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N-GROUP SI-ACTION AND ITS APPLICATIONS TO N-GROUP THEORY

Abstract. In this paper, we define a new concept, called N-group soft intersection action (SI) on a soft set. This new notion gathers soft set theory, set theory and N-group theory together and it shows how a soft set effects on an N-group structure in the mean of intersection and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical N-group theoretic concepts for N-group SI-actions. Finally, we give the applications of N-group SI-actions to N-group theory.

Key words: soft set, N-group SI-action, N-ideal SI-action, soft image, soft pre-image, α-inclusion.

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1. Introduction

Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagün and Sezgin [4] defined the concepts of soft subrings and ideals of a ring, soft subfields of a field and soft submodules of a module and studied their related properties with respect to soft set operations.

Operations of soft sets have been studied by some authors, too. Maji et al. [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagün [26] studied on soft set operations as well. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29].

In this paper, we first define a new type of N-group action on a soft set, called *N-group soft intersection action* (abbreviated as "*N-group SI-action*"), which is based on the inclusion relation and intersection of sets. Since this new concept can be regarded as a bridge among soft set theory, set theory and N-group theory, it is very functional in the mean of improving the soft set theory with respect to N-group structure. Based on this new notion, we then introduce the concepts of N-ideal SI-action and show that if N is a zero-symmetric near-ring, then every N-ideal SI-action over U is an N-group SI-action over U. Moreover, we investigate these notions with respect to soft image, soft pre-image and α-inclusion of soft sets and give their applications to N-group theory.

2. Preliminaries

In this section, we recall some basic notions relevant to N-groups and soft sets. By a *near-ring*, we shall mean an algebraic system $(N, +, .)$, where

($N1$) $(N, +)$ forms a group (not necessarily abelian)

($N2$) $(N, \cdot)$ forms a semigroup and

($N3$) $(a+b)c = ac+bc$ for all $a, b, c \in N$ (i.e. we study on right N-groups.)

Throughout this paper, N will always denote a right near-ring. A normal subgroup I of N is called a left ideal of N if $n(s+i)-ns \in I$ for all $n, s \in N$ and $i \in I$ and denoted by $I \lhd_\ell N$. For a near-ring N, the zero-symmetric part of N denoted by N_0 is defined by $N_0 = \{n \in \Gamma \mid n0 = 0\}$.
Let $(\Gamma, +)$ be a group and

$$\begin{array}{rccc} \mu: & N \times \Gamma & \rightarrow & \Gamma \\ & (n, \gamma) & \rightarrow & n\gamma \end{array}$$

(Γ, μ) is called an *N-group* or *near-ring module* if $\forall x, y \in N$, $\forall \gamma \in \Gamma$,

(i) $x(y\gamma) = (xy)\gamma$ and

(ii) $(x + y)\gamma = x\gamma + y\gamma$.

It is denoted by N^Γ. Clearly N itself is an N-group by natural operation. A subgroup Δ of N^Γ with $N\Delta \subseteq \Delta$ is said to be an *N-subgroup* of Γ and denoted by $\Delta \leq_N \Gamma$. A normal subgroup Δ of Γ is called an *N-ideal* of N^Γ and denoted by $\Delta \trianglelefteq_N \Gamma$, if $\forall \gamma \in \Gamma$, $\forall \delta \in \Delta$, $\forall n \in N$, $n(\gamma + \delta) - n\gamma \in \Delta$. Let N be a near-ring, Γ and Ψ two N-groups. Then, $h : \Gamma \rightarrow \Psi$ is called an *N-homomorphism* if $\forall \gamma, \delta \in \Gamma$, $\forall n \in N$,

(i) $h(\gamma + \delta) = h(\gamma) + h(\delta)$ and

(ii) $h(n\gamma) = nh(\gamma)$.

For all undefined concepts and notions we refer to [24]. From now on, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A, B, C \subseteq E$.

Definition 1 ([7, 22]). *A soft set f_A over U is a set defined by*

$$f_A : E \to P(U) \text{ such that} f_A(x) = \emptyset \text{ if } x \notin A.$$

Here f_A is also called approximate function. A soft set over U can be represented by the set of ordered pairs

$$f_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

It is clear to see that a soft set is a parametrized family of subsets of the set U. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. If we define more than one soft set in a subset A of the set of parameters E, then the soft sets will be denoted by f_A, g_A, h_A etc. If we define more than one soft set in some subsets A, B, C etc. of parameters E, then the soft sets will be denoted by f_A, f_B, f_C etc., respectively. We refer to [7, 12, 13, 19, 22] for further details.

Definition 2 ([7]). *Let f_A and f_B be soft sets over U. Then, union of f_A and f_B, denoted by $f_A \tilde{\cup} f_B$, is defined as $f_A \tilde{\cup} f_B = f_{A \tilde{\cup} B}$, where $f_{A \tilde{\cup} B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.*

Intersection of f_A and f_B, denoted by $f_A \tilde{\cap} f_B$, is defined as $f_A \tilde{\cap} f_B = f_{A \tilde{\cap} B}$, where $f_{A \tilde{\cap} B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

Definition 3 ([7]). *Let f_A and f_B be soft sets over U. Then, $\vee$-product of f_A and f_B, denoted by $f_A \vee f_B$, is defined as $f_A \vee f_B = f_{A \vee B}$, where $f_{A \vee B}(x, y) = f_A(x) \cup f_B(y)$ for all $(x, y) \in E \times E$.*

