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CONDITIONAL RECIPROCAL CONTINUITY AND COMMON FIXED POINTS

Abstract. The aim of the present paper is to obtain common fixed point theorems by employing the recently introduced notion of conditional reciprocal continuity. We demonstrate that conditional reciprocal continuity ensures the existence of fixed points under contractive conditions which otherwise do not ensure the existence of fixed points. Our results generalize and extend several well-known fixed point theorems in the setting of metric spaces. We also provide more answers to the open problem posed by B. E. Rhoades [Contractive Definitions and Continuity, Contemporary Math. 72 (1988), 233-245] regarding existence of a contractive condition which is strong enough to generate a fixed point, but which does not force the map to be continuous at the fixed point.

Key words: fixed point theorem, compatible maps, noncompatible mappings, reciprocal continuity, conditional reciprocal continuity.

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1. Introduction

In 1998, Pant [9] introduced the concept of reciprocal continuity and obtained the first result that established a situation in which a collection of mappings has a fixed point which is a point of discontinuity for all the mappings. The notion of reciprocal continuity has been employed by many researchers in diverse settings to establish fixed point theorems which admit discontinuity at the fixed point. Imdad and Ali [3] have used this concept in the setting of non-self mappings. Singh and Mishra [16] have used reciprocal continuity to establish general fixed point theorems for hybrid pairs of single valued and multi-valued maps. Pant and Pant [14] extended the study of reciprocal continuity to fuzzy metric spaces. Kumar and Pant [6] studied this concept in the setting of probabilistic metric space. Murlishankar and Kalpna [8] established a common fixed point theorem in an intuitionistic fuzzy metric space using contractive condition of integral type. To widen the scope of the study of common fixed points from the class of compatible mappings satisfying contractive conditions to a wider class including compatible as well as noncompatible mappings satisfying contractive, nonexpansive or Lipschitz type condition Pant and Bisht [12] generalized the notion of reciprocal continuity by introducing the new concept of conditional reciprocal continuity which is the weakest form of continuity condition known so far.

In 1986, Jungck [4] generalized the notion of weakly commuting maps by introducing the concept of compatible maps.

Definition 1 ([4]). *Two self-maps f and g of a metric space (X, d) are called compatible iff* $\lim_n d(fgx_n, gfx_n) = 0$, *whenever* $\{x_n\}$ *is a sequence in X such that* $\lim_n fx_n = \lim_n gx_n = t$ *for some t in X.*

The definition of compatibility implies that the mappings f and g will be noncompatible in there exists a sequence $\{x_n\}$ in X such that $\lim_n fx_n = \lim_n gx_n = t$ for some t in X but $\lim_n d(fgx_n, gfx_n)$ is either non zero or nonexistent.

Definition 2 ([10]). *Two self-maps f and g are called pointwise R-weakly commuting on X if given x in X there exists $R \geq 0$ such that $d(fgx, gfx) \leq Rd(fx, gx)$.*

It is well known now that pointwise R-weak commutativity is equivalent to commutativity at coincidence points and in the setting of metric space this notion is equivalent to weak compatibility [5].

Definition 3 ([2]). *Let f and g $(f \neq g)$ be two self maps of a metric space (X, d), then f is called g-absorbing [2] if there exists some positive real number R such that $d(gx, gfx) \leq Rd(fx, gx)$ for all x in X. Similarly g will be called $f-$absorbing if there exists some positive real number R such that $d(fx, fgx) \leq Rd(fx, gx)$ for all x in X.*

It may be observed that the absorbing maps are neither a subclass of compatible maps nor a subclass of noncompatible maps [2].

Definition 4 ([9]). *Two self-mappings f and g of a metric space (X, d) are called reciprocally continuous iff* $\lim_n fgx_n = ft$ *and* $\lim_n gfx_n = gt$, *whenever* $\{x_n\}$ *is a sequence such that* $\lim_n fx_n = \lim_n gx_n = t$ *for some t in X.*

If f and g are both continuous then they are obviously reciprocally continuous but the converse is not true [9].

