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**A GENERAL COINCIDENCE AND FIXED POINT
 THEOREM FOR HYBRID PAIRS OF MAPPINGS
 SATISFYING A COMMON COINCIDENCE
 RANGE PROPERTY**

ABSTRACT. In this paper the notion of common coincidence range property is introduced and a general coincidence and fixed point theorem is proved. As applications, new results for mappings satisfying a contractive condition of integral type and for mappings satisfying a ϕ -contractive condition are obtained.

KEY WORDS: coincidence point, fixed point, hybrid pairs, common coincidence range property, implicit relation, weakly altering distance.

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1. Introduction and preliminaries

Let (X, d) be a metric space. We denote by $CB(X)$ the set of all non-empty closed bounded subsets of (X, d) and by H the Hausdorff-Pompeiu metric on X , i.e.

$$H(A, B) = \max\left\{\sup_{x \in A} \{d(x, B)\}, \sup_{y \in B} \{d(y, A)\}\right\},$$

where $A, B \in CB(X)$ and

$$d(x, A) = \inf_{y \in A} \{d(x, y)\}.$$

Definition 1. Let $f : X \rightarrow X$ and $F : X \rightarrow 2^X$ be.

- 1) A point $x \in X$ is said to be a coincidence point of f and F if $fx \in Fx$. The set of all coincidence points of f and F is denoted by $\mathcal{C}(f, F)$.
- 2) A point $x \in X$ is a common fixed point of f and F if $x = fx \in Fx$.
- 3) The pair of functions f and F is denoted by (f, F) and is named a hybrid pair of mappings.

Definition 2. Let (f, F) be a hybrid pair of mappings.

1) The mapping $f : X \rightarrow X$ is said to be *coincidentally idempotent with respect to F* [3] if $x \in \mathcal{C}(f, F)$ implies $fx = ffx$.

2) The mapping $f : X \rightarrow X$ is said to be *occasionally coincidentally idempotent with respect to F* [8] if there exists $u \in \mathcal{C}(f, F)$ such that $fu = ffu$.

In 2011, Sintunavarat and Kumam [20] introduced the notion of common limit range property for a pair of two single valued mappings. Imdad et al. [4] introduced the notion of common limit range property for a hybrid pair of mappings.

Quite recently, Imdad et al. [5] introduced the notion of joint common limit range property for two pairs of hybrid mappings. Another form of joint common limit range property for hybrid pairs of mappings is introduced in [1].

Definition 3 ([5]). Let $f, g : X \rightarrow X$ be two single valued mappings and $F, G : X \rightarrow CB(X)$ be two multivalued mappings of a metric space (X, d) . The pairs of mappings (f, F) and (g, G) are said to have *joint common limit range property, denoted (JCLR) - property*, if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X and $A, B \in CB(X)$ such that

$$\lim_{n \rightarrow \infty} Fx_n = A, \quad \lim_{n \rightarrow \infty} Gy_n = B,$$

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gy_n = t \in A \cap B \cap f(X) \cap g(X).$$

We introduce the notion of common coincidence range property without the limit of sequences.

Definition 4. Let (X, d) be a metric space, $S, T : X \rightarrow X$ and $A : X \rightarrow 2^X$. The pair (A, S) is said to have a *common coincidence range property with respect to T* , denoted $CP_{(A,S)T}$, if there exists $z = Sx \in Ax$ for some $x \in X$ with $z \in T(X)$.

Example 1. Let $X = [0, \infty)$ with usual metric and $Sx = \frac{x^2 + 1}{2}$, $Tx = x + \frac{1}{4}$ and $Ax = \left[\frac{1}{4}, 1 \right]$. Then $S0 = \frac{1}{2} \in A0 = \left[\frac{1}{4}, 1 \right]$. Hence, there exists $z = \frac{1}{2} \in T(X) = \left[\frac{1}{4}, \infty \right)$.

Remark 1. It is obvious that the Definition 4 is not using the limit of sequences from (X, d) .

Definition 5 ([6]). An *altering distance* is a mapping $\psi : [0, \infty) \rightarrow [0, \infty)$ which satisfies

- $(\psi_1) : \psi$ is increasing and continuous,
- $(\psi_2) : \psi(t) = 0$ if and only if $t = 0$.

Fixed point problems involving altering distances have been studied in [16], [18], [19] and in other papers.

