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**EXISTENCE AND NONEXISTENCE OF POSITIVE SOLUTIONS FOR
NONLINEAR THREE-POINT BOUNDARY-VALUE PROBLEMS***

ABSTRACT: The authors study the existence and nonexistence of positive solutions to the three point boundary-values problem

$$(E) \quad (p(t)u'(t))' + \lambda a(t)f(u(t)) = 0, \quad t \in (0,1),$$

$$(B) \quad u(0) = \alpha u(\eta), \quad \beta u(\eta) = u(1),$$

where $0 < \eta < 1$ and $\beta \geq 0$, $\alpha \geq 0$, $\lambda > 0$. Different conditions for the problem (E) – (B) to have at least one or two positive solutions and sufficient conditions for this problem to have no positive solutions are given, by applying a new Green's function of three point value problem.

KEY WORDS: second-order three-point BVP, positive solution, cone, Green's function, fixed point theorem.

1. INTRODUCTION

In this paper, we study the existence and nonexistence of positive solutions to the boundary-value problem (BVP for short) consisting of the differential equation

$$(1) \quad (p(t)u'(t))' + \lambda a(t)f(u(t)) = 0 \quad t \in (0,1),$$

and the boundary conditions

$$(2) \quad u(0) = \alpha u(\eta), \quad \beta u(\eta) = u(1),$$

where $0 < \eta < 1$ and $\beta \geq 0$, $\alpha \geq 0$, $\lambda > 0$. Our purpose here is to give some existence results for at least one or two positive solutions and nonexistence results to BVP (1) –

(2). Let $I(t) = \int_0^t \frac{ds}{p(s)}$. From now on, we assume the following:

$$(A_1): \quad f \in C((0, +\infty), [0, +\infty));$$

$$(A_2): \quad p \in C((0, 1), (0, +\infty));$$

$$(A_3): \quad a \in C((0, 1), [0, +\infty));$$

$$(A_4): \quad 0 < \int_0^1 (I(1) - I(s))a(s)ds < \infty, \quad I(1) < \infty.$$

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Let $M = (1 - \alpha)I(1) - (\beta - \alpha)I(\eta)$. In the case when $\alpha = \beta = 0$, $p(t) \equiv 1$ BVP (1) - (2) becomes two point boundary value problem, which had been studied by *Argarwal* and *D. O'Regan*^[1].

In [2], Ma studied the case when $p(t) \equiv 1$, $\alpha = 0$, $\beta > 0$. He proved that if $f_0 = 0$, $f_\infty = +\infty$ (suplinear), or $f_0 = +\infty$, $f_\infty = 0$ (sublinear), then BVP (1) - (2) has at least one positive solution, where

$$f_0 = \lim_{x \rightarrow 0} \frac{f(x)}{x}, \quad f_\infty = \lim_{x \rightarrow +\infty} \frac{f(x)}{x}.$$

In the above papers, the concavity of solutions plays an important role since $u''(t) = -a(t)f(u(t)) \leq 0$. Their arguments involve the discussion of positive properties of solutions and fixed point in cone. However, the explicit formulation of Green's function was not given. On the other hand, as $p(t)$ appears in the equation, its solutions may not be concave. The results in this paper extend and complement those in [2]. In addition, we also give a new Green's function for three point boundary value problem.

By a positive solution of BVP (1) - (2) we mean a function $u(t)$ which is positive on $(0,1)$ and satisfies differential equation (1) and the boundary conditions (2).