$\wedge$-product of f_A and f_B, denoted by $f_A \wedge f_B$, is defined as $f_A \wedge f_B = f_{A \wedge B}$, where $f_{A \wedge B}(x, y) = f_A(x) \cap f_B(y)$ for all $(x, y) \in E \times E$

Definition 4 ([8]). *Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, soft image of f_A under Ψ, denoted by $\Psi(f_A)$, is a soft set over U by*

$$(\Psi(f_A))(b) = \begin{cases} \bigcup\{f_A(a) \mid a \in A \text{ and } \Psi(a) = b\}, & if \ \Psi^{-1}(b) \neq \emptyset, \\ \emptyset, & otherwise \end{cases}$$

for all $b \in B$. And soft pre-image (or soft inverse image) of f_B under Ψ, denoted by $\Psi^{-1}(f_B)$, is a soft set over U by $(\Psi^{-1}(f_B))(a) = f_B(\Psi(a))$ for all $a \in A$.

Definition 5 ([10]). *Let f_A be a soft set over U and α be a subset of U. Then, upper α-inclusion of f_A, denoted by $f_A^{\supseteq \alpha}$, and lower α-inclusion of f_A, denoted by $f_A^{\subseteq \alpha}$, are defined as*

$$f_A^{\supseteq \alpha} = \{x \in A \mid f_A(x) \supseteq \alpha\} \text{ and } f_A^{\subseteq \alpha} = \{x \in A \mid f_A(x) \subseteq \alpha\},$$

respectively.

2. N-group SI-actions and N-ideal SI-actions

In this section, we first define *N-group soft intersection action*, abbreviated as *N-group SI-action* and N-ideal SI-action with illustrative examples. We then study their basic properties with respect to soft set operations.

Definition 6. *Let Γ be an N-group and f_Γ be a soft set over U. Then, f_Γ is called an N-group SI-action over U if it satisfies the following conditions:*
(i) $f_\Gamma(x+y) \supseteq f_\Gamma(x) \cap f_\Gamma(y)$
(ii) $f_\Gamma(-x) = f_\Gamma(x)$
(iii) $f_\Gamma(nx) \supseteq f_\Gamma(x)$
for all $x, y \in \Gamma$ and $n \in N$.

Example 1. Consider $N = \{0, a, b, c\}$ be the (right) near-ring per scheme 22 ([24], p. 408) under the operations defined by the following tables:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	0
c	a	a	a	a

Let $\Gamma = N$ and Γ be the set of parameters and $U = \left\{ \begin{bmatrix} x & x \\ 0 & x \end{bmatrix} \mid x, y \in \mathbb{Z}_6 \right\}$, 2×2 matrices with $\mathbb{Z}_6$ terms, is the universal set. We construct a soft set f_Γ over U by

$$f_\Gamma(0) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 5 \\ 0 & 5 \end{bmatrix} \right\},$$

$$f_\Gamma(a) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 5 \\ 0 & 5 \end{bmatrix} \right\},$$

$$f_\Gamma(b) = \left\{ \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} \right\} \quad \text{and} \quad f_\Gamma(c) = \left\{ \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} \right\}$$

Then, one can easily show that the soft set f_Γ is an N-group SI-action over U.

Example 2. In Example 1, assume that $\Gamma = N = \{0, a, b, c\}$ is again the set of parameters and $U = D_3 = \{< x, y >: x^3 = y^2 = e, xy = yx^2\} = \{e, x, x^2, y, yx, yx^2\}$, dihedral group, be the universal set. We define a soft set f_Γ by

$$f_\Gamma(0) = D_3,\ f_\Gamma(a) = \{x, yx\},\ f_\Gamma(b) = \{e, x, x^2, yx\},\ f_\Gamma(c) = \{e, x, x^2, yx\}.$$

Since $f_\Gamma(cb) = f_\Gamma(a) = \{x, yx\} \not\supseteq f_\Gamma(b) = \{e, x, x^2, yx\}$, f_Γ is not an N-group SI-action over U.

It is known that if $N = N_0$, then $n0_\Gamma = 0_\Gamma$ for all $n \in N$. Therefore, if N is a zero-symmetric near-ring and if we take $\Gamma = \{0_\Gamma\}$, then f_Γ is an N-group SI-action over U no matter how f_Γ is defined and and no matter U is.

Proposition 1. *Let f_Γ be an N-group SI-action over U. Then, $f_\Gamma(0_\Gamma) \supseteq f_\Gamma(x)$ for all $x \in \Gamma$.*

Proof. Assume that f_Γ is an N-group SI-action over U. Then, for all $x \in \Gamma$, $f_\Gamma(0_\Gamma) = f_\Gamma(x - x) \supseteq f_\Gamma(x) \cap f_\Gamma(-x) = f_\Gamma(x) \cap f_\Gamma(x) = f_\Gamma(x)$. ■

Theorem 1. *Let Γ be an N-group and f_Γ be a soft set over U. Then, f_Γ is an N-group SI-action over U if and only if*
(i) $f_\Gamma(x - y) \supseteq f_\Gamma(x) \cap f_\Gamma(y)$
(ii) $f_\Gamma(nx) \supseteq f_\Gamma(x)$
for all $x, y \in \Gamma$ and $n \in N$.