Definition 6 ([14]). A weak altering distance is a mapping $\psi : [0, \infty) \rightarrow [0, \infty)$ which satisfies

- $(\psi_1) : \psi$ is increasing,
- $(\psi_2) : \psi(t) = 0$ if and only if $t = 0$.

Example 2. The function ψ defined by $\psi(t) = \begin{cases} t, & t \in [0, 1] \\ e^t, & t \in (1, \infty). \end{cases}$ is a weakly altering distance but it is not an altering distance.

The purpose of this paper is to prove a general coincidence and fixed point theorem for two hybrid pairs of mappings (A, S) and (B, T) satisfying an implicit relation without any form of compatibility and without limits of any sequence of points of (X, d) , using $CP_{(A,S)T}$ -property.

2. Implicit relations

Several fixed point theorems and common fixed theorems have been unified considering a general condition by an implicit relation in [9], [10] and in other papers. The study of fixed points for hybrid pairs of mappings satisfying implicit relations is initiated in [11] - [13] and in other papers.

Let \mathcal{F}_c be the set of all continuous functions $F : \mathbb{R}_+^6 \rightarrow \mathbb{R}$ satisfying the following conditions [15]:

- $(F_1) : F$ is nondecreasing in variable t_1 ,
- $(F_2) : F(t, 0, 0, t, t, 0) > 0, \forall t > 0$,
- $(F_3) : F(t, 0, t, 0, 0, t) > 0, \forall t > 0$.

Example 3. $F(t_1, \dots, t_6) = t_1 - k \max\{t_2, t_3, \dots, t_6\}$, where $k \in [0, 1)$.

Example 4. $F(t_1, \dots, t_6) = t_1 - k \max\left\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\right\}$, where $k \in [0, 1)$.

Example 5. $F(t_1, \dots, t_6) = t_1 - k \max\left\{t_2, \frac{k}{2}(t_3 + t_4), \frac{k}{2}(t_5 + t_6)\right\}$, where $k \in [0, 2)$.

Example 6. $F(t_1, \dots, t_6) = t_1 - k \max\{t_2, k(t_3 + t_4), k(t_5 + t_6)\}$, where $k \in [0, 1)$.

Example 7. $F(t_1, \dots, t_6) = t_1 - at_2 - b(t_3 + t_4) - c \max\{t_3 + t_4, t_5 + t_6\}$, where $a, b, c \geq 0$ and $a + b + c < 1$.

Example 8. $F(t_1, \dots, t_6) = t_1^2 - c_1 \max\{t_2^2, t_3^2, t_4^2\} - c_2 \max\{t_3 t_5, t_4 t_6\} - c_3 t_5 t_6$, where $c_1, c_2, c_3 \geq 0$ and $c_1 < 1$.

Example 9. $F(t_1, \dots, t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, where $a, b, c, d, e \geq 0$ and $c + d < 1, b + e < 1$.

Example 10. $F(t_1, \dots, t_6) = t_1 - \alpha \max\{t_2, t_3, t_4\} - (1 - \alpha)(at_5 + bt_6)$, where $\alpha \in (0, 1)$, $a, b \geq 0$ and $\max\{a, b\} < 1$.

Example 11. $F(t_1, \dots, t_6) = t_1 - at_2 - \frac{b(t_5 + t_6)}{1 + t_3 + t_4}$, where $a, b \geq 0$ and $a + 2b < 1$.

Example 12. $F(t_1, \dots, t_6) = t_1 - \max\{ct_2, ct_3, ct_4, at_5 + bt_6\}$, where $c \in (0, 1)$, $a, b \geq 0$ and $\max\{a, b\} < 1$.

3. Main results

Theorem 1. Let (X, d) be a metric space, $A, B : X \rightarrow CB(X)$ and $S, T : X \rightarrow X$ such that for all $x, y \in X$

$$(1) \quad F \left(\begin{array}{l} \psi(H(Ax, By)), \psi(d(Sx, Ty)), \psi(d(Sx, Ax)), \\ \psi(d(Ty, By)), \psi(d(Sx, By)), \psi(d(Ty, Ax)) \end{array} \right) \leq 0$$

where ψ is a weak altering distance and some $F \in \mathcal{F}_c$.

If (A, S) and T satisfy $CP_{(A,S)T}$ - property, then $\mathcal{C}(B, T) \neq \emptyset$.