2. LEMMAS AND MAIN RESULTS

In this section, we present our main theorems which extend, improve, and complement the known results. Firstly, we give a lemma which will be used in theorem 1-7. Let

$$(3) \quad G(t, s) = \frac{1}{M} \begin{cases} I(s)(I(1) - I(t)) + \beta I(s)(I(t) - I(\eta)), & 0 \leq s \leq t < \eta < 1 \text{ or} \\ & 0 \leq s \leq \eta \leq t \leq 1, \\ I(t)(I(1) - I(s)) + \beta I(t)(I(s) - I(\eta)) \\ \quad + \alpha(I(1) - I(\eta))(I(s) - I(t)), & 0 \leq t \leq s \leq \eta < 1, \\ I(s)(I(1) - I(t)) + \beta I(\eta)(I(t) - I(s)) \\ \quad + \alpha(I(1) - I(t))(I(\eta) - I(s)), & 0 \leq \eta \leq s \leq t \leq 1, \\ (I(1) - I(s))(I(t) - \alpha I(t) + \alpha I(\eta)), & 0 < \eta \leq t \leq s \leq 1 \text{ or} \\ & 0 \leq t < \eta \leq s \leq 1, \end{cases}$$

$$(4) \quad A = \max_{t \in [0,1]} \int_0^1 G(t, s)a(s)ds, \quad B = \max_{t \in [\delta, 1-\delta]} \int_\delta^{1-\delta} G(t, s)a(s)ds,$$

and

$$(5) \quad \mu = \min \left\{ \frac{I(\delta)}{I(1)}, 1 - \frac{I(1-\delta)}{I(1)} \right\}.$$

LEMMA 1. Suppose $y(t) \in C([0,1], [-\infty, +\infty))$, and $M \neq 0$. $G(t, s)$ is defined in (3) and $u(t)$ is a solution of the boundary value problem

$$(6) \quad \begin{cases} (p(t)u'(t))' + y(t) = 0, & t \in (0,1), \\ u(0) = \alpha u(\eta), & u(1) = \beta u(\eta), \end{cases}$$

Then we have

$$u(t) = \int_0^1 G(t, s)y(s)ds, \quad t \in [0,1],$$

Furthermore, if $M > 0$ and $y(t) \geq 0$ for every $t \in [0,1]$, then $G(t, s) \geq 0$ for $(t, s) \in [0,1] \times [0,1]$.

PROOF. Suppose

$$(7) \quad u(t) = -\int_0^t (I(t) - I(s))y(s)ds + AI(t) + B, \quad t \in [0,1].$$

Since $u(0) = \alpha u(\eta)$, $u(1) = \beta u(\eta)$, we get

$$B = -\alpha \int_0^\eta (I(\eta) - I(s))y(s)ds + \alpha AI(\eta) + \alpha B,$$

$$-\int_0^1 (I(1) - I(s))y(s)ds + I(1)A + B = -\beta \int_0^\eta (I(\eta) - I(s))y(s)ds + \beta I(\eta)A + \beta B.$$

Solving A and B from the above equations and substituting them into (7), we get

$$\begin{aligned} u(t) &= -\int_0^t (I(t) - I(s))y(s)ds - \frac{(\beta - \alpha)I(t)}{M} \int_0^\eta (I(\eta) - I(s))y(s)ds \\ &\quad + \frac{(1 - \alpha)I(t)}{M} \int_0^1 (I(1) - I(s))y(s)ds + \frac{\alpha I(\eta)}{M} \int_0^1 (I(1) - I(s))y(s)ds \\ &\quad - \frac{\alpha I(1)}{M} \int_0^\eta (I(\eta) - I(s))y(s)ds = \int_0^1 G(t, s)y(s)ds. \end{aligned}$$

It is easy to prove that $G(t, s) \geq 0$ for all $(t, s) \in [0,1] \times [0,1]$.

We illustrate our main results as follows.

THEOREM 1. Assume $(A_1) - (A_4)$ hold, $M > 0$. A , B and μ are defined in (4) and (5) respectively. If λ satisfies

$$(8) \quad \frac{1}{\mu B f_\infty} < \lambda < \frac{1}{A f_0},$$

then BVP (1) - (2) has at least one positive solution.

If f is superlinear, i.e. $f_0 = 0$, $f_\infty = +\infty$, then Theorem 1 can be applied. If f is sublinear, i.e. $f_0 = +\infty$, $f_\infty = 0$, then the following theorem is needed.

