FASCICULI MATHEMATICI

Nr 33 2002

BARBARA TAL-FIGIEL

GREEN FUNCTION AND A NONLINEAR PROBLEM FOR THE DIFFUSION EQUATION IN A CYLINDRICAL RING

ABSTRACT: The subject of the paper is the construction of the solution $(r,t)\to U(r,t)$ to the parabolic nonlinear partial differential equation (I) $\Delta u(x,t)-D_t u(x,t)-c(x,t)u(x,t)=f(x,t,u(x,t),D_x u(x,t))$, $x=(x_1,x_2)$, $\Delta=D_{x_1}^2+D_{x_2}^2$ for the cylindrical ring $D_1=\{(r,t):r=(x_1^2+x_2^2)^{1/2},\ 0<a<r<b,\ t\in(0,T]\}$. In radial coordinates, consider the equation (II) $D_r^2U(r,t)-D_tU(r,t)+r^{-1}D_rU(r,t)-$ $-C(r,t)U(r,t)=f(r,t,U(r,t),D_rU(r,t))$, $(r,t)\in D_1$, the initial condition $U(r,0)=$ $=f(r)$, $r\in(a,b)$, and the boundary-value conditions $U(a,t)=H(t)$, $U(b,t)=K(t)$, $t\in(0,T]$.

KEY WORDS: a limit problem, Green finction, the Banach fixed point method.

1. INTRODUCTION

The subject of the paper is the construction of the solution $(r,t)\to U(r,t)$ to the parabolic equation

(1) $$PU(r,t)+r^{-1}D_rU(r,t)-C(r,t)U(r,t)=$$
$$=F(r,t,U(r,t),D_rU(r,t)),$$

where

$$PU(r,t)=(D_r^2-D_t)U(r,t)$$

in the domain

$$D=\{(r,t):0<a<r<b,\ t\in(0,T]\},$$

satisfying the initial condition

(2) $$U(r,0)=f(r),\qquad r\in(a,b),$$

and the boundary-value conditions

(3) $$U(a,t)=H(t),\qquad t\in(0,T],$$

(4) $$U(b,t)=K(t),\qquad t\in(0,T].$$

The functions c, F, $f(r)$, H, K are given functions and U is the unknown one. To the construction of the solution U we apply the suitable Green function G, the method of the change of the leading part of differential operator and the Banach fixed point method. In [3], the similar problem for spherical shell is treated.

2. GREEN FUNCTION *G*

To sole problem (1) – (4) we apply the Green function $(r,t,p,s) \to G(r,t,p,s)$ to the equation

(5) $$(D_r^2 - D_t)\, G(r,t,p,s) = 0$$

with the Dirichlet boundary-value conditions

(6) $$G(a,t,p,s) = G(b,t,p,s) = G(r,t,a,s)\; G(r,t,b,s) = 0.$$

Let

$$U(r,t,p,s) = A(t-s)^{-1/2} \exp(B(t,s)(r-p)^2),$$

$$A = (2\sqrt{\pi})^{-1}, \quad B(t,s) = (-4(t-s))^{-1}, \quad 0 \le s < t \le T, \quad r \in (a,b),$$
$$p \in (a,b), \quad r \ne p,$$

denote the fundamental solution to the equation

$$(D_r^2 - D_t)\, U(r,t,p,s) = 0.$$

Let us consider the sequences

$$r_0^1 = r_0^2\, r, \quad r_{2n}^1 = r + 2n(b-a), \quad r_{2n}^2 = r - 2n(b-a), \quad n = 0,1,...$$

$$r_{2n+1}^1 = -r - 2n(b-a) + 2a, \quad r_{2n+1}^2 = -r + 2n(b-a) + 2b, \quad n = 0,1,...$$

Consider the function G defined by the formulas:

$$G(r,t,p,s) = -K_1(r,t,p,s) - K_3(r,t,p,s) + K_2(r,t,p,s) + K_4(r,t,p,s),$$

where

$$K_1(r,t,p,s) = (t-s)^{-1/2} \sum_{n=0}^{\infty} \exp(B(t,s)(-r-2n(b-a)+2a-p)^2),$$

$$K_2(r,t,p,s) = (t-s)^{-1/2} \sum_{n=0}^{\infty} \exp(B(t,s)(r+2n(b-a)-p)^2),$$

$$K_3(r,t,p,s) = (t-s)^{-1/2}\sum_{n=0}^{\infty} \exp(B(t,s)(-r+2n(b-a)+2b-p)^2),$$

$$K_4(r,t,p,s) = (t-s)^{-1/2}\sum_{n=0}^{\infty} \exp(B(t,s)(r-2n(b-a)-p)^2),$$

$$(r,t,p,s)\in D_1 = \{(r,t,p,s) : 0\le s<t\le T,\ r\in J=(a,b),\ p\in J=(a,b),\ r\ne p\}.$$