Moreover,

a) if S is an occasionally coincidentally idempotent with respect to A , then S and A have a common fixed point,

b) if T is an occasionally coincidentally idempotent with respect to B , then T and B have a common fixed point,

c) if the hypothesis of a) and b) hold, then S, T, A and B have a common fixed point.

Proof. Since (A, S) and T satisfy $CP_{(A,S)T}$ - property, there exists $z = Su \in Au = D$ and $z \in T(X)$. Hence, there exists $v \in X$ such that $z = Tv$. By (1) for $x = u$ and $y = v$ we obtain

$$F \left(\begin{array}{l} \psi(H(Au, Bv)), \psi(d(Su, Tv)), \psi(d(Su, Au)), \\ \psi(d(Tv, Bv)), \psi(d(Su, Bv)), \psi(d(Tv, Au)) \end{array} \right) \leq 0,$$

$$(2) \quad F(\psi(H(D, Bv)), 0, 0, \psi(d(z, Bv)), \psi(d(z, Bv)), 0) \leq 0.$$

Since $z \in D$, $d(z, Bv) \leq H(D, Bv)$ which implies $\psi(H(z, Bv)) \leq \psi(H(D, Bv))$.

By (F_1) and (2) we obtain

$$F(\psi(d(z, Bv)), 0, 0, \psi(d(z, Bv)), \psi(d(z, Bv)), 0) \leq 0,$$

a contradiction of (F_2) if $\psi(d(z, Bv)) > 0$. Hence $\psi(d(z, Bv)) = 0$, which implies $d(z, Bv) = 0$. Hence $z = Tv \in Bv$ and $C(B, T) \neq \emptyset$.

Moreover,

a) if S is an occasionally coincidentally idempotent with respect to A , $z = Su = SSu = Sz$ and z is a fixed point of S .

By (1) for $x = z$ and $y = v$ we obtain

$$(3) \quad F\left(\begin{array}{l} \psi(H(Az, Bv)), \psi(d(Sz, Tv)), \psi(d(Sz, Az)), \\ \psi(d(Tv, Bv)), \psi(d(Sz, Bv)), \psi(d(Tv, Az)) \end{array}\right) \leq 0$$

Since $d(z, Az) = d(Su, Az) = d(Tv, Az) \leq \psi(H(Az, Bv))$, it follows that $\psi(d(z, Az)) \leq \psi(H(Az, Bv))$.

By (F_1) and (3) we obtain

$$F(\psi(d(z, Az)), 0, \psi(d(z, Az)), 0, 0, \psi(d(z, Az))) \leq 0,$$

a contradiction of (F_3) if $\psi(d(z, Az)) > 0$. Hence $\psi(d(z, Az)) = 0$, which implies $d(z, Az) = 0$. Hence $z = Sz \in Az$ and z is a common fixed point of A and S .

b) If T is occasionally coincidentally idempotent with respect to B , then $z = Tv = TTv = Tz$ and z is a fixed point of T .

By (1) for $x = u$ and $y = z$ we obtain

$$(4) \quad F\left(\begin{array}{l} \psi(H(Au, Bz)), \psi(d(Su, Tz)), \psi(d(Su, Au)), \\ \psi(d(Tz, Bz)), \psi(d(Su, Bz)), \psi(d(Tz, Au)) \end{array}\right) \leq 0.$$

Since $d(z, Bz) = d(Su, Bz) \leq H(Au, Bz)$, it follows $\psi(d(z, Bz)) \leq \psi(d(Au, Bz))$.

Then by (4) and (F_1) we obtain

$$F(\psi(d(z, Bz)), 0, 0, \psi(d(z, Bz)), \psi(d(z, Bz)), 0) \leq 0,$$

a contradiction of (F_2) if $\psi(d(z, Bz)) > 0$. Hence $\psi(d(z, Bz)) = 0$, which implies $d(z, Bz) = 0$. Hence $z = Tz \in Bz$ and z is a common fixed point of T and B .

c) If the hypothesis of a) and b) hold, then A, B, S and T have a common fixed point z . ■

If $\psi(t) = t$, then by Theorem 1 we obtain

Theorem 2. *Let (X, d) be a metric space, $A, B : X \rightarrow CB(X)$ and $S, T : X \rightarrow X$ such that for all $x, y \in X$*

$$F \left(\begin{array}{l} H(Ax, By), d(Sx, Ty), d(Sx, Ax), \\ d(Ty, By), d(Sx, By), d(Ty, Ax) \end{array} \right) \leq 0$$

for some $F \in \mathcal{F}_c$.

