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A NEW VIEW TO N-GROUP THEORY: SOFT N-GROUPS

Abstract. Molodtsov introduced the concept of soft sets. In this paper, we present the definition of soft N-groups and construct some basic properties by using N-groups and Molodtsov's definition of soft sets. We introduce the notions of soft N-subgroups, soft N-ideal, N-idealistic soft N-groups and soft N-group homomorphisms. Moreover, the relations between N-idealistic soft N-groups and soft N-groups are investigated under certain conditions of the near-ring N and these relations are illustrated by many examples.

Key words: uncertainty modeling, soft sets, soft N-groups, soft N-ideal, N-idealistic soft N-groups.

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1. Introduction

Molodtsov [18] introduced soft set theory in 1999 by for dealing with uncertainties and it has not continued to experience tremendous growth and diversification in the mean of algebraic structures as in [1, 2, 4, 8, 9, 10, 11, 12, 13, 14, 21, 24, 25, 26, 23, 27] but also operations of soft sets as in [3, 15, 22]. Furthermore, soft set relations and functions [5] and soft mappings [17] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 16, 19].

In this paper, we introduce a basic version of soft N-group theory, which extends the notion of N-group by including some algebraic structures in soft set theory. A soft N-group defined in this paper is actually a parametrized family of N-subgroups, and has some properties similar to those of N-groups.

2. Preliminaries

By a near-ring, we shall mean an algebraic system $(N, +, .)$, where

- $(N, +)$ forms a group (not necessarily abelian)

- $(N, .)$ forms a semi-group and
- $(a+b)c = ac+bc$ for all $a, b, c \in N$ (i.e. we study on right near-rings.)

Throughout this paper, N will always denote a right near-ring. For a near-ring N, the zero-symmetric part of N denoted by N_0 is defined by $N_0 = \{n \in N \mid n0 = 0\}$. If $N = N_0$, then N is called a zero-symmetric near-ring. A normal subgroup I of N is called a left ideal of N if $n(s+i) - ns \in I$ for all $n, s \in N$ and $i \in I$ and denoted by $I \lhd_\ell N$.

Let $(\Gamma, +)$ be a group and

$$\begin{aligned} \mu : \quad N \times \Gamma &\rightarrow \Gamma \\ (n, \gamma) &\rightarrow n\gamma \,. \end{aligned}$$

(Γ, μ) is called an N-group if $\forall x, y \in N$, $\forall \gamma \in \Gamma$,

(i) $x(y\gamma) = (xy)\gamma$ and

(ii) $(x+y)\gamma = x\gamma + y\gamma$.

It is denoted by N^Γ. Clearly N itself is an N-group. Let Γ be a group and $M(\Gamma) = \{f \mid f : \Gamma \rightarrow \Gamma\}$. Then Γ is an $M(\Gamma)$-group, with

$$\begin{aligned} \mu : \quad M(\Gamma) \times \Gamma &\rightarrow \Gamma \\ (f, \gamma) &\rightarrow f(\gamma) \,. \end{aligned}$$

A subgroup Δ of N^Γ with $N\Delta \subseteq \Delta$ is said to be an N-subgroup of Γ and denoted by $\Delta \leq_N \Gamma$. A normal subgroup Δ of Γ is called an ideal of N^Γ and denoted by $\Delta \trianglelefteq_N \Gamma$, if $\forall \gamma \in \Gamma$, $\forall \delta \in \Delta$, $\forall n \in N$, $n(\gamma+\delta) - n\gamma \in \Delta$. It is obvious that when we take $\Gamma = N$, the ideals of N^N coincide with the left ideals of N. Let N be a near-ring, Γ and Ψ two N- groups. Then $h : \Gamma \rightarrow \Psi$ is called an N-homomorphism if $\forall \gamma, \delta \in \Gamma$, $\forall n \in N$,

(i) $h(\gamma+\delta) = h(\gamma) + h(\delta)$ and

(ii) $h(n\gamma) = nh(\gamma)$.

N^Γ is said to be a monogenic N-group if and only if there exists a $\gamma \in \Gamma$ such that $N\gamma = \Gamma$. In this case we say that N^Γ is monogenic by γ and γ is a generator for N^Γ. It is well-known that if Γ is a monogenic N-group by γ, then

$$\begin{aligned} h_\gamma : \quad N &\rightarrow \Gamma \\ n &\rightarrow n\gamma \end{aligned}$$

is an N-group epimorphism. For all undefined concepts and notions we refer to Pilz [20].

Molodtsov [18] defined the soft set in the following manner:

Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U and $A \subseteq E$.

Definition 1 ([18]). *A pair (F, A) is called a soft set over U, where F is a mapping given by*

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2 ([15]). *The bi-intersection of two soft sets (F, A) and(G, B) over a common universe U is defined to be the soft set (H, C), where $C = A \cap B$ and $H : C \to P(U)$ is a mapping given by $H(x) = F(x) \cap G(x)$ for all $x \in C$. This is denoted by $(F, A)\widetilde{\sqcap}(G, B) = (H, C)$.*

Definition 3 ([3]). *Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \Cap (G, B)$, and is defined as $(F, A) \Cap (G, B) = (H, C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.*

Definition 4 ([3]). *Let (F, A) and (G, B) be two soft sets over a common universe U. The extended intersection of (F, A) and (G, B) is defined to be the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } \ e \in A \setminus B, \\ G(e) & \text{if } \ e \in B \setminus A, \\ F(e) \cap G(e) & \text{if } \ e \in A \cap B. \end{cases}$$

This relation is denoted by $(F, A) \sqcap_{\varepsilon} (G, B) = (H, C)$.

Definition 5 ([15]). *Let (F, A) and (G, B) be two soft sets over a common universe U. The union of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } \ e \in A \setminus B, \\ G(e) & \text{if } \ e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } \ e \in A \cap B. \end{cases}$$

This relation is denoted by $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 6 ([15]). *If (F, A) and (G, B) are two soft sets over a common universe U, then "(F, A) AND (G, B)" denoted by $(F, A)\widetilde{\wedge}(G, B)$ is defined by $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$, where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$.*

Definition 7 ([8]). *Let $(F_i, A_i)_{i \in I}$ be a nonempty family of soft sets over a common universe U. The union of these soft sets is defined to be the soft set (G, B) such that $B = \bigcup_{i \in I} A_i$ and for all $x \in B$, $G(x) = \bigcup_{i \in I(x)} F_i(x)$ where $I(x) = \{i \in I \mid x \in A_i\}$. In this case we write $\widetilde{\bigcup}_{i \in I}(F_i, A_i) = (G, B)$.*

Definition 8 ([8]). *Let* $(F_i, A_i)_{i\in I}$ *be a nonempty family of soft sets over a common universe set* U. *The AND-soft set* $\widetilde{\bigwedge}_{i\in I}(F_i, A_i)$ *of these soft sets is defined to be the soft set* (H, B) *such that* $B = \prod_{i\in I} A_i$ *and* $H(x) = \bigcap_{i\in I(x)} F_i(x)$ *for all* $x = (x_i)_{i\in I} \in B$.
Note that if $A_i = A$ *and* $F_i = F$ *for all* $i \in I$, *then* $\widetilde{\bigwedge}_{i\in I}(F_i, A_i)$ *is denoted by* $\widetilde{\bigwedge}_{i\in I}(F, A)$. *In this case,* $\prod_{i\in I} A_i = \prod_{i\in I} A$ *means the direct power* A^I.