Proof. Suppose that f_Γ is an N-group SI-action over. Then, by Definition 6, $f_\Gamma(xy) \supseteq f_\Gamma(y)$ and $f_\Gamma(x - y) \supseteq f_\Gamma(x) \cap f_\Gamma(-y) = f_\Gamma(x) \cap f_\Gamma(y)$ for all $x, y \in \Gamma$.
Conversely, assume that $f_\Gamma(xy) \supseteq f_\Gamma(y)$ and $f_\Gamma(x - y) \supseteq f_\Gamma(x) \cap f_\Gamma(y)$ for all $x, y \in \Gamma$. If we choose $x = 0_\Gamma$, then

$$f_\Gamma(0_\Gamma - y) = f_\Gamma(-y) \supseteq f_\Gamma(0_\Gamma) \cap f_\Gamma(y) = f_\Gamma(y)$$

by Proposition 1. Similarly, $f_\Gamma(y) = f_\Gamma(-(-y)) \supseteq f_\Gamma(-y)$, thus $f_\Gamma(-y) = f_\Gamma(y)$ for all $y \in \Gamma$. Also, by assumption $f_\Gamma(x + y) \supseteq f_\Gamma(x) \cap f_\Gamma(-y) = f_\Gamma(x) \cap f_\Gamma(y)$. This completes the proof. ■

Theorem 2. *Let f_Γ be an N-group SI-action over U. If $f_\Gamma(x - y) = f_\Gamma(0_\Gamma)$ for any $x, y \in \Gamma$, then $f_\Gamma(x) = f_\Gamma(y)$.*

Proof. Assume that $f_\Gamma(x - y) = f_\Gamma(0_\Gamma)$ for any $x, y \in \Gamma$. Then,

$$\begin{aligned} f_\Gamma(x) &= f_\Gamma(x - y + y) \supseteq f_\Gamma(x - y) \cap f_\Gamma(y) \\ &= f_\Gamma(0_\Gamma) \cap f_\Gamma(y) = f_\Gamma(y) \end{aligned}$$

and similarly

$$\begin{aligned} f_\Gamma(y) &= f_\Gamma((y - x) + x) \supseteq f_\Gamma(y - x) \cap f_\Gamma(x) \\ &= f_\Gamma(-(y - x)) \cap f_\Gamma(x) \\ &= f_\Gamma(0_\Gamma) \cap f_\Gamma(x) = f_\Gamma(x). \end{aligned}$$

Thus, $f_\Gamma(x) = f_\Gamma(y)$ which completes the proof. ■

It is known that if Γ is an N-group, then $(\Gamma, +)$ is a group but not necessarily abelian. That is, for any $x, y \in \Gamma$, $x + y$ needs not be equal to $y + x$. However, we have the following:

Theorem 3. *Let f_Γ be an N-group SI-action over U and $x \in \Gamma$. Then,*

$$f_\Gamma(x) = f_\Gamma(0_\Gamma) \Leftrightarrow f_\Gamma(x+y) = f_\Gamma(y+x) = f_\Gamma(y)$$

for all $y \in \Gamma$.

Proof. Suppose that $f_\Gamma(x+y) = f_\Gamma(y+x) = f_\Gamma(y)$ for all $y \in \Gamma$. Then, by choosing $y = 0_\Gamma$, we obtain that $f_\Gamma(x) = f_\Gamma(0_\Gamma)$. Conversely, assume that $f_\Gamma(x) = f_\Gamma(0_\Gamma)$. Then, by Proposition 1, we have

$$f_\Gamma(0_\Gamma) = f_\Gamma(x) \supseteq f_\Gamma(y), \quad \forall y \in \Gamma. \tag{1}$$

Since f_Γ is an N-group SI-action over U, then

$$f_\Gamma(x+y) \supseteq f_\Gamma(x) \cap f_\Gamma(y) = f_\Gamma(y), \quad \forall y \in \Gamma.$$

Moreover, for all $y \in \Gamma$

$$\begin{aligned} f_\Gamma(y) &= f_\Gamma((-x)+x)+y) = f_\Gamma(-x+(x+y)) \\ &\supseteq f_\Gamma(-x) \cap f_\Gamma(x+y) = f_\Gamma(x) \cap f_\Gamma(x+y) = f_\Gamma(x+y) \end{aligned}$$

since by (1), $f_\Gamma(x) \supseteq f_\Gamma(y)$ for all $y \in \Gamma$ and $x, y \in \Gamma$ implies that $x+y \in \Gamma$. Thus, it follows that $f_\Gamma(x) \supseteq f_\Gamma(x+y)$, so $f_\Gamma(x+y) = f_\Gamma(y)$ for all $y \in \Gamma$. Now, let $x \in \Gamma$. Then, for all $y \in \Gamma$

$$\begin{aligned} f_\Gamma(y+x) &= f_\Gamma(y+x+(y-y)) = f_\Gamma(y+(x+y)-y) \\ &\supseteq f_\Gamma(y) \cap f_\Gamma(x+y) \cap f_\Gamma(y) \\ &= f_\Gamma(y) \cap f_\Gamma(x+y) = f_\Gamma(y) \end{aligned}$$

since $f_\Gamma(x+y) = f_\Gamma(y)$. Furthermore, for all $y \in \Gamma$,

$$\begin{aligned} f_\Gamma(y) &= f_\Gamma(y+(x-x)) = f_\Gamma((y+x)-x) \\ &\supseteq f_\Gamma(y+x) \cap f_\Gamma(x) = f_\Gamma(y+x) \end{aligned}$$

by (1). It follows that $f_\Gamma(y+x) = f_\Gamma(y)$ and so $f_\Gamma(x+y) = f_\Gamma(y+x) = f_\Gamma(y)$ for all $y \in \Gamma$, which completes the proof. ■