If (A, S) and T satisfy $CP_{(A,S)T}$ - property, then $\mathcal{C}(B, T) \neq \emptyset$.

Moreover,

a) if S is an occasionally coincidentally idempotent with respect to A , then A and S have a common fixed point,

b) if T is an occasionally coincidentally idempotent with respect to B , then B and T have a common fixed point,

c) if the hypothesis of a) and b) hold, then A, B, S and T have a common fixed point.

4. Applications

4.1. Coincidence and common fixed points for hybrid pair of mappings satisfying contractive condition of integral type

In [2], Branciari established the following theorem, which opened the way to the study of fixed points for mappings satisfying a contractive condition of integral type.

Theorem 3 ([2]). *Let (X, d) be a complete metric space, $c \in (0, 1)$ and $f : X \rightarrow X$ be a mapping such that for each $x, y \in X$*

$$\int_0^{d(fx, fy)} h(t) dt \leq c \int_0^{d(x, y)} h(t) dt,$$

where $h : [0, \infty) \rightarrow [0, \infty)$ is a Lebesgue measurable mapping which is summable (i.e. with finite integral) on each compact subset of $[0, \infty)$ such that for $\varepsilon > 0$, $\int_0^\varepsilon h(t) dt > 0$.

Then, f has a unique fixed point $z \in X$ such that for all $x \in X$, $z = \lim_{n \rightarrow \infty} f^n x$.

Some fixed point results for mappings satisfying contractive conditions of integral type are obtained in [16], [17] and in other papers.

Lemma 1. *Let $h : [0, \infty) \rightarrow [0, \infty)$ be as in Theorem 3. Then $\psi(t) = \int_0^t h(x) dx$ is a weakly altering distance.*

Proof. The proof follows by Lemma 2.5 [16]. ■

Theorem 4. *Let (X, d) be a metric space, $f, g : X \rightarrow X$ and $A, B : X \rightarrow CB(X)$, $h : [0, \infty) \rightarrow [0, \infty)$ as in Theorem 3 such that for all $x, y \in X$*

$$(5) \quad F \left(\int_0^{H(Ax,By)} h(t) dt, \int_0^{d(Sx,Ty)} h(t) dt, \int_0^{d(Sx,Ax)} h(t) dt, \int_0^{d(Ty,By)} h(t) dt, \int_0^{d(Sx,By)} h(t) dt, \int_0^{d(Ty,Ax)} h(t) dt \right) \leq 0$$

for some $F \in \mathcal{F}_c$.

If (A, S) and T satisfy $CP_{(A,S)T}$ - property, then $\mathcal{C}(B, T) \neq \emptyset$.

Moreover,

a) if S is an occasionally coincidentally idempotent with respect to A , then S and A have a common fixed point,

b) if T is an occasionally coincidentally idempotent with respect to B , then T and B have a common fixed point,

c) if the hypothesis of a) and b) hold, then A, B, S and T have a common fixed point.

Proof. Let $\psi(t)$ as in Lemma 1. Then

$$\begin{aligned} \psi(H(Ax, By)) &= \int_0^{H(Ax,By)} h(t) dt, \quad \psi(d(Sx, Ty)) = \int_0^{d(Sx,Ty)} h(t) dt, \\ \psi(d(Sx, Ax)) &= \int_0^{d(Sx,Ax)} h(t) dt, \quad \psi(d(Ty, By)) = \int_0^{d(Ty,By)} h(t) dt, \\ \psi(d(Sx, By)) &= \int_0^{d(Sx,By)} h(t) dt, \quad \psi(d(Ty, Ax)) = \int_0^{d(Ty,Ax)} h(t) dt. \end{aligned}$$