Definition 9. *Let* $(F_i, A_i)_{i\in I}$ *be a nonempty family of soft sets over a common universe set* U. *The restricted intersection of these soft sets is defined to be the soft set* (G, B) *such that* $B = \bigcap_{i\in I} A_i \neq \emptyset$ *and for all* $x \in B$, $G(x) = \bigcap_{i\in I} F_i(x)$. *In this case we write* $⋒_{i\in I}(F_i, A_i) = (G, B)$.

3. Soft N-groups

In the sequel, let N be a near-ring, Γ be an N-group and A be a nonempty set. R will refer to an arbitrary binary relation between an element of A and an element of Γ, that is, R is a subset of $A \times \Gamma$ without otherwise specified. A set-valued function $F : A \to P(\Gamma)$ can be defined as $F(x) = \{y \in \Gamma \mid (x, y) \in R\}$ for all $x \in A$. Then the pair (F, A) is a soft set over N, which is derived from the relation R. For a soft set (F, A), the set $Supp(F, A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the support of the soft set (F, A). The null soft set is a soft set with an empty support, and a soft set (F, A) is non-null if $Supp(F, A) \neq \emptyset$ [8].

Now we are ready to give the definition of soft N-groups.

Definition 10. *Let* (F, A) *be a non-null soft set over an* N*-group* Γ. *Then* (F, A) *is called a* soft N-group *over* Γ *if* $F(x)$ *is an* N*-subgroup of* Γ *for all* $x \in Supp(F, A)$.

Example 1 (cf.,[21]). Let the additive group $(\mathbb{Z}_6, +)$. Under a multiplication defined by following table, $(\mathbb{Z}_6, +, \cdot)$ is a (right) near-ring.

$\cdot$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	1	5	3	1	5
2	0	2	4	0	2	4
3	3	3	3	3	3	3
4	0	4	2	0	4	2
5	3	5	1	3	5	1

Let $\Gamma = \mathbb{Z}_6$ and (F, A) be a soft set over Γ, where $A = \mathbb{Z}_6$ and $F : A \to P(\Gamma)$ is a set-valued function defined by

$$F(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy \in \{0, 3\}\}$$

for all $x \in A$. Then $F(0) = F(3) = \mathbb{Z}_6$ and $F(1) = F(2) = F(4) = F(5) = \{0, 3\}$. Since $\mathbb{Z}_6$ and $\{0, 3\}$ are both N-subgroups of $\mathbb{Z}_6$, (F, A) is a soft N-group over $\mathbb{Z}_6$.

Let $\Gamma = \mathbb{Z}_6$, $I = \{0, 2, 4\}$ and $G : I \to P(\Gamma)$ be a set-valued function defined by

$$G(x) = \{y \in I \mid xRy \Leftrightarrow xy \in \{0, 2, 4\}\}$$

for all $x \in I$. Then we have $G(0) = G(2) = G(4) = \{0, 2, 4\}$. Since $N\{0, 2, 4\} \nsubseteq \{0, 2, 4\}$, $\{0, 2, 4\}$ is not an N-subgroup of $\mathbb{Z}_6$, therefore (G, I) is not a soft N-group over $\mathbb{Z}_6$.

Example 2. Let $\Gamma = \mathbb{Z}_2$. It is well-known that $\mathbb{Z}_2$ is an $M(\mathbb{Z}_2)$-group. Let (F, A) be a soft set over $\mathbb{Z}_2$, where $A = \mathbb{Z}_2$ and $F : A \to P(\Gamma)$ is a set-valued function defined by $F(0) = F(1) = \mathbb{Z}_2$. Since $\mathbb{Z}_2$ is an $M(\mathbb{Z}_2)$-subgroup of $\mathbb{Z}_2$, (F, A) is a soft $M(\mathbb{Z}_2)$-group over $\mathbb{Z}_2$.

Let $\Gamma = \mathbb{Z}_2$ and $K = \{0, I\} \subseteq M(\mathbb{Z}_2)$, where I is the identity function and 0 is the zero function. It is obvious that K is a near-ring with the operations of usual addition and composition of functions, also it is seen that $\mathbb{Z}_2$ is a K-group. Let (G, B) be a soft set over $\mathbb{Z}_2$, where $B = \mathbb{Z}_2$ and $G : B \to P(\Gamma)$ is a set-valued function defined by

$$G(x) = \{y \in \Gamma \mid x\alpha y \Leftrightarrow y = nx \; for\; some\; n \in N\}$$

for all $x \in A$. Here $nx = x + x ... + x$ means the n -fold sum of x and $0x = 0$. Then $G(0) = \{0\}$, $G(1) = \{0, 1\}$. Since $\{0\}$ and $\{0, 1\}$ are both K-subgroup of $\mathbb{Z}_2$, (G, B) is a soft K-group over $\mathbb{Z}_2$. Note that, if we defined above F as G, then $F(0) = \{0\}$, $F(1) = \{0, 1\}$. Since $\{0\}$ is not an $M(\mathbb{Z}_2)$-subgroup of $\mathbb{Z}_2$, then (F, A) would not be a soft $M(\mathbb{Z}_2)$-group over $\mathbb{Z}_2$.

Theorem 1. *Let (F, A), (G, B) and (K, A) be soft N-groups over Γ. Then*

a)If it is non-null, then the soft set $(F, A)\widetilde{\wedge}(G, B)$ is a soft N-group over Γ.

b) If it is non-null, then the bi-intersection $(F, A)\widetilde{\sqcap}(K, A)$ is a soft N-group over Γ.

c) If it is non-null, then the restricted intersection $(F, A) \Cap (G, B)$ is a soft N-group over Γ.

d) If it is non-null, then the soft set $(F, A) \sqcap_{\varepsilon} (G, B)$ is a soft N-group over Γ.

e) If A and B are disjoint, then $(F, A)\widetilde{\cup}(G, B)$ is a soft N-group over Γ.

Proof. *a*) Let $(F, A)\widetilde{\wedge}(G, B) = (Q, A \times B)$, where $Q(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$. Then by hypothesis, $(Q, A \times B)$ is a non-null soft set over Γ. If $(x, y) \in Supp(Q, A \times B)$, then $Q(x, y) = F(x) \cap G(y) \neq \emptyset$.