THEOREM 2. Assume $(A_1) - (A_4)$ hold, $M > 0$. A , B and μ are defined in (4) and (5) respectively. If λ satisfies

$$(9) \quad \frac{1}{\mu B f_0} < \lambda < \frac{1}{A f_\infty}$$

then BVP (1) - (2) has at least one positive solution.

THEOREM 3. Assume $(A_1) - (A_4)$ hold, $M > 0$. A , B and μ are defined in (4) and (5) respectively. Furthermore, if

$$(10) \quad \liminf_{x \rightarrow 0} \frac{f(x)}{x} > \frac{1}{\mu \lambda B}, \quad \liminf_{x \rightarrow +\infty} \frac{f(x)}{x} > \frac{1}{\mu B \lambda},$$

then, if λ satisfies

$$(11) \quad \lambda \leq \frac{1}{A} \sup_{s>0} \frac{s}{\max_{x \in [\mu s, s]} f(x)},$$

then BVP (1) - (2) has at least two positive solutions.

THEOREM 4. Assume $(A_1) - (A_4)$ hold, $M > 0$. A , B and μ were defined in (4) and (5) respectively. Furthermore, if

$$(12) \quad \limsup_{x \rightarrow 0} \frac{f(x)}{x} < \frac{1}{\lambda A}, \quad \limsup_{x \rightarrow +\infty} \frac{f(x)}{x} > \frac{1}{A \lambda},$$

and if λ satisfies

$$(13) \quad \lambda \geq \frac{1}{\mu B} \inf_{s>0} \frac{s}{\min_{x \in [\mu s, s]} f(x)},$$

then BVP (1) - (2) has at least two positive solutions.

THEOREM 5. Assume $(A_1) - (A_4)$ hold, $M > 0$. A , B and μ were defined in (4) and (5) respectively. If

$$(14) \quad f(x) > \frac{x}{\lambda\mu A}, \quad \text{for } x \in (0, +\infty),$$

then BVP (1) – (2) has no positive solution.

THEOREM 6. Assume $(A_1) - (A_4)$ hold, $M > 0$. A , B and μ were defined in (4) and (5) respectively. If

$$(15) \quad f(x) < \frac{x}{\lambda B}, \quad \text{for } x \in (0, +\infty),$$

then BVP (1) – (2) has no positive solution.

THEOREM 7. Assume $(A_1) - (A_4)$ hold, $M < 0$, then BVP (1) – (2) has no positive solution.

We also need the following lemmas.

LEMMA 2. If $y \in C([0, 1], [0, +\infty))$, $M > 0$, $u(t)$ is the unique solution of (6), then

$$\frac{u(t) - u(0)}{I(t) - I(0)} \leq \frac{u(\eta) - u(0)}{I(\eta) - I(0)}, \quad t \in [\eta, 1]; \quad \frac{u(t) - u(0)}{I(t) - I(0)} \geq \frac{u(\eta) - u(0)}{I(\eta) - I(0)}, \quad t \in [0, \eta],$$

and

$$\frac{u(t) - u(1)}{I(t) - I(1)} \leq \frac{u(\eta) - u(1)}{I(\eta) - I(1)}, \quad t \in [\eta, 1]; \quad \frac{u(t) - u(1)}{I(t) - I(1)} \geq \frac{u(\eta) - u(1)}{I(\eta) - I(1)}, \quad t \in [0, \eta].$$

PROOF. The proofs is simple and hence omitted.

LEMMA 3. Let $y \in C([0, 1], [0, +\infty))$, $M > 0$, $\delta \in (0, \frac{1}{2})$, $u(t)$ be the unique solution of (6). Suppose $u(r) = \|u\| = \max_{t \in [0, 1]} |u(t)|t$. We have

$$(16) \quad u(t) \geq \begin{cases} \frac{I(t)}{I(1)} \|u\|, & 0 \leq t \leq r, \\ \left(1 - \frac{I(t)}{I(1)}\right) \|u\|, & r \leq t \leq 1, \end{cases}$$

when $ar \in (0, 1)$,

$$(17) \quad u(t) \geq \frac{I(t)}{I(1)} \|u\|, \quad t \in [0, 1],$$

when $r=1$ and

$$(18) \quad u(t) \geq \left(1 - \frac{I(t)}{I(1)}\right) \|u\|, \quad t \in [0, 1],$$

when $r=0$. Furthermore, it holds that

$$(19) \quad u(t) \geq \mu \|u\|, \quad t \in [\delta, 1-\delta],$$

where

$$\mu = \min \left\{ \frac{I(\delta)}{I(1)}, 1 - \frac{I(1-\delta)}{I(1)} \right\}.$$