By (5) we obtain

$$F \left(\begin{matrix} \psi(H(Ax, By)), \psi(d(Sx, Ty)), \psi(d(Sx, Ax)), \\ \psi(d(Ty, By)), \psi(d(Sx, By)), \psi(d(Ty, Ax)) \end{matrix} \right) \leq 0,$$

which is inequality (1) from Theorem 1. Theorem 4 follows from Theorem 1. ■

By Theorem 4 and Example 3 we obtain

Theorem 5. *Let (X, d) be a metric space and $h(t)$ as in Theorem 3 such that for all $x, y \in X$*

$$\int_0^{H(Ax,By)} h(t) dt \leq k \max \left\{ \int_0^{d(Sx,Ty)} h(t) dt, \int_0^{d(Sx,Ax)} h(t) dt, \int_0^{d(Ty,By)} h(t) dt \right\}, \int_0^{d(Sx,By)} h(t) dt, b \int_0^{d(Ty,Ax)} h(t) dt \Big\},$$

where $k \in [0, 1)$.

Moreover, if

a) if S is an occasionally coincidentally idempotent with respect to A , then S and A have a common fixed point,

b) if T is an occasionally coincidentally idempotent with respect to B , then T and B have a common fixed point,

c) if the hypothesis of a) and b) hold, then A, B, S and T have a common fixed point.

Remark 2. *By Theorem 4 and Examples 4 - 12 we obtain new particular results.*

4.2. Fixed points for hybrid pairs of mappings satisfying φ -contractive conditions

As in [7], let Φ be the set of all functions $\varphi : [0, \infty) \rightarrow [0, \infty)$ such that

$$(\varphi_1) : \varphi(t) < t \text{ for } t > 0,$$

$$(\varphi_2) : \varphi(0) = 0.$$

In the following, we denote by Φ_c the set of all nondecreasing functions $\varphi : [0, \infty) \rightarrow [0, \infty)$ satisfying conditions (φ_1) and (φ_2) .

The following functions $F : \mathbb{R}_+^6 \rightarrow \mathbb{R}_+$ satisfy conditions (F_1) , (F_2) , (F_3) .

Example 13. $F(t_1, \dots, t_6) = t_1 - \varphi(\max\{t_2, t_3, \dots, t_6\})$.

Example 14. $F(t_1, \dots, t_6) = t_1 - \varphi\left(\max\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\}\right)$.

Example 15. $F(t_1, \dots, t_6) = t_1 - \varphi\left(\max\{t_2, \frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\}\right)$.

Example 16. $F(t_1, \dots, t_6) = t_1 - \varphi(\max\{t_2, \sqrt{t_3 t_4}, \sqrt{t_5 t_6}, \sqrt{t_3 t_5}, \sqrt{t_4 t_6}\})$.

Example 17. $F(t_1, \dots, t_6) = t_1 - \varphi\left(at_2 + \frac{b\sqrt{t_5 t_6}}{1 + t_3 + t_4}\right)$, where $a, b \geq 0$ and $a + b \leq 1$.

Example 18. $F(t_1, \dots, t_6) = t_1 - \varphi(at_2 + bt_3 + ct_4 + dt_5 + et_6)$, where $a, b, c, d, e \geq 0$ and $c + d \leq 1, b + e \leq 1$.

Example 19. $F(t_1, \dots, t_6) = t_1 - \varphi\left(at_2 + b \max\{t_3, t_4\} + c \max\left\{\frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\right\}\right)$, where $a, b, c \geq 0$ and $a + b + c \leq 1$.

Example 20. $F(t_1, \dots, t_6) = t_1 - \varphi(at_2 + b \max\{2t_4 + t_5, 2t_4 + t_6, t_3 + t_5 + t_6\})$, where $a, b \geq 0$ and $a + b \leq 1$.

By Theorem 2 and Example 13 we obtain

Theorem 6. *Let (X, d) be a metric space, $A, B : X \rightarrow CB(X)$ and $S, T : X \rightarrow X$ such that for all $x, y \in X$*

$$H(Ax, By) \leq \varphi(\max\{d(Sx, Ty), d(Sx, Ax), d(Ty, By), d(Sx, By), d(Ty, Ax)\})$$

for some $\varphi \in \Phi_c$.

If (A, S) and T satisfy $CP_{(A,S)T}$ - property, then $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if

a) if S is an occasionally coincidentally idempotent with respect to A , then A and S have a common fixed point,

b) if T is an occasionally coincidentally idempotent with respect to B , then B and T have a common fixed point,

c) if the hypothesis of a) and b) hold, then A, B, S and T have a common fixed point.

Remark 3. By Theorem 2 and Examples 14 - 20 we obtain new particular results.

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