It follows that $\emptyset \neq F(x)$ and $\emptyset \neq G(y)$ are both N-subgroups of Γ. Hence $Q(x,y)$ is an N-subgroup of Γ for all $(x,y) \in Supp(Q, A \times B)$. Therefore $(Q, A \times B)$ is a soft N-group over Γ.

b) Let $(F,A)\widetilde{\sqcap}(K,A) = (W,A)$, where $W(x) = F(x) \cap K(x)$ for all $x \in A$. Suppose that (W,A) is a non-null soft set over Γ. If $x \in Supp(W,A)$, then $W(x) = F(x) \cap K(x) \neq \emptyset$. Thus $\emptyset \neq F(x)$ and $\emptyset \neq K(x)$ are both N-subgroups of Γ. Hence $W(x)$ is an N-subgroup of Γ for all $x \in Supp(W,A)$. Therefore (W,A) is a soft N-group over Γ, as required.

c) Let $(F,A) \Cap (G,B) = (H,C)$, where $H(x) = F(x) \cap G(x)$ for all $x \in C = A \cap B \neq \emptyset$. Suppose that (H,C) is a non-null soft set over Γ. If $x \in Supp(H,C)$, then $H(x) = F(x) \cap G(x) \neq \emptyset$. It follows that $\emptyset \neq F(x)$ and $\emptyset \neq G(x)$ are both N-subgroups of Γ. Hence $H(x)$ is an N-subgroup of Γ for all $x \in Supp(H,C)$. Thus, (H,C) is a soft N-group over Γ.

d) Let $(F,A) \sqcap_{\varepsilon} (G,B) = (K, A \cup B)$, where

$$K(x) = \begin{cases} F(x) & \text{if } \ x \in A \setminus B, \\ G(x) & \text{if } \ x \in B \setminus A, \\ F(x) \cap G(x) & \text{if } \ x \in A \cap B \end{cases}$$

for all $x \in A \cup B$. Suppose that $(K, A \cup B)$ is a non-null soft set over Γ. Let $x \in Supp(K, A \cup B)$. If $x \in A \setminus B$, then $\emptyset \neq K(x) = F(x) \leq_N \Gamma$. If $x \in B \setminus A$, then $\emptyset \neq K(x) = G(x) \leq_N \Gamma$ and if $x \in A \cap B$, then $K(x) = F(x) \cap G(x) \neq \emptyset$. Since $\emptyset \neq F(x) \leq_N \Gamma$ and $\emptyset \neq G(x) \leq_N \Gamma$, it follows that $K(x) \leq_N \Gamma$ for all $x \in Supp(K, A \cup B)$. Therefore $(F,A) \sqcap_{\varepsilon} (G,B) = (K, A \cup B)$ is a soft N-group over Γ.

e) Let $(F,A)\widetilde{\cup}(G,B) = (T, A \cup B)$, where

$$T(x) = \begin{cases} F(x) & \text{if } \ x \in A \setminus B, \\ G(x) & \text{if } \ x \in B \setminus A, \\ F(x) \cup G(x) & \text{if } \ x \in A \cap B \end{cases}$$

for all $x \in A \cup B$. Since $A \cap B = \emptyset$, it follows that either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in A \cup B$. If $x \in A \setminus B$, then $T(x) = F(x)$ is an N-subgroup of Γ and if $x \in B \setminus A$, then $T(x) = G(x)$ is an N-subgroup of Γ. Thus, $(T, A \cup B)$ is a soft N-group over Γ. ∎

Definition 11. *Let (F,A) and (G,B) be two soft N-groups over Γ and Ψ, respectively. The product of soft N-groups (F,A) and (G,B) is defined as $(F,A) \times (G,B) = (U, A \times B)$, where $U(x,y) = F(x) \times G(y)$ for all $(x,y) \in A \times B$.*

Proposition 1. *Let (F,A) and (G,B) be two soft N-groups over Γ and Ψ, respectively. Then if it is non-null, the product $(F,A) \times (G,B)$ is a soft N-group over $\Gamma \times \Psi$.*

Proof. Let $(F,A)\times(G,B)=(U,A\times B)$, where $U(x,y)=F(x)\times G(y)$ for all $(x,y)\in A\times B$. Then by hypothesis, $(U,A\times B)$ is a non-null soft set over $\Gamma\times\Psi$. If $(x,y)\in Supp(U,A\times B)$, then $U(x,y)=F(x)\times G(y)\neq\emptyset$. Since $\emptyset\neq F(x)$ is an N-subgroup of Γ and $\emptyset\neq G(y)$ is an N-subgroup of Ψ, it follows that $U(x,y)$ is an N-subgroup of $\Gamma\times\Psi$ for all $(x,y)\in Supp(U,A\times B)$. Therefore $(U,A\times B)$ is a soft N-group over $\Gamma\times\Psi$. ■

Example 3. Let N be the near-ring on S_3 with two binary operations as given in table below (cf.,[20] No 11 on S_3).

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	5	4	3	2
2	2	4	0	5	1	3
3	3	5	4	0	2	1
4	4	2	3	1	5	0
5	5	3	1	2	0	4

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	1	1	3	2	2	3
3	1	1	2	3	3	2
4	0	0	5	4	4	5
5	0	0	4	5	5	4

Let $\Gamma=N$ and the soft set (F,A) over Γ, where $A=\{0,3,5\}$ and $F:A\rightarrow P(\Gamma)$ is a set-valued function defined by

$$F(x)=\{y\in N\mid xRy\Leftrightarrow xy\in\{0,1\}\}$$

for all $x\in A$. Then $F(0)=N$ and $F(3)=F(5)=\{0,1\}$. Since N and $\{0,1\}$ are both N-subgroups of Γ, (F,A) is a soft N-group over Γ.

Let $\Gamma=N$ and the soft set (G,B) over Γ, where $B=\{0,4,5\}$ and $G:B\rightarrow P(\Gamma)$ is a set-valued function defined by

$$G(x)=\{y\in N\mid xRy\Leftrightarrow xy\in\{0,4,5\}\}$$

for all $x\in B$. Then $G(0)=G(4)=G(5)=N$. Since N is an N-subgroup of Γ, (G,B) is a soft N-group over Γ.

Let $(F,A)\widetilde{\wedge}(G,B)=(Q,A\times B)$, where $Q(x)=F(x)\cap G(y)$ for all $(x,y)\in A\times B$. Then $Q(0,0)=Q(0,4)=Q(0,5)=N$, $Q(3,0)=Q(3,4)=Q(3,5)=Q(5,0)=Q(5,4)=Q(5,5)=\{0,1\}$. Since N and $\{0,1\}$ are both N-subgroups of Γ, $(Q,A\times B)$ is a soft N-group over Γ.

Let $(F,A)\Cap(G,B)=(H,C)$, where $H(x)=F(x)\cap G(x)$ for all $x\in C=A\cap B=\{0,5\}$. Since $H(0)=F(0)\cap G(0)=N$ and $H(5)=F(5)\cap G(5)=\{0,1\}$ are both N-subgroups of Γ, (H,C) is a soft N-group over Γ.

Assume that $(F,A)\sqcap_{\varepsilon}(G,B)=(T,A\cup B)$, where

$$T(x)=\begin{cases} F(x) & \text{if } x\in A\setminus B=\{3\},\\ G(x) & \text{if } x\in B\setminus A=\{4\},\\ F(x)\cap G(x) & \text{if } x\in A\cap B=\{0,5\}\end{cases}$$

for all $x \in A \cup B$. Then $Supp(T, A \cup B) = \{0, 3, 4, 5\}$ and $T(0) = N$, $T(3) = \{0, 1\}$, $T(4) = N$ and $T(5) = \{0, 1\}$. Since $T(x) \leq_N \Gamma$ for all $x \in Supp(T, A \cup B)$, $(T, A \cup B)$ is a soft N-group over Γ.

Let $(F, A) \times (G, B) = (W, A \times B)$, where $W(x) = F(x) \times G(y)$ for all $(x, y) \in A \times B$. Then $W(0, 0) = W(0, 4) = W(0, 5) = N \times N$, $W(3, 0) = W(3, 4) = W(3, 5) = W(5, 0) = W(5, 4) = W(5, 5) = \{0, 1\} \times N$. Since $N \times N \leq_N N \times N$ and $\{0, 1\} \times N \leq_N N \times N$, $(W, A \times B)$ is a soft N-group over $N \times N$.