Definition 5 ([12]). *Two selfmappings f and g of a set X are called conditionally reciprocal continuous (CRC) iff whenever the set of sequences* $\{x_n\}$ *satisfying* $\lim_n fx_n = \lim_n gx_n$ *is nonempty, there exists a sequence*

$\{y_n\}$ satisfying $\lim_n fy_n = \lim_n gy_n = t(say)$ *for some t in X such that* $\lim_n fgy_n = ft$ *and* $\lim_n gfy_n = gt$.

If f and g are either continuous or reciprocally continuous then they are obviously conditionally reciprocally continuous but, as shown in Example 1 below, the converse is not true (see also [12]).

As an application of conditional reciprocal continuity we prove common fixed point theorems under contractive conditions that extend the scope of the study of common fixed point theorems from the class of compatible continuous mappings to a wider class of mappings which also includes non-compatible and discontinuous mappings. Our results also demonstrate the usefulness of the notion of the absorbing maps in fixed point considerations.

The question whether there exists a contractive definition which is strong enough to generate a fixed point but which does not force the map to be continuous at the fixed point was reiterated by Rhoades in [15] as an existing open problem. Gopat et al. [2], Imdad and Ali [3], Pant [9], Bisht [1] and Pant and Bisht [12] have provided some solutions to this problem. In the followings theorems we have provided more answers to this problem. It may be observed that in the examples illustrating the theorems, none of the mapping is continuous at their common fixed point.

2. Main results

Theorem 1. *Let f and g be conditionally reciprocally continuous pointwise R-weakly commuting self-mappings of a complete metric space (X, d) such that*

(i) $fX \subseteq gX$

(ii) $d(fx, fy) \le kd(gx, gy), \; k \in [0, 1)$.

If g is f-absorbing or f is g-absorbing then f and g have a unique common fixed point.

Proof. Let x_0 be any point in X. Define sequences $\{x_n\}$ and $\{y_n\}$ in X by

$$y_n = fx_n = gx_{n+1}, \quad n = 0, 1, 2, \dots . \tag{1}$$

We claim that $\{y_n\}$ is a Cauchy sequence. Using *(ii)* we obtain

$$d(y_n, y_{n+1}) = d(fx_n, fx_{n+1}) \le kd(gx_n, gx_{n+1}) = kd(y_{n-1}, y_n),$$

i.e., $d(y_n, y_{n+1}) \le k^n d(y_0, y_1)$.

Moreover, for every $p > 0$, we get

$$\begin{aligned} d(y_n, y_{n+p}) &\le d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{n+p-1}, y_{n+p}) \\ &\le (k^n + k^{n+1} + \dots + k^{n+p-1}) d(y_0, y_1) \le \frac{k^n}{1-k} d(y_0, y_1). \end{aligned}$$

This means that $d(y_n, y_{n+p}) \to 0$ as $n \to \infty$. Therefore $\{y_n\}$ is a Cauchy sequence. Since X is complete, there exists a point t in X such that $y_n \to t$ as $n \to \infty$. Moreover, $y_n = fx_n = gx_{n+1} \to t$.

Since f and g are conditionally reciprocally continuous and $\lim_n fx_n = t$, $\lim_n gx_n = t$, there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = \lim_n gy_n = u(say)$ such that $\lim_n fgy_n = fu$ and $\lim_n gfy_n = gu$. Since $fX \subseteq gX$, for each y_n there exists a z_n in X such that $fy_n = gz_n$. thus $fy_n \to u$, $gy_n \to u$, and $gz_n \to u$ as $n \to \infty$. By virtue of this and using (ii) we obtain $fz_n \to u$. Therefore, we have

$$fy_n = gz_n \to u, \quad gy_n \to u, \quad fz_n \to u. \tag{2}$$

Suppose that g is f-absorbing, $d(fz_n, fgz_n) \leq Rd(fz_n, gz_n)$. On letting $n \to \infty$, we obtain $fgz_n \to u$. Using (ii) we get $d(fu, fgz_n) \leq kd(gu, ggz_n)$. On making $n \to \infty$ and in view of $gfy_n = ggz_n \to gu$ we get $u = fu$. Since $fX \subseteq gX$, there exists v in X such that $u = fu = gv$. Now using (ii),we obtain $d(fy_n, fv) \leq kd(gy_n, gv)$. On letting $n \to \infty$, we get $fv = gv$. Pointwise R-weak commutativity of f and g implies that $d(fgu, gfu) \leq R_1 d(fu, gu)$ for some $R_1 > 0$, that is, $fgv = gfv$. Thus $fgv = gfv = ggv = ffv$. Finally using (ii), we obtain $d(fv, ffv) \leq kd(gv, gfv) = kd(fv, ffv)$, that is, $(1-k)d(fv, ffv) = 0$. Hence $fv = ffv = gfv$ and fv is a common fixed point of f and g.