By [1] (p. 476), the function G is the Green function to the equation

$$PG(r,t,p,s) = 0, \quad (r,t,p,s)\in D_1,$$

to the domain D_1 and to the homogeneous boundary-value conditions of the Dirichlet type

$$G(a,t,p,s) = G(b,t,p,s) = G(r,t,a,s) = G(r,t,b,s) = 0.$$

3. GREEN POTENTIALS

Let us consider the Green potentials of the double layer of the from

$$D_pG(r,t,a,s),\ D_pG(r,t,b,s).$$

In the sequel, we shall calculate the last potentials by:

THEOREM 1. $D_pG(r,t,a,s) = Q_1(r,t,s) + Q_2(r,t,s) + Q_3(r,t,s)$,

$$D_pG(r,t,b,s) = R_1(r,t,s) + R_2(r,t,s) + R_3(r,t,s),$$

where

(7) $$Q_1(r,t,s) = A(t-s)^{-3/2}(r-a)\exp(B(t,s)(r-a)^2),$$

$$Q_2(r,t,s)=A(t-s)^{-3/2}\sum_{n=0}^{\infty}(r+2n(b-a)-a)\exp(B(t,s)(r+2n(b-a)-a)^2),$$

$$Q_3(r,t,s)=A(t-s)^{-3/2}\sum_{n=0}^{\infty}(-r-2n(b-a)+a)\exp(B(t,s)(-r-2n(b-a)+a)^2),$$

(8) $$R_1(r,t,s) = A(t-s)^{-3/2}(b-r)\exp(B(t,s)(r-b)^2),$$

$$R_2(r,t,s)=-A(t-s)^{-3/2}\sum_{n=0}^{\infty}(r+2n(b-a)-b)\exp(B(t,s)(r+2n(b-a)-b)^2),$$

$$R_3(r,t,s)=-A(t-s)^{-3/2}\sum_{n=0}^{\infty}(-r-2n(b-a)+b)\exp(B(t,s)(r+2n(b-a)+b)^2),$$

Proof. By easy calculus, we obtain

$$-D_pK_1(r,t,a,s) = \tfrac{A}{2}(t-s)^{-3/2}(r-a)\exp(B(t,s)(r-a)^2)+$$

$$+\tfrac{A}{2}(t-s)^{-3/2}\sum_{n=0}^{\infty}(r+2n(b-a)+a)\exp(B(t,s)(r+2n(b-a)-a)^2),$$

$$-D_pK_3(r,t,a,s) =$$

$$= \tfrac{A}{2}(t-s)^{-3/2}\sum_{n=0}^{\infty}(r-2n(b-a)-2b+a)\exp(B(t,s)(r-2n(b-a)-2b+a)^2),$$

$$-D_pK_2(r,t,a,s) =$$

$$= \tfrac{A}{2}(t-s)^{-3/2}\sum_{n=0}^{\infty}(r+2n(b-a)-a)\exp(B(t,s)(r+2n(b-a)-a)^2),$$

$$-D_pK_4(r,t,a,s) =$$

$$= \tfrac{A}{2}(t-s)^{-3/2}\sum_{n=0}^{\infty}(r-2n(b-a)-2b+a)\exp(B(t,s)(r-2n(b-a)-2b+a)^2),$$

4. Some class of the functions applied in the sequel and Green potentials

Let (K_3) denote the class of all functions $f \in C^1([a,b])$ such that

$$D_p^i f(a) = D_p^i f(b) = 0 \quad (i=0,1), \quad f(p)=0 \quad \text{for} \quad p \in (R \setminus [a,b]),$$

and let (K_2) denote the class of all functions $H \in C^1([0,T])$ such that

$$H(0) = D_t H(0) = 0.$$

Let us consider the Green potentials

$$U_1 = U_1^1 + U_1^2 + U_1^3,$$

$$U_1^i = \int_0^t Q_i(r,t,s)H(s)\,ds \quad (i=1,2,3),$$

(4b)
$$U_1 = U_2^1 + U_2^2 + U_2^3,$$

$$U_2^i = \int_s^t R_i(r,t,p)K(s)\,ds \quad (i=1,2,3),$$

$$U_3 = \int_a^b f(p)G(r,t,p,0)\,dp.$$

Next, we shall give the properties of the potentials U_i $(i=1,2,3)$.