Definition 12. *Let (F, A) and (G, B) be two N-groups over Γ. Then (F, A) is called a* soft N-subgroup *of (G, B) if it satisfies:*
(i) $A \subseteq B$
(ii) $F(x)$ is an N-subgroup of $G(x)$ for all $x \in Supp(F, A)$.

Proposition 2. *Let (F, A), (G, A) and (H, B) be soft N-groups over Γ. Then we have the following:*
a) If $F(x) \subset G(x)$ for all $x \in A$, then (F, A) is a soft N-subgroup of (G, A).
b) $(F, A) \tilde{\sqcap} (G, A)$ is a soft N-subgroup of both (F, A) and (G, A) if it is non-null.
c) $(F, A) \Cap (H, B)$ is a soft N-subgroup of both (F, A) and (H, B) if it is non-null.
d) $(F, A) \sqcap_\varepsilon (G, A)$ is a soft N-subgroup of both (F, A) and (G, A) if it is non-null.

Proof. *a)* If $F(x) \subseteq G(x)$ for all $x \in A$, it is clear that $F(x)$ is an N-subgroup of $G(x)$. Thus, the proof is obvious.

b) It follows from (a) and Theorem 1(b).

c) Since $A \cap B \subseteq A$ (and $A \cap B \subseteq B$), the first condition of Definition 12 is satisfied. Let $(F, A) \Cap (H, B) = (K, C)$, where $C = A \cap B$ and $K(x) = F(x) \cap H(x)$ for all $x \in C$. Since $K(x) = F(x) \cap H(x) \subseteq F(x)$ and $K(x) = F(x) \cap H(x) \subseteq H(x)$ for all $x \in C$, the proof is completed from Theorem 1(a).

d) Let $(F, A) \sqcap_\varepsilon (G, A) = (Q, A)$ where $Q(x) = F(x) \cap G(x)$ for all $x \in A$. Since $Q(x) = F(x) \cap G(x) \subseteq F(x)$ and $Q(x) = F(x) \cap G(x) \subseteq G(x)$ for all $x \in A$, the proof is completed from Theorem 1(a). ■

Theorem 2. *Let (F, A) be a soft N-group over Γ and $(F_i, A_i)_{i \in I}$ be a nonempty family of soft N-subgroups of (F, A). Then we have the following:*
a) $\Cap_{i \in I} (F_i, A_i)$ is a soft N-subgroup of (F, A), if it is non-null.
b) $\widetilde{\bigwedge}_{i \in I} (F_i, A_i)$ is a soft N-subgroup of $\widetilde{\bigwedge}_{i \in I} (F, A)$, if it is non-null.
c) If $\{A_i \mid i \in I\}$ are pairwise disjoint, i.e., $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\widetilde{\bigcup}_{i \in I} (F_i, A_i)$ is soft N-subgroup of (F, A).

Proposition 3. *Let (F, A) be a soft N-group over Γ and $(F_i, A_i)_{i\in I}$ be a nonempty family of soft N-subgroups of (F, A). Then $⋒_{i\in I}(F_i, A_i)$ is a soft N-subgroup of (F_i, A_i) for each $i \in I$, if it is non-null.*

Proof. Let $⋒_{i\in I}(F_i, A_i) = (H, C)$, where $C = \bigcap_{i\in I} A_i \neq \emptyset$ and $H(x) = \bigcap_{i\in I} F_i(x)$ for all $x \in C$. The parameter set of the soft set $⋒_{i\in I}(F_i, A_i)$, that is, $\bigcap_{i\in I} A_i$ is a subset of the parameter set of the soft set $(F_i, A_i)_{i\in I}$ for all $i \in I$. Suppose that (H, C) is a non-null soft set over N. If $x \in Supp(H, C)$, then $H(x) = \bigcap_{i\in I} F_i(x) \neq \emptyset$. Thus $\emptyset \neq F_i(x)$ are N-subgroups of Γ for all $i \in I$. Therefore $H(x) = \bigcap_{i\in I} F_i(x)$ is an N-subgroup of Γ. Moreover, since $\bigcap_{i\in I} F_i(x) \subset F_i(x)$, for all $i \in I$ and for all $x \in \bigcap_{i\in I} A_i$, the rest of the proof is obvious. ■

Definition 13. *Let (F, A) be a soft N-group over Γ and (H, B) be a soft N-subgroup of (F, A). Then we say that (H, B) is a soft N-ideal of (F, A), written $(H, B)\widetilde{\lhd_N}(F, A)$, if $H(x)$ is an ideal of $F(x)$; i.e., $H(x) \trianglelefteq_N F(x)$ for all $x \in B$.*

Theorem 3. *Let (F, A) be a soft N-group over Γ and $(F_i, A_i)_{i\in I}$ be a nonempty family of soft N-ideals of (F, A). Then we have the following:*

a) $⋒_{i\in I}(F_i, A_i)$ is a soft N-ideal of (F, A), if it is non-null.

b) $\widetilde{\bigwedge}_{i\in I}(F_i, A_i)$ is a soft N-ideal of $\widetilde{\bigwedge}_{i\in I}(F, A)$, if it is non-null.

c) If $\{A_i \mid i \in I\}$ are pairwise disjoint, i.e., $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\widetilde{\bigcup}_{i\in I}(F_i, A_i)$ is soft N-ideal of (F, A).

Proposition 4. *Let (F, A) be a soft N-group over Γ and $(F_i, A_i)_{i\in I}$ be a nonempty family of soft N-ideals of (F, A). Then $⋒_{i\in I}(F_i, A_i)$ is a soft N-ideal of (F_i, A_i) for each $i \in I$, if it is non-null.*

Proposition 5. *Let (F, A) and (G, A) be two soft N-ideals over Γ. Then $(F, A) \sqcap_\varepsilon (G, A)$ is a soft N-ideal of both (F, A) and (G, A), if it is non-null.*

4. Soft N-ideals

Definition 14. *Let (F, A) be a soft N-group over Γ. A non-null soft set (G, I) over Γ is called a* soft N-ideal *of (F, A) denoted by $(G, I)\widetilde{\trianglelefteq_N}(F, A)$ if it satisfies:*

(i)] $I \subset A$

(ii)] $G(x) \trianglelefteq_N F(x)$ for all $x \in Supp(G, I)$.

Example 4. Let $\Gamma = \mathbb{Z}_6$ and (F, A) be a soft set over Γ, where $A = \{0, 2, 4\}$ and $F : A \to P(\Gamma)$ is a set-valued function defined by

$$F(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy \in \{0, 3\}\}$$

for all $x \in A$. Then $F(0) = \mathbb{Z}_6$ and $F(2) = F(4) = \{0, 3\}$. It can be easily seen that (F, A) is a soft N-group over $\mathbb{Z}_6$.

Let $\Gamma = \mathbb{Z}_6$ and $G : A \to P(\Gamma)$ be a set-valued function defined by

$$G(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy \in \{0, 2, 4\}\}$$

for all $x \in A = \{0, 2, 4\}$. Then, $G(0) = G(2) = G(4) = \mathbb{Z}_6$. It is easily seen that $F(x) \trianglelefteq_N G(x)$ for all $x \in Supp(F, A) = \{0, 2, 4\}$, hence $(F, A) \widetilde{\triangleleft_N} (G, A)$.