PROOF. By Lemma 2, if $0 < r < 1$ and $0 < t \leq r$, then

$$(20) \quad u(t) \geq \frac{I(t)}{I(1)} u(r) + \left(1 - \frac{I(t)}{I(1)}\right) u(0) \geq \frac{I(t)}{I(1)} \|u\|.$$

Similarly, if $0 < r < 1$, $0 \leq t \leq r$, we get

$$(21) \quad u(t) \geq \left(1 - \frac{I(t)}{I(1)}\right) \|u\| + \frac{I(t)}{I(1)} u(1) \geq \left(1 - \frac{I(t)}{I(1)}\right) \|u\|.$$

Hence (20) and (21) imply that (16) is valid. The proofs of (17), (18) and (19) are similar and hence omitted.

LEMMA 4. Let X be a Banach space, and K be a cone of X . Assume Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1, \bar{\Omega}_1 \subseteq \Omega_2$. And let $A: K \cap (\bar{\Omega}_2 - \Omega_1) \rightarrow K$ be a completely continuous operator such that

$$(i) \quad \|Ax\| \leq \|x\| \text{ for every } x \in K \cap \partial\Omega_1$$

and

$$\|Ax\| \geq \|x\| \text{ for every } x \in K \cap \partial\Omega_2$$

or

$$(ii) \quad \|Ax\| \geq \|x\| \text{ for every } x \in K \cap \partial\Omega_1$$

and

$$\|Ax\| \leq \|x\| \text{ for every } x \in K \cap \partial\Omega_2.$$

Then A has at least one fixed point in $A: K \cap (\bar{\Omega}_2 - \Omega_1)$.

3. PROOF OF THEOREMS

In this section, we prove the theorem 1-7 which are presented in section 2. Define the classical Banach space $X = C[0,1]$ endowed with the norm $\|u\| = \max_{t \in [0,1]} |u(t)|$. Define cone P in X by

$$P = \{x \in X : (p(t)u'(t))' \leq 0, \text{ and } u(t) \geq 0 \text{ for } t \in [0,1], \\ u(0) = \alpha u(\eta), \quad u(1) = \beta u(\eta)\},$$

and the integral operator $T : P \rightarrow X$ by

$$Tu(t) = \lambda \int_0^1 G(t,s) f(u(s)) ds, \quad t \in [0,1].$$

It is easy to prove that T is completely continuous and $TP \subseteq P$.

PROOF OF THEOREM 1. We now define Ω_1 and prove that for $u \in P \cap \partial\Omega_1$, $\|Tu\| \leq \|u\|$. By inequality (8), there exists a $\varepsilon > 0$ such that

$$(22) \quad \frac{1}{B\mu(f_\infty - \varepsilon)} \leq \lambda \leq \frac{1}{A(f_0 + \varepsilon)}.$$

For this fixed ε , there exists a $D_1 > 0$ such that for $u \in P$ and $0 \leq s \leq 1$, satisfying $0 < u(s) < D_1$, $f(u(s)) \leq (f_0 + \varepsilon)u(s)$. Take

$$\Omega_1 = \{u \in X : \|u\| < D_1\}.$$

Let $u \in P \cap \partial\Omega_1$, then using (22), we have

$$\begin{aligned} \|Tu\| &= \max_{t \in [0,1]} \lambda \int_0^1 G(t,s) a(s) f(u(s)) ds \leq \\ &\leq \lambda \max_{t \in [0,1]} \int_0^1 G(t,s) a(s) (f_0 + \varepsilon) u(s) ds \leq \\ &\leq \lambda \max_{t \in [0,1]} \int_0^1 G(t,s) a(s) (f_0 + \varepsilon) \|u\| ds \leq \|u\|. \end{aligned}$$

