl(U)=\emptyset$.

4. $\Rightarrow$ 1. Let A and B be any two disjoint $\hat{g}$-closed sets of (X,τ). By assumption, there exist open sets U containing A and V containing B such that $cl(U)\cap cl(V)=\emptyset$, we have $U\cap V=\emptyset$ and thus (X,τ) is $\hat{g}$-normal. ■

Theorem 18. *If $f:(X,\tau)\to(Y,\sigma)$ is bijective, pre-semi-open, $\hat{g}$-continuous and open and (X,τ) is $\hat{g}$-normal, then (Y,σ) is $\hat{g}$-normal.*

Proof. Let A and B be any disjoint $\hat{g}$-closed sets of (Y,σ). The function f is $\hat{g}$-irresolute by Theorem 11 and so $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\hat{g}$-closed sets of (X,τ). Since (X,τ) is $\hat{g}$-normal, there exist disjoint open sets U and V such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$. Since f is open and bijective, we have $f(U)$ and $f(V)$ are open in (Y,σ) such that $A\subseteq f(U)$, $B\subseteq f(V)$ and $f(U)\cap f(V)=\emptyset$. Therefore, (Y,σ) is $\hat{g}$-normal. ■

Theorem 19. *If $f:(X,\tau)\to(Y,\sigma)$ is irresolute $\hat{g}$-closed continuous injection and (Y,σ) is $\hat{g}$-normal, then (X,τ) is $\hat{g}$-normal.*

Proof. Let A and B be any disjoint $\hat{g}$-closed subsets of (X,τ). Since f is irresolute $\hat{g}$-closed, $f(A)$ and $f(B)$ are disjoint $\hat{g}$-closed sets of (Y,σ) by Theorem 13. Since (Y,σ) is $\hat{g}$-normal, there exist disjoint open sets U and V such that $f(A)\subseteq U$ and $f(B)\subseteq V$. i.e., $A\subseteq f^{-1}(U)$, $B\subseteq f^{-1}(V)$ and $f^{-1}(U)\cap f^{-1}(V)=\emptyset$. Since f is continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are open in (X,τ), we have (X,τ) is $\hat{g}$-normal. ■

Theorem 20. *If $f:(X,\tau)\to(Y,\sigma)$ is weakly continuous $\hat{g}$-closed injection and (Y,σ) is $\hat{g}$-normal, then (X,τ) is normal.*

Proof. Let A and B be any two disjoint closed sets of (X,τ). Since f is injective and $\hat{g}$-closed, $f(A)$ and $f(B)$ are disjoint $\hat{g}$-closed sets of (Y,σ). Since (Y,σ) is $\hat{g}$-normal, by Theorem 17, there exist open sets U and V such that $f(A)\subseteq U$, $f(B)\subseteq V$ and $cl(U)\cap cl(V)=\emptyset$. Since f is weakly continuous, it follows that [5, Theorem 1], $A\subseteq f^{-1}(U)\subseteq int(f^{-1}(cl(U)))$, $B\subseteq f^{-1}(V)\subseteq int(f^{-1}(cl(V)))$ and $int(f^{-1}(cl(U)))\cap int(f^{-1}(cl(V)))=\emptyset$. Therefore, (X,τ) is normal. ■

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