Next suppose that f is g-absorbing, $d(gy_n, gfy_n) \leq Rd(fy_n, gy_n)$. On making $n \to \infty$ we get $u = gu$. Using (ii) we get $d(fy_n, fu) \leq kd(gy_n, gu)$. On letting $n \to \infty$ we get $fy_n \to fu$. Hence $u = fu = gu$ and u is a common fixed point of f and g. ■

Uniqueness of the common fixed point theorem follows easily in each of the two cases. We now give example to illustrate the above theorem:

Example 1 ([12])**.** Let $X = [2, 20]$ and d be the usual metric on X. Define $f, g : X \to X$ as follows

$$fx = 2 \text{ if } x = 2 \text{ or } x > 5, \quad fx = 6 \text{ if } 2 < x \leq 5,$$
$$g2 = 2, \ \ gx = 12, \text{ if } 2 < x \leq 5, \quad gx = \frac{(x+1)}{3} \text{ if } x > 5.$$

Then f and g satisfy all the condition of Theorem 1 and have a unique common fixed point at $x = 2$. It can be verified in this example that f and g satisfy the contraction condition (ii) for $k = \frac{4}{5}$. The mapping f and g are pointwise R-weakly commuting maps as they commute at their only coincidence point $x = 2$. Furthermore, f is g-absorbing with $R = \frac{29}{18}$. It can also be noted that f and g are conditionally reciprocally continuous. To see this, let $\{x_n\}$ be the constant sequence given by x_n. Then $fx_n \to 2$,$gx_n \to 2$.

Also $fgx_n \to 2 = f2$ and $gfx_n \to 2 = g2$. Hence f and g are conditionally reciprocally continuous. It is also obvious that f and g are not reciprocally continuous. To see this, $\{y_n\}$ be the sequence in X given by $y_n = 5+\epsilon_n$ where $\epsilon_n \to 0$ as $n \to \infty$. Then $fy_n \to 2$,$gy_n \to 2$, $\lim_n fgy_n = f(2+\frac{\epsilon_n}{3}) = 6 \neq f2$ and $\lim_n gfy_n = g2 = 2$. Thus $\lim_n gfy_n = g2$ but $\lim_{n\to\infty} fgy_n \neq f2$. Hence f and g are not reciprocally continuous mappings.

Example 2. Let $X = [1, \infty)$ and d be the usual metric on X. Define $f, g : X \to X$ by

$$fx = 2x - 1, \quad gx = 3x - 2.$$

Then f and g satisfy all the conditions of Theorem 1 and have a unique common fixed point at $x = 1$. It can be verified in this example that f and g satisfy the contraction condition (ii) for $k = \frac{2}{3}$. Furthermore, g is f-absorbing with $R = 4$. The mappings f and g are pointwise R-weakly commuting since they commute at their only coincidence point $x = 1$. It can be noted that f and g are condittionally reciprocally continuous since both f and g are continuous.

We now prove a common fixed point theorem for a pair of mappings satisfy an (ϵ, δ) type contractive condition. It is now well known (e.g., Example 3 below) that an (ϵ, δ) contrative condition does not ensure the existence of a fixed point.

Example 3 ([13])**.** Let $X = [0, 2]$ and d be the Euclidean metric on X. Define $f, : X \to X$ by

$$fx = \frac{(1+x)}{2} \text{ if } x < 1, \quad fx = 0 \text{ if } x \geq 1.$$

Then f satisfy the contractive condition

$$\epsilon \leq \max\{d(x,y), d(x,fx), d(y,fy)\} < \epsilon + \delta \Rightarrow d(fx,fy) < \epsilon$$

with $\delta(\epsilon) = 1$ for $\epsilon \geq 1$ and $\delta(\epsilon) = 1 - \epsilon$ for $\epsilon < 1$ but f does not have a fixed point.