LEMMA 1. *If* $H,K \in (K_2)$ *then the potentials* U_i $(i=1,2)$ *satisfy the conditions:*

1^0 $PU_i(r,t) = 0, \quad (r,t)\in D \quad (i=1,2,3),$

2^0 $U_1^1(r,t) \to H(t)$ as $(r,t)\to(a,t), \quad t\in(0,T],$

3^0 $U_1^2(r,t) + U_1^3(r,t) \to 0$ as $(r,t)\to(a,t), \quad t\in(0,T],$

4^0 $U_1(r,t)\to 0$ as $(r,t)\to(b,t), \quad t\in(0,T],$

5^0 $U_2^1(r,t)\to K(t)$ as $(r,t)\to(b,t), \quad t\in(0,T],$

6^0 $U_2^3(r,t)\to 0$ as $(r,t)\to(b,t), \quad t\in(0,T],$

7^0 $U_2(r,t)\to 0$ as $(r,t)\to(a,t), \quad t\in(0,T].$

PROOF. By [1] (p. 472) and by the properties of the potential of the double layer, we obtain $1^0 - 7^0$.

From Lemma 1, we get.

LEMMA 2. *The compatibility conditons*

$$U_1(a,0) = U(b,0) = U_3(a,0) = U_3(b,0) = 0$$

holds.

In the sequel, we shall apply the class (K). Let (K) denote the class of all functions $(y,s,W,V)\to F(y,s,W,V)$ sastisfying the conditions:

(a) the functions F are defined and continuous in the set

$$D_1 = \{(y,s,W,V):(y,s)\in D,\ W,V\in R\times R\},$$

(b) the functions F are boundary, i.e. $|F|\le M$, where $M>0$ is a positive constant,

(c) the functions F satisfy the Lipshitz condition with the Lipschitz constats $q\in(0,1)$, i.e.

$$\|F(W^1,V^1)-F(W^2,V^2)\| \le q(\|W^1-W^2\| + \|V^1-V^2\|),$$
$$W^i,V^i\in R \quad (i=1,2),$$

unformly with respect to $(y,s)\in \overline{D}$ with the supermum norm.

5. The change of the unknown function

Let $(r,t) \to \overline{U}(r,t)$ denote the solution of the equation

(10) $$P\overline{U}(r,t) = 0, \qquad (r,t) \in D,$$

with the limit conditions

(11) $$\overline{U}(r,0) = f(r), \qquad r \in (a,b),$$

(12) $$\overline{U}(a,t) = H(t), \qquad t \in (0,T],$$

(13) $$\overline{U}(a,t) = H(t), \qquad t \in (0,T],$$

By [3], the solution $\overline{U}$ of the last problems is of the from

$$\overline{U}(r,t) = U_1(r,t) + U_2(r,t) + U_3(r,t),$$

where

$$U_1(r,t) = \int_0^t H(s)\, D_p G(r,t,a,s) ds,$$

$$U_2(r,t) = \int_0^t K(s)\, D_p G(r,t,b,s) ds,$$

$$U_3(r,t) = \int_0^t f(p) G(r,t,p,0) dp$$

or

$$U_1(r,t) = \int_0^t H(s)(Q_1(r,t,s) + Q_2(r,t,s) + Q_3(r,t,s))ds,$$

$$U_2(r,t) = \int_0^t K(s)(R_1(r,t,s) + R_2(r,t,s) + R_3(r,t,s))ds.$$

Let us consider the transformation

(14) $$(r,t) \to U(r,t) = \overline{U}(r,t) + W(r,t), \quad (r,t) \in D,$$

where W is new unknown function.