Theorem 4. *Let (F, A) be a soft N-group Γ, (G_1, I_1) and (G_2, I_2) be soft N-ideals of (F, A). Then the soft set $(G_1, I_1) \Cap (G_2, I_2)$ is a soft N-ideal of (F, A) if it is non-null.*

Proof. Assume that $(G_1, I_1) \widetilde{\trianglelefteq_N} (F, A)$ and $(G_2, I_2) \widetilde{\trianglelefteq_N} (F, A)$. Let $(G_1, I_1) \Cap (G_2, I_2) = (G, I)$, where $I = I_1 \cap I_2 \neq \emptyset$ and $G(x) = G_1(x) \cap G_2(x)$ for all $x \in I$. Since $I_1 \subset A$ and $I_2 \subset A$, it is clear that $I \subset A$. Suppose that the soft set (G, I) is non-null. If $x \in Supp(G, I)$, then $G(x) = G_1(x) \cap G_2(x) \neq \emptyset$. Since $G_1(x) \trianglelefteq_N F(x)$, $G_2(x) \trianglelefteq_N F(x)$ and the intersection of ideals of Γ is an ideal of Γ, it follows that $G(x) \trianglelefteq_N F(x)$ for all $x \in Supp(G, I)$. Therefore $(G_1, I_1) \Cap (G_2, I_2) \widetilde{\trianglelefteq_N} (F, A)$. ∎

Theorem 5. *Let (F, A) be a soft N-group Γ, (G_1, I_1) and (G_2, I_2) be soft N-ideals of (F, A). Then the soft set $(G_1, I_1) \widetilde{\cup} (G_2, I_2)$ is a soft N-ideal of (F, A) if I_1 and I_2 are disjoint.*

Proof. Assume that $(G_1, I_1) \widetilde{\trianglelefteq_N} (F, A)$ and $(G_2, I_2) \widetilde{\trianglelefteq_N} (F, A)$. Let $(G_1, I_1) \widetilde{\cup} (G_2, I_2) = (G, I)$, where $I = I_1 \cup I_2$ and for all $x \in I$

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in I_1 \setminus I_2, \\ G_2(x) & \text{if } x \in I_2 \setminus I_1, \\ G_1(x) \cup G_2(x) & \text{if } x \in I_1 \cap I_2. \end{cases}$$

Since $I_1 \subset A$ and $I_2 \subset A$, it is obvious that $I \subset A$. If $I_1 \cap I_2 = \emptyset$, then for all $x \in Supp(G, I)$, we know that either $x \in I_1 \setminus I_2$ or $x \in I_2 \setminus I_1$. If $x \in I_1 \setminus I_2$, then $\emptyset \neq G_1(x) = G(x) \trianglelefteq_N F(x)$ and if $x \in I_2 \setminus I_1$, then $\emptyset \neq G_2(x) = G(x) \trianglelefteq_N F(x)$ for all $x \in Supp(G, I)$. Therefore $(G_1, I_1) \widetilde{\cup} (G_2, I_2) \widetilde{\trianglelefteq_N} (F, A)$. ∎

Theorem 6. *Let (F, A) be a soft N-group Γ, (G_1, I_1) and (G_2, I_2) be soft N-ideals of (F, A). Then the soft set $(G_1, I_1) \sqcap_\varepsilon (G_2, I_2)$ is a soft N-ideal of (F, A) if it is non-null.*

Proof. Assume that $(G_1, I_1) \widetilde{\triangleleft_N} (F, A)$ and $(G_2, I_2) \widetilde{\triangleleft_N} (F, A)$. Let $(G_1, I_1) \sqcap_\varepsilon (G_2, I_2) = (G, I)$, where $I = I_1 \cup I_2$ and

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in I_1 \setminus I_2, \\ G_2(x) & \text{if } x \in I_2 \setminus I_1, \\ G_1(x) \cap G_2(x) & \text{if } x \in I_1 \cap I_2 \end{cases}$$

for all $x \in I$. Since $I_1 \subset A$ and $I_2 \subset A$, it is obvious that $I \subset A$. Suppose that the soft set (G, I) is non-null and $x \in Supp(G, I)$. If $x \in I_1 \setminus I_2$, then $\emptyset \neq G_1(x) = G(x) \trianglelefteq_N F(x)$ and if $x \in I_2 \setminus I_1$, then $\emptyset \neq G_2(x) = G(x) \trianglelefteq_N F(x)$. And if $x \in I_1 \cap I_2$, then $\emptyset \neq G(x) = G_1(x) \cap G_2(x)$. Since $(G_1, I_1)\widetilde{\trianglelefteq}_N(F, A)$ and $(G_2, I_2)\widetilde{\trianglelefteq}_N(F, A)$, we know that the nonempty sets $G_1(x)$ and $G_2(x)$ are both ideals of $F(x)$. It follows that $G(x) \trianglelefteq_N F(x)$ for all $x \in Supp(G, I)$. Therefore $(G_1, I_1) \sqcap_\varepsilon (G_2, I_2)\widetilde{\trianglelefteq}_N(F, A)$, as required. ■

Example 5. Let $\Gamma = \mathbb{Z}_6$ and $F : A \to P(\Gamma)$ be a set-valued function defined by $F(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy \in \{0, 2, 4\}\}$ if $x \in \{0, 2, 4\}$ and $F(x) = \mathbb{Z}_6$ if $x \in \{1, 3, 5\}$. Then, we have $F(0) = F(1) = F(2) = F(3) = F(4) = F(5) = \mathbb{Z}_6$.

Let $\Gamma = \mathbb{Z}_6$ and (G, B) be a soft set over Γ, where $B = \{2, 3, 4\}$ and $G : B \to P(\Gamma)$ is a set-valued function defined by

$$G(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy \in \{0, 3\}\}$$

for all $x \in B$. Then $G(3) = \mathbb{Z}_6$ and $G(2) = G(4) = \{0, 3\}$. Since $G(2) \trianglelefteq_N F(2)$, $G(3) \trianglelefteq_N F(3)$ and $G(4) \trianglelefteq_N F(4)$, it follows that $(G, B)\widetilde{\triangleleft}_N(F, A)$.

Let $\Gamma = \mathbb{Z}_6$ and (H, C) be a soft set over $\mathbb{Z}_6$, where $C = \{0, 2, 4\}$ and $H : C \to P(\Gamma)$ is a set-valued function defined by

$$H(x) = \{y \in C \mid xRy \Leftrightarrow xy \in \{0, 2, 4\}\}$$

for all $x \in C$. Then $H(0) = H(2) = H(4) = \{0, 2, 4\}$. Since $H(x) \trianglelefteq_N F(x)$ for all $x \in \{0, 2, 4\}$, it follows that $(H, C)\widetilde{\trianglelefteq}_N(F, A)$.

Now we consider the restricted intersection of soft N-ideals (G, B) and (H, C) of (F, A). Let $(G, B)\Cap(H, C) = (T, B \cap C)$ where $T(x) = G(x) \cap H(x)$ for all $x \in B \cap C = \{2, 4\}$. Then we have $T(2) = \{0\} \trianglelefteq_N F(2)$, $T(4) = \{0\} \trianglelefteq_N F(4)$ for all $x \in Supp(T, B \cap C)$, which means that $(G, B)\Cap(H, C)\widetilde{\trianglelefteq}_N(F, A)$.