In view of the above example, the next theorem demonstrates the usefulness of be conditional reciprocal continuity and shows that the new notion ensure the existence of a common fixed point under an (ϵ, δ) contractive condition.

Theorem 2. *Let f and g be conditionally reciprocally continuous pointwise R-weakly commuting self-mappings of a complete metric space (X, d) such that*

(i) $fX \subseteq gX$

(ii) $d(fx, fy) < d(gx, gy)$, *whenever* $gx \neq gy$

(iii) given $\epsilon > 0$ *there exists* $\delta > 0$ *such that*

$$\epsilon < d(gx, gy) < \epsilon + \delta \Rightarrow d(fx, fy) \leq \epsilon.$$

If g *is* f*-absorbing or* f *is* g*-absorbing then* f *and* g *have a unique common fixed point.*

Proof. Let x_0 be any point in X. Define sequences $\{x_n\}$ and $\{y_n\}$ in X by

$$y_n = fx_n = gx_{n+1}. \tag{3}$$

We claim that $\{y_n\}$ is a Cauchy sequence. Using (ii) we obtain

$$d(y_n, y_{n+1}) = d(fx_n, fx_{n+1}) < d(gx_n, gx_{n+1}) = d(y_{n-1}, y_n).$$

Thus $\{d(y_n, y_{n+1})\}$ is a strictly decreasing sequence of positive real numbers and, tends to a limit $r \geq 0$, that is, $\lim_{n\to\infty} d(y_n, y_{n+1}) = r$, $r \geq 0$.

We assert that $r = 0$. For, if not, suppose that $r > 0$. Then given $\delta > 0$, however small δ may be, there exists a positive integer N such that for each $n \geq N$, we have

$$r < d(y_n, y_{n+1}) = d(fx_n, fx_{n+1}) < r + \delta,$$

that is,

$$r < d(gx_{n+1}, gx_{n+2}) < r + \delta. \tag{4}$$

Selecting δ in (4) accordance with (iii), for each $n \geq N$ we get $d(fx_{n+1}, fx_{n+2}) \leq r$, that is, $d(y_{n+1}, y_{n+2}) \leq r$, a contradiction to (4). Therefore $\lim_{n\to\infty} d(y_n, y_{n+1}) = 0$. We now show that $\{y_n\}$ is a Cauchy sequence. Suppose it is not. Then there exists an $\epsilon > 0$ and a subsequence $\{y_{n_i}\}$ of $\{y_n\}$ such that $d(y_{n_i}, y_{n_{i+1}}) \geq 2\epsilon$. Select δ in (iii) so that $0 < \delta \leq \epsilon$. Since $\lim_{n\to\infty} d(y_n, y_{n+1}) = 0$, there exists an integer N such that $d(y_n, y_{n+1}) < \frac{\delta}{6}$ whenever $n \geq N$.

Let $n_i \geq N$. Then, there exists integers m_i satisfying $n_i < m_i < n_{i+1}$ such that $d(y_{n_i}, y_{m_i}) \geq \epsilon + \frac{\delta}{3}$. If not, then

$$d(y_{n_i}, y_{n_{i+1}}) \leq d(y_{n_i}, y_{n_{i+1}-1}) + d(y_{n_{i+1}-1}, y_{n_{i+1}}) < \epsilon + \frac{\delta}{3} + \frac{\delta}{6} < 2\epsilon,$$

a contradiction. Let m_i be the smallest integer such that $d(y_{n_i}, y_{m_i}) \geq \epsilon + \frac{\delta}{3}$. Then $d(y_{n_i}, y_{m_i-2}) < \epsilon + \frac{\delta}{3}$ and

$$\begin{aligned} \epsilon + \frac{\delta}{3} &\leq d(y_{n_i}, y_{m_i}) \leq d(y_{n_i}, y_{m_i-2}) + d(y_{m_i-2}, y_{m_i-1}) + d(y_{m_i-1}, y_{m_i}) \\ &< \epsilon + \frac{\delta}{3} + \frac{\delta}{6} + \frac{\delta}{6} < \epsilon + \frac{2\delta}{3}, \end{aligned}$$