Next, we define Ω_2 and show that for $u \in P \cap \partial\Omega_2$, $\|Tu\| \geq \|u\|$. By the definition of f_∞ , there exists a $\bar{D}_2 > 0$ such that $f(u(s)) \geq (f_\infty - \varepsilon)u(s)$ for $u \in X$ and $u(s) \geq \bar{D}_2$, $0 \leq s \leq 1$. Take $D_2 = \max\{\bar{D}_2, 2D_1\}$ and set

$$\Omega_2 = \left\{ x \in X : \|u\| < \frac{D_2}{\mu} \right\}.$$

If $u \in P \cap \partial\Omega_2$, by Lemma 3, we get $u(t) \geq \mu \|u\| = D_2 \geq \bar{D}_2$ for $t \in [\delta, 1 - \delta]$. Hence using (22) one has

$$\begin{aligned} \|Tu\| &= \max_{t \in [0,1]} \lambda \int_0^1 G(t,s) a(s) f(u(s)) ds \geq \\ &\geq \lambda \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) f(u(s)) ds \geq \\ &\geq \lambda \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) (f_{\infty} - \varepsilon) u(s) ds \geq \\ &\geq \lambda (f_{\infty} - \varepsilon) \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) \mu \|u\| ds \geq \|u\|. \end{aligned}$$

By Lemma 4, it follows that T has a fixed point $u \in PI(\bar{\Omega}_2 - \Omega_1)$ and so u is a positive solution of (1) - (2).

PROOF OF THEOREM 2. It is similar to that of Theorem 1 and hence omitted.

PROOF OF THEOREM 3. Define function $q(s)$ by

$$q(s) = \frac{s}{\max_{x \in [\mu s, s]} f(x)}, \quad s \in (0, +\infty),$$

we see that $q(s)$ is continuous in $(0, +\infty)$. By (10) together with $q(s) \leq \frac{s}{f(s)}$, we have

$$\limsup_{s \rightarrow 0} q(s) < \mu \lambda B, \quad \limsup_{s \rightarrow +\infty} q(s) < \mu \lambda B.$$

By (11), $\lambda < \frac{1}{A} \sup_{s>0} q(s)$, $A > \mu B$. Then there exists $0 < s_1 < s_2$ such that $q(s_i) = \lambda A$ for $i=1,2$, and thus

$$\max_{x \in [\mu s_i, s_i]} f(x) = \frac{s_i}{\lambda A}, \quad i=1,2,$$

i.e.,

$$f(x) \leq \frac{s_i}{\lambda A}, \quad x \in [\mu s_i, s_i] \quad i=1,2.$$

Take

$$\Omega_i^0 = \{u \in X : \|u\| < s_i\}, \quad i=1,2,$$

if $u_i \in PI \partial\Omega_i^0$, then

$$\|Tu_i\| \leq \lambda \max_{t \in [0,1]} \int_0^1 G(t,s) a(s) \frac{s_i}{\lambda A} ds = s_i = \|u_i\|, \quad i=1,2.$$

On the other hand, by (10), there exist $0 < H_1 < \mu s_1 < s_1 < s_2 < \mu H_2 < H_2$ such that

$$f(u(s)) > \frac{u(s)}{\mu \lambda B}, \quad 0 < u(s) \leq H_1, \quad \delta \leq s \leq 1 - \delta,$$

$$f(u(s)) > \frac{u(s)}{\mu \lambda B}, \quad u(s) > \mu H_2, \quad \delta \leq s \leq 1 - \delta.$$

Take

$$\Omega_i = \{u \in H : \|u\| < H_i\}, \quad i=1,2,$$

if $u_i \in PI \partial\Omega_i$, then

$$\begin{aligned} \|Tu\| &= \max_{t \in [0,1]} \lambda \int_0^1 G(t,s) a(s) f(u_i(s)) ds \geq \\ &\geq \lambda \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) f(u_i(s)) ds \geq \\ &\geq \lambda \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) \frac{u_i(s)}{\mu \lambda B} ds \geq \\ &\geq \frac{1}{\mu B} \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) \mu \|u_i\| ds \geq \|u_i\|. \end{aligned}$$