By transfromation (14), we obtain the equation

(15) $$PW(r,t) = m(r,t) + F(r,t,W(r,t),D_r W(r,t)), \quad (r,t) \in D,$$

with

$$m(r,t) = C(r,t)\overline{U}(r,t),$$

$$F(r,t,W(r,t),D_r W(r,t)) = r^{-1} D_r W(r,t) + C(r,t)(W(r,t) - \overline{U}(r,t) +$$

$$+ f(r,t,W(r,t)-\overline{U}(r,t), D_rW(r,t)-D_r\overline{U}(r,t))$$

and with homogeneous limit conditions

(16) $$W(r,0) = W(a,t) = W(b,t) = 0, \qquad (r,t)\in D.$$

6. The system of the integral equations resulting from problem (15) - (16)

Let

$$D_rW(r,t) = V(r,t).$$

Applying transformation (14) and the Green function G, we get the system of the integral equations to the unknown functions W, V of the form

(18) $$W(r,t) = \int_0^t\int_a^b m(p,s)G(r,t,p,s)\,dpds + \int_0^t\int_a^b G(r,t,p,s)F(p,s,w(p,s),V(p,s))\,dpds,$$

(19) $$V(r,t) = \int_0^t\int_a^b (D_rG(r,t,p,s))\,dpds + \int_0^t\int_a^b (D_rG(r,t,p,s)F(p,s,w(p,s),V(p,s))\,dpds.$$

7. Some Banach spaces and the solution of system (18), (19)

In the sequel by C^i $(i=1,2,\ldots)$ we shall denote positive constants.

To solve system (18), (19), let us consider the system of integro-differential equations

$$W(r,t) = f_1(r,t) + \int_0^t\int_a^b G(r,t,p,s)F(p,s,W(p,s),V(p,s))dpds, \quad (r,t)\in D,$$

$$V(r,t) = D_rf_1(r,t) + \int_0^t\int_a^b (D_rG(r,t,p,s))F(p,s,W(p,s),V(p,s))dpds, \quad (r,t)\in D,$$

where

$$f_1(r,t) = \int_0^t \int_a^b C(p,s)(U_1(p,s)+U_2(p,s)+U_3(p,s))G(r,t,p,s)dpds,$$

$$f_2(r,t) = \int_0^t \int_a^b C(p,s)(U_1(p,s)+U_2(p,s)+U_3(p,s))D_r G(r,t,p,s)dpds.$$

Let

$$\overline{F} = (f_1, f_2),$$

$$C \in C(\overline{D}), \quad C(p,s) \ge 0, \quad (p,s) \in \overline{D}, \quad \sup_{(p,s)\in\overline{D}} C(p,s) = M_1,$$

$$M_2 = \max\{\sup|H(t)| + \sup|K(t)| + \sup|f(p)| : (p,t) \in \overline{D}\}.$$

From the inequality

$$G(r,t,p,s) \ge 0$$

if follows that

$$G(r,t,p,s) \le U(r,t,p,s)$$

and we obtain:

LEMMA 3. *The inequality*

(20) $$\| f_1 \| \le M_1 M_2 (b-a) t^{1/2} \le C^1 T^{1/2}$$

holds.

To estimate $\| f_2 \|$ let us consider the integrals $S_1(r,t)$, $S_2(r,t)$, where

$$S_1(r,t) = M \int_0^t \int_a^b \frac{r-p}{(t-s)^{-3/2}} \exp(B(t,s)(r-p)^2)dpds,$$

$$S_2(r,t) = M\left(\sum_{n=1}^{\infty} B_n(r,t)\right),$$

$$B_n(r,t) = \int_0^t \int_a^b (A_n^1(r,t,p,s) + A_n^t(r,t,p,s) + A_n^3(r,t,p,s) + A_n^4(r,t,p,s))dpds$$

and

$$A_n^1 = (t-s)^{-3/2}[-r+2n(b-a)+2a-p]\exp(B(t,s)(r+2n(b-a)-p)^2),$$

$$A_n^2 = (t-s)^{-3/2}[r+2n(b-a)-p]\exp(B(t,s)(r+2n(b-a)-p)^2),$$

$$A_n^3 = (t-s)^{-3/2}[-r+2n(b-a)+2a-p]\exp(B(t,s)(-r+2n(b-a)+2b-p)^2),$$

$$A_n^4 = (t-s)^{-3/2}[-r+2n(b-a)+2a-p]\exp(B(t,s)(r+2n(b-a)-p)^2).$$

Let us consider the indentity

$$\frac{r-p}{(t-s)^{3/2}} = \frac{(r-p)^{1,8}}{(t-s)^{0,9}} \frac{1}{(r-p)^{0,9}(t-s)^{0,6}}.$$

Moreover, let

$$z = \frac{r-p}{(t-s)^{1/2}}, \qquad z^{1,8} = \frac{(r-p)^{1,8}}{(t-s)^{0,9}}.$$

We have the inequality ([2]):

$$z^{1,8}\exp(-z^2) \leq C_1$$

and, by [1] (p. 476), we obtain the estimate

(21) $$S_2(r,t) \leq C_2 \int_0^t \int_a^b dpds \leq C_2 t|_0^T (b-a) = C^3 T.$$

From Lemma 3 and by (21), we get:

Lemma 4. *The inequality*

$$\| \overline{F} \| = \| f_1 \| + \| f_2 \| \leq C^1 T^{1/2} + C^3 T$$

holds.