that is, $\epsilon < \epsilon + \frac{\delta}{3} \leq d(gx_{n_i+1}, gx_{m_i+1}) < \epsilon + \frac{2\delta}{3}$. In view of (iii), this yields $d(y_{n_i+1}, y_{m_i+1}) \leq \epsilon$. But then

$$\begin{aligned} d(y_{n_i}, y_{m_i}) &\leq d(y_{n_i}, y_{n_i+1}) + d(y_{n_i+1}, y_{m_i+1}) + d(y_{m_i+1}, y_{m_i}) \\ &< \frac{\delta}{6} + \epsilon + \frac{\delta}{6} = \epsilon + \frac{\delta}{3}, \end{aligned}$$

which contradicts the previous said statement. Hence $\{y_n\}$ is a Cauchy sequence. Since X is a complete, there exists a point t in X such that $y_n \to t$. Moreover, $y_n = fx_n = gx_{n+1} \to t$.

Since f and g are conditionally reciprocally continuous and $\lim_{n\to\infty} fx_n = t$, $\lim_{n\to\infty} gx_n = t$ there exists a sequence $\{y_n\}$ satisfying $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = u(say)$ such that $\lim_{n\to\infty} fgy_n = fu$ and $\lim_{n\to\infty} gfy_n = gu$. Since $fX \subseteq gX$, for each y_n there exists a z_n in X such that $fy_n = gz_n$. Thus $fy_n \to u$,$gy_n \to u$ and $gz_n \to u$ as $n \to \infty$. By virtue of this and using (ii) we obtain $fz_n \to u$. Therefore, we have

$$gy_n \to u, \quad fz_n \to u, \quad fy_n = gz_n \to u.$$

Suppose that g is f-absorbing, $d(fz_n, fgz_n) \leq Rd(fz_n, gz_n)$. On letting $n \to \infty$, we obtain $fgz_n \to u$. Using (ii) we get $d(fu, fgz_n) < d(gu, ggz_n)$. On makking $n \to \infty$ and in view of $gfy_n = ggz_n \to gu$ we get $u = fu$. Since $fX \subseteq gX$, there exists v in X such that $u = fu = gv$. Now using (ii) we obtain $d(fy_n, fv) < d(gy_n, gv)$. On letting $n \to \infty$, we get $fv = gv$. Pointwise R-weak commutativity of f and g implies that $d(fgu, gfu) \leq R_1 d(fu, gu)$ for some $R_1 > 0$, that is, $fgv = gfv$. Thus $fgv = gfv = ggv = ffv$. If $fv \neq ffv$ then using (ii) we obtain $d(fv, ffv) < d(gv, gfv) = d(fv, ffv)$, a contradiction. Hence $fv = ffv = gfv$ and fv is a common fixed point of f and g.

Next suppose that f is g-absorbing, $d(gy_n, gfy_n) \leq Rd(fy_n, gy_n)$. On making $n \to \infty$ we get $u = gu$. Using (ii) we get $d(fy_n, fu) < d(gy_n, gu)$. On letting $n \to \infty$ we get $fy_n \to fu$. Hence $u = fu = gu$ and u is a common fixed point of f and g. ■

We now give an example to illustrate Theorem 2.

Example 4 ([12]). Let $X = [2, 20]$ and d be the usual metric on X. Define $f, g : X \to X$ as follows

$$\begin{aligned} &fx = 2 \text{ if } x = 2 \text{ or } x > 5, \quad fx = 6 \text{ if } 2 < x \leq 5, \\ &g2 = 2, \;\; gx = \frac{(x+31)}{3} \text{ if } 2 < x \leq 5, \quad gx = \frac{(x+1)}{3} \text{ if } x > 5. \end{aligned}$$