Theorem 4. *If f_Γ and f_Δ are N-group SI-actions over U, then so is $f_\Gamma \wedge f_\Delta$ over U.*

Proof. By Definition 3, let $f_\Gamma \wedge f_\Delta = f_{\Gamma\wedge\Delta}$, where $f_{\Gamma\wedge\Delta}(x,y) = f_\Gamma(x) \cap f_\Delta(y)$ for all $(x,y) \in E \times E$. Since Γ and Δ are N-groups, then $\Gamma \times \Delta$ is an $N \times N$ group. So, let $(x_1, y_1), (x_2, y_2) \in \Gamma \times \Delta$ and $(n_1, n_2) \in N \times N$. Then,

$$\begin{aligned} f_{\Gamma\wedge\Delta}((x_1,y_1)-(x_2,y_2)) &= f_{\Gamma\wedge\Delta}(x_1-x_2, y_1-y_2) \\ &= f_\Gamma(x_1-x_2) \cap f_\Delta(y_1-y_2) \\ &\supseteq (f_\Gamma(x_1) \cap f_\Gamma(x_2)) \cap (f_\Delta(y_1) \cap f_\Delta(y_2)) \\ &= (f_\Gamma(x_1) \cap f_\Delta(y_1)) \cap (f_\Gamma(x_2) \cap f_\Delta(y_2)) \\ &= f_{\Gamma\wedge\Delta}(x_1,y_1) \cap f_{\Gamma\wedge\Delta}(x_2,y_2), \end{aligned}$$

$$\begin{aligned} f_{\Gamma\wedge\Delta}((n_1,n_2)(x_2,y_2)) &= f_{\Gamma\wedge\Delta}(n_1x_2,n_2y_2) = f_\Gamma(n_1x_2)\cap f_\Delta(n_2y_2) \\ &\supseteq f_\Gamma(x_2)\cap f_\Delta(y_2) = f_{\Gamma\wedge\Delta}(x_2,y_2). \end{aligned}$$

Thus, $f_\Gamma \wedge f_\Delta$ is an N-group SI-action over U. ■

Definition 7. *Let f_Γ, g_Δ be N-group SI-actions over U. Then, product of N-group SI-actions f_Γ and g_Δ is defined as $f_\Gamma \times g_\Delta = h_{\Gamma\times\Delta}$, where $h_{\Gamma\times\Delta}(x,y) = f_\Gamma(x)\times g_\Delta(y)$ for all $(x,y)\in\Gamma\times\Delta$.*

Theorem 5. *If f_Γ and g_Δ are N-group SI-actions over U, then so is $f_\Gamma \times g_\Delta$ over $U\times U$.*

Proof. By Definition 7, let $f_\Gamma\times g_\Delta = h_{\Gamma\times\Delta}$, where $h_{\Gamma\times\Delta}(x,y) = f_\Gamma(x)\times g_\Delta(y)$ for all $(x,y)\in\Gamma\times\Delta$. Then, for all $(x_1,y_1),(x_2,y_2)\in\Gamma\times\Delta$ and $(n_1,n_2)\in N\times N$,

$$\begin{aligned} h_{\Gamma\times\Delta}((x_1,y_1)-(x_2,y_2)) &= h_{\Gamma\times\Delta}(x_1-x_2,y_1-y_2) \\ &= f_\Gamma(x_1-x_2)\times g_\Delta(y_1-y_2) \\ &\supseteq (f_\Gamma(x_1)\cap f_\Delta(x_2))\times(g_\Delta(y_1)\cap g_\Delta(y_2)) \\ &= (f_\Gamma(x_1)\times g_\Delta(y_1))\cap(f_\Gamma(x_2)\times g_\Delta(y_2)) \\ &= h_{\Gamma\times\Delta}(x_1,y_1)\cap h_{\Gamma\times\Delta}(x_2,y_2), \end{aligned}$$

$$\begin{aligned} h_{\Gamma\times\Delta}((n_1,n_2)(x_2,y_2)) &= h_{\Gamma\times\Delta}(n_1x_2,n_2y_2) = f_\Gamma(n_1x_2)\times g_\Delta(n_2y_2) \\ &\supseteq f_\Gamma(x_2)\times g_\Delta(y_2) = h_{\Gamma\times\Delta}(x_2,y_2). \end{aligned}$$

Hence, $f_\Gamma\times g_\Delta = h_{\Gamma\times\Delta}$ is an N-group SI-action over $U\times U$. ■

Theorem 6. *If f_Γ and h_Γ are two N-group SI-actions over U, then so is $f_\Gamma\widetilde{\cap}h_\Gamma$ over U.*

Proof. Let $x,y\in\Gamma$ and $n\in N$, then

$$\begin{aligned} (f_\Gamma\widetilde{\cap}h_\Gamma)(x-y) &= f_\Gamma(x-y)\cap h_\Gamma(x-y) \\ &\supseteq (f_\Gamma(x)\cap f_\Gamma(y))\cap(h_\Gamma(x)\cap h_\Gamma(y)) \\ &= (f_\Gamma(x)\cap h_\Gamma(x))\cap(f_\Gamma(y)\cap h_\Gamma(y)) \\ &= (f_\Gamma\widetilde{\cap}h_\Gamma)(x)\cap(f_\Gamma\widetilde{\cap}h_\Gamma)(y), \end{aligned}$$

$$(f_\Gamma\widetilde{\cap}h_\Gamma)(nx) = f_\Gamma(nx)\cap h_\Gamma(nx)\supseteq f_\Gamma(x)\cap h_\Gamma(x) = (f_\Gamma\widetilde{\cap}h_\Gamma)(x).$$