Next, let us consider the Banach space

$$B^2 = \{\overline{Q} = (Q^1, Q^2)\}$$

with

$$Q^1(r,t) = \int_0^t \int_a^b F(y,s,W(p,s),V(p,s))G(r,t,p,s)dpds$$

and

$$Q^2(r,t) = \int_0^t \int_a^b F(p,W(p,s),V(p,s))D_r G(r,t,p,s)dpds$$

and with the norm

$$\|\overline{Q}\| \leq \|Q^1\| + \|Q^2\| \leq C^3 T^{1/2} + C^4 T.$$

8. The balls in Banach spaces

Let $\overline{O}$ denote the vector function identically equal zero.
Let

$$R_1 = (C^1 + C^3)T^{1/2} + C^2 T.$$

Consider three balls: $K(\overline{O},R_1)$ – the ball with the center $\overline{O}$ and radius R_1 being the set of all vector functions $(\overline{F},\overline{Q})$ for which

$$\|(\overline{F},\overline{Q}\| \leq \|\overline{F}\| + \|\overline{Q}\| \leq R_1,$$

$K(\overline{O},qR_1)$ – the ball with the center $\overline{O}$ and radius qR_1 being the set of the functions $\overline{F}$ for which

$$\|\overline{F}\| \leq qR_1,$$

$K(\overline{O},(1-q)R_1)$ – the ball with the center $\overline{O}$ and radius $(1-q)R_1$ being the set of the functions $\overline{Q}$ for which

$$\|\overline{Q}\| \leq (1-q)R_1.$$

9. THE TRANSFORMATION TO THE BANACH FIXED POINT METHOD

Consider the transfirmation

$$(S) \qquad \overline{D} \ni (r,t) \to S(r,t,\overline{F},\overline{Q}) = \overline{F}(r,t) + \overline{Q}(r,t,W(r,t),V(r,t)).$$

LEMMA 5. *If* $q \in (0,1)$ *then transformation* (*S*) satisfies the conditions:

1^0 (*S*) *is the contraction with the coefficient* q,

2^0 (*S*) *transforms the ball* $K(\overline{O},R_1)$ *into itself.*

PROOF. 1^0 : Let $\overline{F} \in K(\overline{O},qR_1)$ and $\overline{Q} \in K((1-q)R_1)$. Then

$$\begin{aligned}
&\|\overline{S}(\overline{F},W^1,V^1) - S(\overline{F},W^2,V^2)\| \leq \\
&\quad \leq \|\overline{F} + \overline{Q}(W^1,V^1) - \overline{F} - \overline{Q}(W^2,V^2)\| \leq \\
&\quad \leq \|\overline{Q}(W^1,V^1) - \overline{Q}(W^2,V^2)\| \leq \\
&\quad \leq q\|(W^1 - W^2) + (V^1 - V^2)\|, \qquad W^i,V^i \in R \quad (i=1,2).
\end{aligned}$$

2^0 : We have

$$\|\overline{F} + \overline{Q}(W,V)\| = \|\overline{F}\| + \|\overline{Q}(W,V)\| \leq qR_1 + (1-q)R_1 = R_1.$$

By the Banach fixed point theorem we obtain:

THEOREM 1. *If* $\overline{F} \in (K(\overline{O}, qR_1)$, $\overline{Q}(r,t,W(r,t),V(r,t)) \in K(\overline{O},(1-q)R_1)$, $q \in (0,1)$, *then there exists the fixed point* $\overline{Z}(r,t) = (W(r,t),V(r,t))$ *to the transformation* (*S*) *for which*

$$(S_1) \qquad \{\overline{Z}(r,t)\} = \overline{F}(r,t) + \overline{Q}(r,t,\overline{Z}(r,t)), \quad (r,t) \in D.$$