Then f and g satisfy all the condition of Theorem 2 and have a unique common fixed point at $x = 2$. It can be seen in this example that f and g satisfy the condition (ii) and the condition

$$\epsilon < d(gx, gy) < \epsilon + \delta \Rightarrow d(fx, fy) \leq \epsilon$$

with $\delta(\epsilon) = 1$ for $\epsilon \geq 4$ and $\delta(\epsilon) = 4 - \epsilon$ for $\epsilon < 4$. Furthermore, f is g-absorbing with $R = 2$. It can also be noted that f and g are conditionally reciprocally continuous. To see this, let$\{x_n\}$ be the constant sequence given by $x_n = 2$. Then $fx_n \to 2$, $gx_n \to 2$. Also $fgx_n \to 2 = f2$ and $gfx_n \to 2 = g2$. Hence f and g are conditionally reciprocally continuous. It is also obvious that f and g are not reciprocally continuous. To see this,let $\{y_n\}$ be the sequence in X given by $y_n = 5 + \epsilon_n$ where $\epsilon_n \to 0$ as $n \to \infty$. Then $fy_n \to 2$, $gy_n \to 2$, $\lim_n fgy_n = f\left(2 + \frac{\epsilon_n}{3}\right) = 6 \neq f2$ and $\lim_n gfy_n = g2 = 2$. Thus $\lim_n gfy_n = g2$ but $\lim_n fgy_n \neq f2$. Hence f and g are not reciprocally continuous mappings. Further, f and g are pointwise R-weakly commuting maps as they commute at their only coincidence point $x = 2$.

Remark 1. By putting $g = I_X$, i.e., identity mapping in Theorem 2 we get the generalized version of the main theorem of Meir and Keeler [7].

We now prove a common fixed point theorem for a noncompatible pair of self-mappings satisfying a nonexpansive type condition.

Theorem 3. *Let f and g be conditionally reciprocally continuous noncompatible self-mappings of a metric space (X, d) satisfying*
(i) $fX \subseteq gX$
(ii) $d(fx, fy) \leq d(gx, gy)$.
If f and g are pointwise R-weakly commuting and g is f-absorbing or f is g-absorbing then f and g have a common fixed point.

Proof. Since f and g are noncompatible maps, there exists a sequence $\{x_n\}$ in X such that $fx_n \to t$ and $gx_n \to t$ for some t in X but either $\lim_n d(fgx_n, gfx_n) \neq 0$ or the limit does not exist. Also, since f and g are conditionally reciprocally continuous and $fx_n \to t$ and $gx_n \to t$, there exists a sequence $\{y_n\}$ satisfying $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = u(say)$ such that $\lim_{n\to\infty} fgy_n = fu$ and $\lim_{n\to\infty} gfy_n = gu$. Since $fX \subseteq gX$, for each y_n there exists z_n in X such that $fy_n = gz_n$. Thus $fy_n \to u$, $gy_n \to u$ and $gz_n \to u$ as $n \to \infty$. By virtue of this and using (ii) we obtain $fz_n \to u$. Therefore, we have

$$fy_n = gz_n \to u, \quad gy_n \to u, \quad fz_n \to u. \tag{5}$$

Suppose that g is f-absorbing, then $d(fy_n, fgy_n) \leq Rd(fy_n, gy_n)$. On letting $n \to \infty$, we get $u = fu$. Since $fX \subseteq gX$, there exists v in X such that $u = fu = gv$. Now using (ii), we obtain $d(fy_n, fv) \leq d(gy_n, gv)$. On letting $n \to \infty$, we get $fv = gv$. Pointwise R-weak commutativity of f and g implies that $d(fgv, gfv) \leq R_1 d(fv, gv)$ for some $R_1 > 0$, that is, $fgv = gfv$ and hence $fgv = gfv = ggv = ffv$. Since g is f-absorbing, $d(fv, fgv) \leq Rd(fv, gv)$. This yields $fv = fgv$. Hence $gv = fgv = ggv$ and gv is a common fixed point of f and g.

Finally suppose that f is g-absorbing, $d(gy_n, gfy_n) \leq Rd(fy_n, gy_n)$. On making $n \to \infty$ we get $u = gu$. Using (ii) we get $d(fy_n, fu) \leq (gy_n, gu)$. On letting $n \to \infty$ we get $fy_n \to fu$. Hence $u = fu = gu$ and u is a common fixed point of f and g. ■

Remark 2. In all the results proved in this paper, we have not assumed any mappings to be continuous. In fact the mappings assumed by us become discontinuous at their common fixed point. Thus we provide more answers to the problem posed by Rhoades [15] regarding existence a contractive condition which is strong enough to generate a fixed point, but which does not force the map to be continuous at the fixed point.

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