Therefore, $f_\Gamma\widetilde{\cap}h_\Gamma$ is an N-group SI-action over U. ■

Definition 8. *Let Γ be an N-group and f_Γ be a soft set over U. Then, f_Γ is called an N-ideal SI-action of Γ over U if the following conditions are satisfied:*

(*i*) $f_\Gamma(x+y) \supseteq f_\Gamma(x) \cap f_\Gamma(y)$,

(*ii*) $f_\Gamma(-x) = f_\Gamma(x)$,

(*iii*) $f_\Gamma(x+y-x) \supseteq f_\Gamma(y)$,

(*iv*) $f_\Gamma(n(x+y)-nx) \supseteq f_\Gamma(y)$

for all $x, y \in \Gamma$ and $n \in N$. Here, note that $f_\Gamma(x+y) \supseteq f_\Gamma(x) \cap f_\Gamma(y)$ and $f_\Gamma(-x) = f_\Gamma(x)$ imply $f_\Gamma(x-y) \supseteq f_\Gamma(x) \cap f_\Gamma(y)$.

Example 3. Consider the (right) near-ring $N = \{0, a, b, c\}$ with the following tables [27]:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	0	a	b	b
c	0	a	b	c

Let $\Gamma = N$ be the set of parameters and $U = D_2$, dihedral group, be the universal set. We define a soft set f_Γ over U by

$$f_\Gamma(0) = D_2, \quad f_\Gamma(a) = \{e, y, yx\}, \quad f_\Gamma(b) = \{x, y\}, \quad f_N(c) = \{y\}.$$

Then, one can show that f_Γ is an N-ideal SI-action of Γ over U.

Example 4. Consider the (right) near-ring $N = \{0, 1, 2, 3\}$ ([24], p. 407) with the following tables:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	3	0	3
3	0	2	0	2

Let $\Gamma = N$ be the set of parameters and $U = \mathbb{Z}^+$ be the universal set. We define a soft set f_Γ over U by

$$\begin{aligned} f_\Gamma(0) &= \{1, 2, 3, 5, 6, 7, 9, 10, 11, 17\}, \\ f_\Gamma(1) &= f_\Gamma(3) = \{1, 3, 5, 7, 9, 11\}, \\ f_\Gamma(2) &= \{1, 5, 7, 9, 11\}. \end{aligned}$$

Since $f_\Gamma(3 \cdot (1+1) - 3 \cdot 1) = f_\Gamma(3 \cdot 2 - 3 \cdot 1) = f_\Gamma(0-2) = f_\Gamma(0+2) = f_\Gamma(2) \not\supseteq f_\Gamma(1)$, f_Γ is not an N-ideal SI-action of Γ over U.

It is known that if N is a zero-symmetric near-ring, then every N-ideal of Γ is also an N-subgroup of Γ [24]. Here, we have an analog for this case:

Theorem 7. *Let N be a zero-symmetric near-ring. Then, every N-ideal SI-action over U is an N-group SI-action over U.*

Proof. Let f_Γ be an N-ideal SI-action of Γ over U. Since $f_\Gamma(n(x+y)-nx) \supseteq f_\Gamma(y)$, for all $x, y \in \Gamma$ and $n \in N$, in particular for $x = 0_\Gamma$, it follows that $f_\Gamma(n(0_\Gamma + y) - n0_\Gamma) = f_\Gamma(ny - 0_\Gamma) = f_\Gamma(ny) \supseteq f_\Gamma(y)$. Since the other conditions is satisfied by Definition 8, f_Γ is an N-group SI-action over U. ■

Theorem 8. *Let f_Γ be an N-ideal SI-action of Γ and f_Δ be an N-ideal SI-action of Δ over U. Then, $f_\Gamma \wedge f_\Delta$ is an N-ideal SI-action of $\Gamma \times \Delta$ over U.*

Proof. Let $(x_1, y_1), (x_2, y_2) \in \Gamma \times \Delta$ and $(n_1, n_2) \in N \times N$. Then, $f_{\Gamma\wedge\Delta}((x_1, y_1) - (x_2, y_2)) \supseteq f_{\Gamma\wedge\Delta}(x_1, y_1) \cap f_{\Gamma\wedge\Delta}(x_2, y_2)$ can be shown similar to Theorem 4. Moreover,

$$\begin{aligned} f_{\Gamma\wedge\Delta}((x_1, y_1) + (x_2, y_2) - (x_1, y_1)) &= f_{\Gamma\wedge\Delta}(x_1 + x_2 - x_1, y_1 + y_2 - y_1) \\ &= f_\Gamma(x_1 + x_2 - x_1) \cap f_\Delta(y_1 + y_2 - y_1) \\ &\supseteq f_\Gamma(x_2) \cap f_\Delta(y_2) = f_{\Gamma\wedge\Delta}(x_2, y_2) \end{aligned}$$

and

$$\begin{aligned} & f_{\Gamma\wedge\Delta}((n_1, n_2)((x_1, y_1) + (x_2, y_2)) - (n_1, n_2)(x_1, y_1)) \\ &\quad = f_{\Gamma\wedge\Delta}(n_1(x_1 + x_2) - n_1x_1, n_2(y_1 + y_2) - n_2y_1) \\ &\quad = f_\Gamma(n_1(x_1 + x_2) - n_1x_1) \cap f_\Delta(n_2(y_1 + y_2) - n_2y_1) \\ &\quad \supseteq f_\Gamma(x_2) \cap f_\Delta(y_2) = f_{\Gamma\wedge\Delta}(x_2, y_2). \end{aligned}$$