Now we consider $(G, B)\widetilde{\cup}(H, C)$. Let $(G, B)\widetilde{\cup}(H, C) = (W, B \cup C)$, where

$$W(x) = \begin{cases} G(x) & \text{if} \quad x \in B \setminus C = \{3\}, \\ H(x) & \text{if} \quad x \in C \setminus B = \{0\}, \\ G(x) \cup H(x) & \text{if} \quad x \in B \cap C = \{2, 4\} \end{cases}$$

for all $x \in B \cup C = \{0, 2, 3, 4\}$. Then, $W(0) = \{0, 2, 4\}$, $W(2) = W(4) = \{0, 2, 3, 4\}$ and $W(3) = \mathbb{Z}_6$. Since $W(2)$ and $W(4)$ is not an ideal of $F(2)$ and $F(4)$ respectively, $(G, B)\widetilde{\cup}(H, C)$ is not a soft N-ideal of (F, A). That is to say, the condition 'disjoint' can not be removed from this theorem.

Furthermore, since $W(0) = \{0, 2, 4\} \trianglelefteq_N F(0)$, and $W(2) = \{0\} \trianglelefteq_N F(2)$, $W(3) = \mathbb{Z}_6 \trianglelefteq_N F(3)$ and $W(4) = \{0\} \trianglelefteq_N F(4)$, it is easy to see that $(G, B) \sqcap_\varepsilon (H, C)$ is a soft N-ideal of (F, A).

5. N-idealistic soft N-groups and the relationships between soft N-groups and N-idealistic soft N-groups

Definition 15. *Let (F, A) be a soft N-group over Γ. If $F(x)\widetilde{\trianglelefteq_N}\Gamma$ for all $x \in Supp(F, A)$, then (F, A) is called an* N-idealistic soft N-group *over Γ. Here, (F, A) should be a non-null soft set over Γ.*

Example 6. Let $\Gamma = \mathbb{Z}_6$ and the soft N-group (F, A) of Γ be the one given in Example 1. Since $F(x) \trianglelefteq_N \Gamma$ for all $x \in Supp(F, A) = \mathbb{Z}_6$, (F, A) is an N-idealistic soft N-group over Γ.

Theorem 7. *Let (F, A) and (G, B) be two N-idealistic soft N-groups over Γ. Then we have the following:*

a) If it is non-null, $(F, A) \Cap (G, B)$ is an N-idealistic soft N-group over Γ.

b) If A and B are disjoint, then $(F, A)\widetilde{\cup}(G, B)$ is an N-idealistic soft N-group over Γ.

c) If it is non-null, $(F, A)\widetilde{\wedge}(G, B)$ is an N-idealistic soft N-group over Γ.

d) If it is non-null, $(F, A) \sqcap_{\varepsilon} (G, B)$ is an N-idealistic soft N-group over Γ.

Proof. Straightforward, hence is omitted. ■

Definition 16. *An N-group Γ is said to satisfy the condition (N) if $\Delta \trianglelefteq_N \Theta \trianglelefteq_N \Lambda$, then $\Delta \trianglelefteq_N \Lambda$.*

Proposition 6 ([20], 1.34 Proposition)**.** *If $N = N_0$, then every ideal of Γ is also an N-subgroup of Γ.*

Proposition 7. *Let $N = N_0$, Γ be an N-group which satisfies the condition (N) and let (F, A) be an N-idealistic soft N-group over Γ. If (G, I) is a soft N-ideal of (F, A), then (G, I) is also N-idealistic soft N-group over Γ.*

Proof. If $(G, I)\widetilde{\trianglelefteq_N}(F, A)$, then for all $x \in Supp(G, I)$, $G(x) \trianglelefteq_N F(x)$. Since (F, A) is an N-idealistic soft N-group over Γ, then for all $x \in Supp(F, A)$, $F(x)\trianglelefteq_N\Gamma$. Thus we have $G(x)\trianglelefteq_N F(x)\trianglelefteq_N\Gamma$ for all $x \in Supp(G, I)$. Since Γ satisfies condition (N), $G(x)\trianglelefteq_N\Gamma$ for all $x \in Supp(G, I)$. Because of the fact that every ideal of Γ is also an N-subgroup of Γ when N is a zero-symmetric near-ring, then $G(x)$ is also an N-subgroup of Γ for all $x \in Supp(G, I)$. This means that (G, I) is a soft N-group over Γ. Moreover, (G, I) is an N-idealistic soft N-group over Γ. ■

Proposition 8. *If N is a zero-symmetric near-ring, then every N-idealistic soft N-group over an N-group Γ is a soft N-group over Γ, however the converse is not true in general.*

The following example shows that the converse of Proposition 8 is not true in general.

Example 7 (cf.,[20]). Let Klein-4 group $N = \{0,1,2,3\}$. Under the operations defined by the following tables, $(N,+,.)$ is a (right) near-ring. It is easily seen that N is not a zero-symmetric near-ring.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	0	0	0	2
3	1	1	1	3

Let $\Gamma = N$, $B = \{0,1\}$ and (F,A) be a soft set over Γ, where $A = \{0,2\}$ and assume that $F : A \to P(\Gamma)$ is a set-valued function defined by

$$F(x) = \{0\} \cup \{y \in B \mid xRy \Leftrightarrow xy = 0\}$$

for all $x \in A$. Then $F(0) = F(2) = \{0,1\}$. It can be easily shown that $\{0,1\} \leq_N \Gamma$. Hence (F,A) is a soft N-group over Γ. Nevertheless $F(0) = F(2) = \{0,1\}$ is not an ideal of Γ, since $2.(3+1) - 2.3 = 2 \notin \{0,1\}$. It follows that (F,A) is not an N-idealistic soft N-group over Γ.

And let $\Gamma = N$ and (G,C) be a soft set over Γ, where $C = \{1,3\}$ and assume that $G : C \to P(\Gamma)$ is a set-valued function defined by

$$G(x) = \{0\} \cup \{y \in N \setminus C \mid xRy \Leftrightarrow xy = 1\}$$

for all $x \in C$. Then we have $G(1) = G(3) = \{0,2\}$. It can be easily illustrated that $\{0,2\} \trianglelefteq_N \Gamma$. Hence (G,C) is an N-idealistic soft N-group over Γ. However $\{0,2\}$ is not an N-subgroup of Γ, since $N\{0,2\} \nsubseteq \{0,2\}$. It follows that (G,C) is not a soft N-group over Γ.

Similarly, let the soft set (G,I) in Example 1. It is obvious that $N = \mathbb{Z}_6$ is not a zero-symmetric near-ring, and since $\{0,2,4\}$ is an ideal of Γ; but not an N-subgroup of Γ, it follows that (G,I) is an N-idealistic soft N-group but not a soft N-group over Γ.

Proposition 9. *Let (F,A) be a soft set over Γ and $B \subset A$. If (F,A) is an N-idealistic soft N-group over Γ, then so is (F,B), whenever (F,B) is non-null.*

Proof. It is obvious, hence omitted. ■

As can be seen from the following example, the converse of Proposition 8 is not true in general.