10. CONSTRUCTION OF THE FIXED POINT $\overline{Z}(r,t)$ BY THE METHOD OF THE SUCCESSIVE APPROXIMATIONS

Let us consider the sequence:

$$\{\overline{Z}_n(r,t)\} = \{(W_n(r,t),V_n(r,t))\}, \quad (r,t) \in \overline{D}.$$

$$\overline{Z}_0(r,t) \in K(\overline{O},R_1), \overline{Q}(r,t,\overline{Z}(r,t)) \in K((1-q)R),$$

$$\overline{Z}_1(r,t) = \overline{Z}_0 K(r,t) + \overline{Q}(r,t,\overline{Z}(r,t)),$$

....

$$\overline{Z}_n(r,t) = \overline{Z}_0(r,t) + \overline{Q}(r,t,\overline{Z}_{n-1}(r,t)), \quad n = 1,2,...$$

and the Cauchy sequence

$$\overline{Z}_{m,n}(r,t) = \overline{Z}_n(r,t) - \overline{Z}_m(r,t) \quad m,n = 1,2,..., \quad n > m.$$

LEMMA 6. *If* $\overline{F} \in K(\overline{O}, qR_1)$, $\overline{Q} = (Q^1(r,t,W,V), Q^2(r,t,W,V)) \in K(\overline{O}, (1-q)R)$, $q \in (0,1)$, *then*

$$(S_2) \qquad \| \overline{Z}_n - \overline{Z}_m \| \le q^m (1-q)^{-1} \| \overline{Z}_1 - \overline{Z}_0 \|, \quad n > m, \quad m,n = 1,2,... \,.$$

PROOF. Observe that

$$(S_3) \qquad \| \overline{Z}_n - \overline{Z}_m \| \le q \| \overline{Z}_{n-1} - \overline{Z}_{n-2} \| \le ... \le q^n \| \overline{Z}_1 - \overline{Z}_0 \|.$$

We can write

$$\overline{Z}_n(r,t) - \overline{Z}_m(r,t) = \sum_{j=m+1}^{n} (\overline{Z}_j(r,t)) - \overline{Z}_{j-1}(r,t)).$$

By the last formula, we obtain the estimate

$$\| \overline{Z}_{n,m} \| \le \sum_{j=m+1}^{n} q^{j+1} \| \overline{Z}_1 - \overline{Z}_0 \| \le q^m (1-q)^{-1} \| \overline{Z}_1 - \overline{Z}_0 \|.$$

Since $C(\overline{D})$ is a complete space and $\overline{Z}_n \in C(\overline{D})$ thus there exists

$$\lim_{n\to\infty} \overline{Z}_n(r,t) = \overline{Z}(r,t), \quad \overline{Z} \in C(\overline{D}).$$

THEOREM 2. *The function*

$$(r,t) \to \overline{Z}(r,t) = (W(r,t), V(r,t))$$

satisfies the equation

$$(S_4) \qquad \overline{Z}(r,t) = \overline{F}(r,t) + \overline{Q}(r,t,\overline{Z}(r,t))$$

or the system of the equations

$$(S_5) \qquad (W(r,t), V(r,t)) = (f_1(r,t), f_2(r,t)) + \overline{Q}(r,t,W(r,t),V(r,t)).$$

11. UNIQUENESS THEOREM

The solution (W,V) of system (18), (19) solved by the fixed point method is unique. On the other hand, the function.

$$(y,s) \to F^1(y,s) = F(y,s,W(y,s),V(y,s))$$

satisfies the Lipschitz condition. By the Poisson theorem, from the system integro–differential equations follows diffrential equation (1) and therefore, both problems are equivalent. Since the solution $\overline{U}$ to problem (10) – (13) is unique thus we obtain:

THEOREM 3. *The function*

$$(r,t) \to U(r,t) = \overline{U}(r,t) + W(r,t)$$

is the unique solution to problem (1) – (4).

REFERENCES

[1] M. Krzyżański, *Partial differential equations of second order*, vol. I, PWN – Polish Scientific Publishers, Warszawa, 1971.

[2] W. Pogorzelski, *Równania całkowe i ich zastosowania*, vol. I, PWN, Warszawa, 1958.

[3] B. Tal-Figiel, Green function and some nonlinear problem for spherical shell solved by the change of leading part of the differential operator, *unpublished.*

(Institute of Mathematics, Cracow University of Technology, Warszawska 24, Kraków)

Received on 02.07.2001 and, in revised form, on 08.10.2001.