Therefore, $f_\Gamma \wedge f_\Delta$ is an N-ideal SI-action of $\Gamma \times \Delta$ over U. ■

Definition 9. *Let f_Γ be an N-ideal SI-action of Γ and f_Δ be an N-ideal SI-action of Δ over U. Then, product of N-ideal SI-actions f_Γ and g_Δ is defined as $f_\Gamma \times g_\Delta = h_{\Gamma\times\Delta}$, where $h_{\Gamma\times\Delta}(x, y) = f_\Gamma(x) \times g_\Delta(y)$ for all $(x, y) \in \Gamma \times \Delta$.*

Theorem 9. *If f_Γ is an N-ideal SI-action of Γ and f_Δ is an N-ideal SI-action of Δ over U, then $f_\Gamma \times g_\Delta$ is an N-ideal SI-action of $\Gamma \times \Delta$ over $U \times U$.*

Proof. Let $(x_1, y_1), (x_2, y_2) \in \Gamma \times \Delta$ and $(n_1, n_2) \in N \times N$. Then, $f_{\Gamma\wedge\Delta}((x_1, y_1) - (x_2, y_2)) \supseteq f_{\Gamma\wedge\Delta}(x_1, y_1) \cap f_{\Gamma\wedge\Delta}(x_2, y_2)$ can be shown similar to Theorem 5. Now,

$$\begin{aligned} f_{\Gamma\times\Delta}((x_1, y_1) + (x_2, y_2) - (x_1, y_1)) &= f_{\Gamma\times\Delta}(x_1 + x_2 - x_1, y_1 + y_2 - y_1) \\ &= f_\Gamma(x_1 + x_2 - x_1) \times f_\Delta(y_1 + y_2 - y_1) \\ &\supseteq f_\Gamma(x_2) \times f_\Delta(y_2) = f_{\Gamma\times\Delta}(x_2, y_2) \end{aligned}$$

and

$$\begin{aligned} f_{\Gamma\times\Delta}((n_1,n_2)((x_1,y_1)+(x_2,y_2))-(n_1,n_2)(x_1,y_1)) \\ &= f_{\Gamma\times\Delta}(n_1(x_1+x_2)-n_1x_1, n_2(y_1+y_2)-n_2y_1) \\ &= f_\Gamma(n_1(x_1+x_2)-n_1x_1)\times f_\Delta(n_2(y_1+y_2)-n_2y_1) \\ &\supseteq f_\Gamma(x_2)\times f_\Delta(y_2) = f_{\Gamma\times\Delta}(x_2,y_2). \end{aligned}$$

Hence, $f_\Gamma \times g_\Delta$ is an N-ideal SI-action of $\Gamma\times\Delta$ over $U\times U$. ■

Theorem 10. *If f_Γ and h_Γ are two N-ideal SI-actions of Γ over U, then $f_\Gamma \widetilde{\cap} h_\Gamma$ is an N-ideal SI-action of Γ over U.*

Proof. Let $x,y\in\Gamma$ and $n\in N$. Then, $(f_\Gamma\widetilde{\cap}h_\Gamma)(x-y)\supseteq(f_\Gamma\widetilde{\cap}h_\Gamma)(x)\cap(f_\Gamma\widetilde{\cap}h_\Gamma)(y)$ can be shown similar to Theorem 10. Now,

$$\begin{aligned} (f_\Gamma\widetilde{\cap}h_\Gamma)(x+y-x) &= f_\Gamma(x+y-x)\cap h_\Gamma(x+y-x) \\ &\supseteq f_\Gamma(y)\cap h_\Gamma(y) = (f_\Gamma\widetilde{\cap}h_\Gamma)(y) \end{aligned}$$

and

$$\begin{aligned} (f_\Gamma\widetilde{\cap}h_\Gamma)(n(x+y)-nx) &= f_\Gamma(n(x+y)-nx)\cap h_\Gamma(n(x+y)-nx) \\ &\supseteq f_\Gamma(y)\cap h_\Gamma(y) = (f_\Gamma\widetilde{\cap}h_\Gamma)(y) \end{aligned}$$

Therefore, $f_\Gamma\widetilde{\cap}h_\Gamma$ is an N-ideal SI-action of Γ over U. ■

4. Applications of N-group SI-actions and N-ideal SI-actions

In this section, we give the applications of soft image, soft pre-image, upper α-inclusion of soft sets and N-homomorphism to N-group theory with respect to N-group SI-actions and N-ideal SI-actions.