Example 8. Let $\Gamma = N$ in Example 7, $B = \{0, 1\}$ and (F, A) be a soft set over Γ, where $A = N$ and assume that $F : A \to P(\Gamma)$ is a set-valued function defined by

$$F(x) = \{0\} \cup \{y \in B \mid xRy \Leftrightarrow xy = 0\}$$

for all $x \in A$. Then, $F(0) = F(2) = \{0, 1\}$, $F(1) = F(3) = \{0\}$. Since $\{0, 1\}$ is not an ideal of Γ as shown in Example 7, (F, A) is not an N-idealistic soft N-group over Γ. However, when we take $B = \{1, 3\} \subset A$, then $(F\mid_B, B)$ is an N-idealistic soft N-group over Γ, where $F\mid_B$ is the restriction of F to B.

Definition 17. *Let (F, A) be an N-idealistic soft N-group over an N-group Γ. Then,*

a) (F, A) is called trivial if $F(x) = \{0_\Gamma\}$ for all $x \in Supp(F, A)$.

b) (F, A) is said to be whole if $F(x) = \Gamma$ for all $x \in Supp(F, A)$.

Example 9. The soft set (F, A) in Example 2 is a whole $M(\mathbb{Z}_2)$-idealistic soft $M(\mathbb{Z}_2)$-group over $\mathbb{Z}_2$.

Let (F, A) be a soft N-group over an N-group Γ and let $f : \Gamma \to \Psi$ be a mapping of N-groups. Then the soft set $(f(F), Supp(F, A))$ over Ψ can be defined, where

$$f(F) : Supp(F, A) \to P(\Psi)$$

is given by $f(F)(x) = f(F(x))$ for all $x \in Supp(F, A)$. It is also worth nothing that $Supp(F, A) = Supp(f(F), Supp(F, A))$.

Proposition 10. *Let $f : \Gamma \to \Psi$ be an epimorphism of N-groups. If (F, A) is an N-idealistic soft N-group over Γ, then $(f(F), Supp(F, A))$ is an N-idealistic soft N-group over Ψ.*

Proof. Note first that since (F, A) is an idealistic N-idealistic soft N-group over Γ, it has to be a non-null soft set over Γ, thus $(f(F), Supp(F, A))$ is a non-null soft set over Ψ, too. We have $f(F)(x) = f(F(x)) \neq \emptyset$ for all $x \in Supp(f(F), Supp(F, A))$. Because of the fact that (F, A) is an N-idealistic soft N-group over Γ, the nonempty set $F(x)$ is an ideal of Γ. Thus, we can conclude that its onto homomorphic image $f(F(x))$ is an ideal of Ψ. So, $f(F(x))$ is an ideal of Ψ for all $x \in Supp(f(F), Supp(F, A))$. It means that $(f(F), Supp(F, A))$ is an N-idealistic soft N-group over Ψ. ∎

Theorem 8. *Let (F, A) be an N-idealistic soft N-group over Γ and let $f : \Gamma \to \Psi$ be an epimorphism of N-groups. Then*

a) If $F(x) = Ker(f)$ for all $x \in Supp(F, A)$, then $(f(F), Supp(F, A))$ is a trivial N-idealistic soft N-group over Ψ.

b) If (F, A) is whole, then $(f(F), Supp(F, A))$ is a whole N-idealistic soft N-group over Ψ.

Proof. *a*) Assume that $F(x) = Ker(f)$ for all $x \in Supp(F, A)$. Then $f(F)(x) = f(F(x)) = 0_\Psi$ for all $x \in Supp(F, A)$. That is to say $(f(F), Supp(F, A))$ is a trivial N-idealistic soft N-group over Ψ.

b) Suppose that (F, A) is whole. Then, $F(x) = \Gamma$ for all $x \in Supp(F, A)$. It follows that $f(F)(x) = f(F(x)) = F(\Gamma) = \Psi$ for all $x \in Supp(F, A)$, which means that $(f(F), Supp(F, A))$ is a whole N-idealistic soft N-group over Ψ. ■

Example 10. *a*) Let $\Gamma = N$ in Example 7, and (F, A) be a soft set over Γ, where $A = \{0, 1, 2\}$ and assume that $F : A \to P(\Gamma)$ is a set-valued function defined by

$$F(x) = \{y \in N \mid xRy \Leftrightarrow 2x = y\}$$

for all $x \in A$. Then $F(0) = F(1) = F(2) = \{0\}$. It is obvious that (F, A) is a trivial N-idealistic soft N-group over Γ.

Let $f : \Gamma \to \Gamma$ be the mapping defined by $f(n) = n\gamma$, where $\gamma = 3 \in \Gamma$. Since Γ is a monogenic N-group by $\gamma = 3$, one can say that f is an epimorphism of N-groups. Also, it is obvious that f is an one-to-one mapping, therefore $Ker(f) = \{0\}$, which means that $F(x) = Ker(f)$ for all $x \in Supp(F, A)$. We need to show that $(f(F), A)$ is a trivial N-idealistic soft N-group over Γ. To see this, we construct the soft set $(f(F), A)$ over Γ, where

$$f(F) : A \to P(\Gamma)$$

is given by $f(F)(x) = f(F(x))$ for all $x \in A$. It follows that, $f(F)(0) = f(F(0)) = f(F)(1) = f(F(1)) = f(F)(2) = f(F(2)) = f(0) = \{0\}$. It is easy to see that $(f(F), A)$ is an N-idealistic soft N-group over Γ, furthermore $(f(F), A)$ is a trivial N-idealistic soft N-group over Γ, as required.

b) Let $\Gamma = N$ in Example 7, and (G, B) be a soft set over Γ, where $B = N$ and assume that $F : B \to P(\Gamma)$ is a set-valued function defined by

$$G(x) = \{3\} \cup \{y \in N \mid xRy \Leftrightarrow xy \in \{0, 1\}\}$$

for all $x \in B$. Then $G(0) = G(1) = G(2) = G(3) = N$. Since $G(x) = \Gamma$ for all $x \in Supp(G, B)$, it follows that (G, B) is a whole N-idealistic soft N-group over Γ.

Let $f : \Gamma \to \Gamma$ be the above near-ring epimorphism. We need to show that $(f(G), Supp(G, B))$ is a whole N-idealistic soft N-group over Γ. To see this, let construct the soft set $(f(G), Supp(G, B))$ over Γ, where

$$f(G) : Supp(G, B) \to P(\Gamma)$$

is given by $f(G)(x) = f(G(x))$ for all $x \in Supp(G, B)$. It follows that $f(G)(0) = f(G(0)) = f(G)(2) = f(G(2)) = f(G)(4) = f(G(4)) = f(\Gamma) = \Gamma$. It is easy to see that $(f(G), Supp(G, B))$ is an N-idealistic soft N-group over Γ, furthermore $(f(G), Supp(G, B))$ is a whole N-id1ealistic soft N-group over Γ, as required.

Definition 18. *Let (F, A) and (G, B) be soft N-groups over Γ_1 and Γ_2, respectively. Let $f : \Gamma_1 \to \Gamma_2$ and $g : A \to B$ be two mappings. Then the pair (f, g) is called a* soft N-group homomorphism *if it satisfies the conditions below:*

(i) f is an epimorphism of N-groups.

(ii) g is a surjective mapping.