Theorem 11. *If f_Γ is an N-ideal SI-action of Γ over U, then $\Gamma_f = \{x\in\Gamma : f_\Gamma(x) = f_\Gamma(0_\Gamma)\}$ is an N-ideal of Γ.*

Proof. It is obvious that $0_\Gamma\in\Gamma_f\subseteq\Gamma$. We need to show that (i) $x-y\in\Gamma_f$, (ii) $\gamma+x-\gamma\in\Gamma_f$ and (iii) $n(\gamma+x)-n\gamma\in\Gamma_f$ for all $x,y\in\Gamma_f$ and $n\in N$ and $\gamma\in\Gamma$. If $x,y\in\Gamma_f$, then $f_\Gamma(x)=f_\Gamma(y)=f_\Gamma(0_\Gamma)$. By Proposition 1,

$$\begin{aligned} &f_\Gamma(0_\Gamma)\supseteq f_\Gamma(x-y),\quad f_\Gamma(0_\Gamma)\supseteq f_\Gamma(\gamma+x-\gamma) \\ &\text{and}\quad f_\Gamma(0_\Gamma)\supseteq f_\Gamma(n(\gamma+x)-n\gamma) \end{aligned}$$

for all $n\in N$, $x,y\in\Gamma_f$ and $\gamma\in\Gamma$. Since f_Γ is an N-ideal SI-action of Γ over U, then for all $n\in N$, $x,y\in\Gamma_f$ and $\gamma\in\Gamma$

(*i*) $f_\Gamma(x-y) \supseteq f_\Gamma(x) \cap f_\Gamma(y) = f_\Gamma(0_\Gamma)$,
(*ii*) $f_\Gamma(\gamma + x - \gamma) \supseteq f_\Gamma(x) = f_\Gamma(0_\Gamma)$
(*iii*) $f_\Gamma(n(\gamma + x) - n\gamma) \supseteq f_\Gamma(x) = f_\Gamma(0_\Gamma)$.

Hence,

$$f_\Gamma(x-y) = f_\Gamma(0_\Gamma), \quad f_\Gamma(\gamma + x - \gamma) = f_\Gamma(0_\Gamma)$$
$$\text{and} \quad f_\Gamma(n(\gamma + x) - n\gamma) = f_\Gamma(0_\Gamma)$$

for all $n \in N$, $x, y \in \Gamma_f$ and $\gamma \in \Gamma$. Therefore Γ_f is an N-ideal of Γ. ■

Theorem 12. *Let f_Γ be a soft set over U and α be a subset of U such that $\emptyset \subseteq \alpha \subseteq f_\Gamma(0_\Gamma)$. If f_Γ is an N-ideal SI-action over U, then $f_\Gamma^{\supseteq\alpha}$ is an N-ideal of Γ.*

Proof. Since $f_\Gamma(0_\Gamma) \supseteq \alpha$, then $0_\Gamma \in f_\Gamma^{\supseteq\alpha}$ and $\emptyset \neq f_\Gamma^{\supseteq\alpha} \subseteq \Gamma$. Let $x, y \in f_\Gamma^{\supseteq\alpha}$, then

$$f_\Gamma(x) \supseteq \alpha \quad \text{and} \quad f_\Gamma(y) \supseteq \alpha.$$

We need to show that
(*i*) $x - y \in f_\Gamma^{\supseteq\alpha}$
(*ii*) $\gamma + x - \gamma \in f_\Gamma^{\supseteq\alpha}$
(*iii*) $n(\gamma + x) - n\gamma \in f_\Gamma^{\supseteq\alpha}$

for all $x, y \in f_\Gamma^{\supseteq\alpha}$, $n \in N$ and $\gamma \in \Gamma$. Since f_Γ is an N-ideal SI-action over U, it follows that

$$f_\Gamma(x-y) \supseteq f_\Gamma(x) \cap f_\Gamma(y) \supseteq \alpha \cap \alpha = \alpha,$$
$$f_\Gamma(\gamma + x - \gamma) \supseteq f_\Gamma(x) \supseteq \alpha$$
$$\text{and} \ f_\Gamma(n(\gamma + x) - n) \supseteq f_\Gamma(x) \supseteq \alpha.$$

Thus, the proof is completed. ■

Theorem 13. *Let f_Γ and f_Δ be soft sets over U and Ψ be an N-isomorphism from Γ to Δ. If f_Γ is an N-ideal SI-action of Γ over U, then $\Psi(f_\Gamma)$ is an N-ideal SI-action of Δ over U.*

Proof. Let δ_1, δ_2 and $n \in N$. Since Ψ is surjective, there exists $\gamma_1, \gamma_2 \in \Gamma$ such that $\Psi(\gamma_1) = \delta_1$ and $\Psi(\gamma_2) = \delta_2$. Then,

$$\begin{aligned}
(\Psi(f_\Gamma))(\delta_1 - \delta_2) &= \bigcup \{f_\Gamma(\gamma) : \gamma \in \Gamma, \Psi(\gamma) = \delta_1 - \delta_2\} \\
&= \bigcup \{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = \Psi^{-1}(\delta_1 - \delta_2)\} \\
&= \bigcup \{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = \Psi^{-1}(\Psi(\gamma_1 - \gamma_2)) = \gamma_1 - \gamma_2\} \\
&= \bigcup \{f_\Gamma(\gamma_1 - \gamma_2) : \gamma_i \in \Gamma, \Psi(\gamma_i) = \delta_i, i = 1, 2\} \\
&\supseteq \bigcup \{f_\Gamma(\gamma_1) \cap f_\Gamma(\gamma_2) : \gamma_i \in \Gamma, \Psi(\gamma_i) = \delta_i, i = 1, 2\}
\end{aligned}$$

$$\begin{aligned}
&= (\bigcup\{f_\Gamma(\gamma_1) : \gamma_1 \in \Gamma, \Psi(\gamma_1) = \delta_1\}) \\
&\cap\ (\bigcup\{f_\Gamma(\gamma_2) : \gamma_2 \in \Gamma, \Psi(\gamma_2) = \delta_2\}) \\
&= (\Psi(f_\Gamma))(\delta_1) \cap (\Psi(f_\Gamma))(\delta_2).
\end{aligned}$$

Also,

$$\begin{aligned}
(\Psi(f_\Gamma))(\delta_1 + \delta_2 - \delta_1) &= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \Psi(\gamma) = \delta_1 + \delta_2 - \delta_1\} \\
&= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = \Psi^{-1}(\delta_1 + \delta_2 - \delta_1)\} \\
&= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = \Psi^{-1}(\Psi(\gamma_1 + \gamma_2 - \gamma_1)) \\
&\qquad\qquad\qquad = \gamma_1 + \gamma_2 - \gamma_1\} \\
&= \bigcup\{f_\Gamma(\gamma_1 + \gamma_2 - \gamma_1) : \gamma_i \in \Gamma, \Psi(\gamma_i) = \delta_i, i = 1, 2\} \\
&\supseteq \bigcup\{f_\Gamma(\gamma_2) : \gamma_2 \in \Gamma, \Psi(\gamma_2) = \delta_2\} = (\Psi(f_\Gamma))(\delta_2).
\end{aligned}$$

Furthermore,

$$\begin{aligned}
(\Psi(f_\Gamma))(n(\delta_1 + \delta_2) - n\delta_1) &= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \Psi(\gamma) = n(\delta_1 + \delta_2) - n\delta_1\} \\
&= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = \Psi^{-1}(n(\delta_1 + \delta_2) - n\delta_1)\} \\
&= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = \Psi^{-1}(\Psi(n(\gamma_1 + \gamma_2) - n\gamma_1)))\} \\
&= \bigcup\{f_\Gamma(\gamma) : \gamma \in \Gamma, \gamma = n(\gamma_1 + \gamma_2) - n\gamma_1\} \\
&= \bigcup\{f_\Gamma(n(\gamma_1 + \gamma_2) - n\gamma_1) : \gamma_i \in \Gamma, \Psi(\gamma_i) = \delta_i, i = 1, 2, 3\} \\
&\supseteq \bigcup\{f_\Gamma(\gamma_2) : \gamma_2 \in \Gamma, \Psi(\gamma_2) = \delta_2\} = (\Psi(f_\Gamma))(\delta_2).
\end{aligned}$$

Hence, $\Psi(f_\Gamma)$ is an N-ideal SI-action of Δ over U. ■

Theorem 14. *Let f_Γ and f_Δ be soft sets over U and Ψ be an N-homomorphism from N to Δ. If f_Δ is an N-ideal SI-action of Δ over U, then $\Psi^{-1}(f_\Delta)$ is an N-ideal SI-action of Γ over U.*

Proof. Let $\gamma_1, \gamma_2 \in \Gamma$ and $n \in N$. Then,

$$\begin{aligned}
(\Psi^{-1}(f_\Delta))(\gamma_1 - \gamma_2) &= f_\Delta(\Psi(\gamma_1 - \gamma_2)) = f_\Delta(\Psi(\gamma_1) - \Psi(\gamma_2)) \\
&\supseteq f_\Delta(\Psi(\gamma_1)) \cap f_\Delta(\Psi(\gamma_2)) \\
&= (\Psi^{-1}(f_\Delta))(\gamma_1) \cap (\Psi^{-1}(f_\Delta))(\gamma_2).
\end{aligned}$$

Also,

$$\begin{aligned}
(\Psi^{-1}(f_\Delta))(\gamma_1 + \gamma_2 - \gamma_1) &= f_\Delta(\Psi(\gamma_1 + \gamma_2 - \gamma_1)) \\
&= f_\Delta(\Psi(\gamma_1) + \Psi(\gamma_2) - \Psi(\gamma_1)) \\
&\supseteq f_\Delta(\Psi(\gamma_2)) = (\Psi^{-1}(f_\Delta))(\gamma_2).
\end{aligned}$$

Furthermore,

$$\begin{aligned}(\Psi^{-1}(f_\Delta))(n(\gamma_1+\gamma_2)-n\gamma_1) &= f_\Delta(\Psi((n(\gamma_1+\gamma_2)-n\gamma_1))) \\ &= f_\Delta(n((\Psi(\gamma_1)+\Psi(\gamma_2))-n\Psi(\gamma_1)) \\ &\supseteq f_\Delta(\Psi(\gamma_2)=(\Psi^{-1}(f_\Delta))(\gamma_2).\end{aligned}$$

Hence, $\Psi^{-1}(f_\Delta)$ is an N-ideal SI-action of Γ over U. ■

5. Conclusion

In this paper, we have defined a new type of N-group action on a soft set, called N-group SI-action by using the soft sets. This new concept picks up the soft set theory, set theory and N-group theory together and therefore, it is very functional for obtaining results in the mean of N-group structure. Based on this definition, we have introduced the concept of N-ideal SI-action of an N-group. We have then investigated these notions with respect to soft image, soft pre-image and upper α-inclusion of soft sets. Finally, we give some applications of N-ideal SI-actions to N-group theory. To extend this study, one can further study the other algebraic structures such as different algebras in view of their SI-actions.

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