(iii) $f(F(x)) = G(g(x))$ for all $x \in A$. If there exists a soft N-group homomorphisms between (F, A) and (G, B), we mention that (F, A) is soft homomorphic *to (G, B), which is denoted by $(F, A) \sim_N (G, B)$. Furthermore, if f is an isomorphism of N-groups and g is a bijective mapping, then (f, g) is said to be a* soft N-group isomorphism. *In this case, we say that (F, A) is* soft isomorphic *to (G, B), which is denoted by $(F, A) \simeq_N (G, B)$.*

Example 11. Let $\Gamma = \mathbb{Z}_6$ and (F, A) be a soft set over Γ, where $F : A \to P(\Gamma)$ is a function by $F(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy \in \{0, 3\}\}$ for all $x \in A = \{0, 1, 2, 4, 5\}$. Then we have $F(0) = \mathbb{Z}_6$, $F(1) = F(2) = F(4) = F(5) = \{0, 3\}$. It is obvious that (F, A) is a soft N-group over Γ. Let (G, B) be a soft set over $\Gamma = \mathbb{Z}_6$, where $G : B \to P(\Gamma)$ is a function defined by $G(x) = \{y \in \mathbb{Z}_6 \mid xRy \Leftrightarrow xy = 0\}$ for all $x \in B = \{0, 2, 4\}$. Then we have $G(0) = \mathbb{Z}_6$, $G(2) = G(4) = \{0, 3\}$. It is obvious that (G, B) is a soft N-group over Γ. Let $f : \mathbb{Z}_6 \to \{0, 2, 4\}$ be the mapping defined by $f(n) = n\gamma$, where $\gamma = 5 \in \mathbb{Z}_6$. Since $\mathbb{Z}_6$ is a monogenic N-group by $\gamma = 5$, one can say that f is an epimorphism of N-groups. Let $g : A \to B$ be the mapping defined by $g(x) = 4x$ for all $x \in \mathbb{Z}_6$. Then one can easily say that g is surjective. Since $f(F(0)) = F(\mathbb{Z}_6) = \mathbb{Z}_6$, $f(F(1)) = f(F(2)) = f(F(4)) = f(F(5)) = f(\{0, 3\}) = \{0, 3\}$ and $G(g(0)) = G(0) = \mathbb{Z}_6$, $G(g(1)) = G(4) = \{0, 3\}$, $G(g(2)) = G(2) = \{0, 3\}$, $G(g(4)) = G(4) = \{0, 3\}$, $G(g(5)) = G(2) = \{0, 3\}$, $f(F(x)) = G(g(x))$ is satisfied for all $x \in A$. Therefore (f, g) is a soft N-homomorphism and $(F, A) \sim_N (G, B)$. Because of the fact that f is isomorphism of N- groups but g is not a bijective mapping, we can not say that (F, A) is soft isomorphic to (G, B).

6. Conclusion

Throughout this paper, in an N-group structure, we have studied the algebraic properties of soft sets which were introduced by Molodtsov as a new mathematical tool for dealing with uncertainty. This work bears on soft N-groups, soft N-subgroups, soft N-ideals, N-idealistic soft N-groups and soft N-group homomorphisms. Moreover, the relation between soft N-groups and N-idealistic soft N-groups are investigated under certain conditions for the near-ring N and they are illustrated by many examples.

References

[1] Acar U., Koyuncu F., Tanay B., Soft sets and soft rings, *Comput. Math. Appl.,* 59(2010), 3458-3463.

[2] Aktaş H, Çağman N., Soft sets and soft groups, *Inform. Sci.,* 177(2007), 2726-2735.

[3] Ali M.I., Feng F., Liu X., Min W.K., Shabir M., On some new operations in soft set theory, *Comput. Math. Appl.,* 57(2009), 1547-1553.

[4] Atagün A.O., Sezgin A., Soft substructures of rings, fields and modules, *Comput. Math. Appl.,* 61(3)(2011), 592-601.

[5] Babitha K.V., Sunil J.J., Soft set relations and functions, *Comput. Math. Appl.,* 60(7)(2010), 1840–1849.

[6] Çağman N., Enginoğlu S., Soft matrix theory and its decision making, *Comput. Math. Appl.,* 59(2010), 3308-3314.

[7] Çağman N., Enginoğlu S., Soft set theory and uni-int decision making, *Eur. J. Oper. Res.,* 207(2010), 848-855.

[8] Feng F., Jun Y.B., Zhao X., Soft semirings, *Comput. Math. Appl.,* 56(2008), 2621-2628.

[9] Feng F., Liu X.Y., Leoreanu-Fotea V., Jun Y.B., Soft sets and soft rough sets, *Inform. Sci.,* 181(6)(2011), 1125-1137.

[10] Feng F., Li C., Davvaz B., Ali M.I., Soft sets combined with fuzzy sets and rough sets: a tentative approach, *Soft Comput.,* 14(6)(2010), 899-911.

[11] Jun Y.B., Soft BCK/BCI-algebras, *Comput. Math. Appl.,* 56(2008), 1408-1413.

[12] Jun Y.B., Park C.H., Applications of soft sets in ideal theory of BCK/ BCI-algebras, *Inform. Sci.,* 178(2008), 2466-2475.

[13] Jun Y.B., Lee K.J., Zhan J., Soft p-ideals of soft BCI-algebras, *Comput. Math. Appl.,* 58(2009), 2060-2068.

[14] Kazanci O., Yilmaz Ş., Yamak S., Soft sets and soft BCH-algebras, *Hacet. J. Math. Stat.,* 39(2)(2010), 205-217.

[15] Maji P.K., Biswas R., Roy A.R., Soft set theory, *Comput. Math. Appl.,* 45(2003), 555-562.

[16] Maji P.K., Roy A.R., Biswas R., An application of soft sets in a decision making problem, *Comput. Math. Appl.,* 44(2002), 1077-1083.

[17] Majumdar P., Samanta S.K., On soft mappings, *Comput. Math. Appl.,* 60 (9)(2010), 2666-2672.

[18] Molodtsov D., Soft set theory-first results, *Comput. Math. Appl.,* 37(1999), 19-31.

[19] Molodtsov D.A., Leonov V.Yu., Kovkov D.V., Soft sets technique and its application, *Nechetkie Sistemy i Myagkie Vychisleniya,* 1(1)(2006), 8-39.

[20] Pilz G., *Near-rings,* North Holland Publishing Company, Amsterdam-New York-Oxford, 1983.
[21] Sezgin A., Atagün A.O., Aygün E., A note on soft near-rings and idealistic soft near-rings, *Filomat,* 25(2)(2011), 53-68.
[22] Sezgin A., Atagün A.O., On operations of soft sets, *Comput. Math. Appl.,* 61(5)(2011), 1457-1467.
[23] Sezgin A., Atagün A.O., Soft groups and normalistic soft groups, *Comput. Math. Appl.,* 62(2)(2011), 685-698.
[24] Sezgin A., Atagün A.O., Çağman N., Union soft substructures of near-rings and N-groups, *Neural Comput. Appl.,* 21 (Issue 1-Supplement) (2012), 133-143.
[25] Sezgin A., Atagün A.O., Çağman N., Soft intersection near-rings with applications, *Neural Comput. Appl.,* 21(Issue 1-Supplement)(2012), 221-229.
[26] Sezer A.S., A new view to ring theory via soft union rings, ideals and bi-ideals, *Knowledge-Based Systems,* 36(2012), 300-314.
[27] Zhan J., Jun Y.B., Soft BL-algebras based on fuzzy sets, *Comput. Math. Appl.,* 59(6)(2010), 2037-2046.

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