Hence it follows from Lemma 4, it follows that T has two fixed point $u_1 \in PI(\bar{\Omega}_1^0 - \Omega_1)$, $u_2 \in PI(\bar{\Omega}_2 - \Omega_2^0)$ and so u_1, u_2 are two positive solution of (1) - (2).

PROOF OF THEOREM 4. It is similar to that of Theorem 3 and omitted.

PROOF OF THEOREM 5. To the contrary assume that $u(t)$ is a positive solution of BVP (1) - (2), then

$$\begin{aligned} \|u\| = \|Tu\| &= \max_{t \in [0,1]} \lambda \int_0^1 G(t,s) a(s) f(u(s)) ds \geq \\ &\geq \lambda \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t,s) a(s) f(u(s)) ds > \end{aligned}$$

$$\begin{aligned}
&> \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t, s) a(s) \frac{u(s)}{\mu A} ds \geq \\
&\geq \max_{t \in [\delta, 1-\delta]} \int_{\delta}^{1-\delta} G(t, s) a(s) \frac{\mu \|u\|}{A \mu} ds \geq \|u\|,
\end{aligned}$$

which is a contradiction.

PROOF OF THEOREM 6. It is similar to that of Theorem 5 and omitted.

PROOF OF THEOREM 7. To the contrary assume that $u(t)$ is a positive solution of BVP (1) – (2), since $M < 0$, then $G(t, s) \leq 0$, thus

$$u(t) = \int_0^1 G(t, s) a(s) f(u(s)) ds \leq 0,$$

which is a contradiction.

Now, we give an example to illustrate our results as follows

EXAMPLE. Consider the following three-point BVP

$$(23) \quad \begin{cases} u''(t) + \lambda a(t) f(u(t)) = 0, & t \in (0, 1), \\ u(0) = u(\frac{1}{2}), & u(1) = \frac{1}{2} u(\frac{1}{2}), \end{cases}$$

with $\lambda > 0$, $\eta = \frac{1}{2}$, $\alpha = 1$, $\beta = \frac{1}{2}$ and $a(t) \equiv 1$, $f(u) = u \frac{400u+100}{u+2}$. It is easy to see that

$$f_{\infty} = 400, \quad f_0 = 50$$

$$G(t, s) = \begin{cases} \frac{3}{4}s - \frac{1}{2}st, & 0 \leq s \leq t < \eta < 1 \text{ or } 0 \leq s \leq \eta \leq t \leq 1, \\ \frac{1}{2}s + \frac{1}{4}t - \frac{1}{2}st, & 0 \leq t \leq s \leq \eta < 1, \\ \frac{1}{2} - \frac{1}{4}s - \frac{1}{4}t, & 0 \leq \eta \leq s \leq t \leq 1, \\ \frac{1}{2} - \frac{1}{2}s, & 0 \leq \eta \leq t \leq s \leq 1 \text{ or } 0 \leq t < \eta \leq s \leq 1. \end{cases}$$

Choosing $\delta = \frac{1}{4}$, we get

$$\mu = \frac{1}{4},$$

$$A = \max_{t \in [0, 1]} \int_0^1 G(t, s) ds = \max_{t \in [0, 1]} \left\{ \frac{1}{4}t^2 + \frac{19}{16}t + \frac{3}{32} \right\} = \frac{49}{32},$$

$$B = \max_{t \in [\frac{1}{4}, \frac{3}{4}]} \int_{\frac{1}{4}}^{\frac{3}{4}} G(t, s) ds = \max_{t \in [\frac{1}{4}, \frac{3}{4}]} \left\{ \frac{7}{128} + \frac{13}{64}t + \frac{3}{4}t^2 \right\} = \frac{25}{32}.$$

By Theorem 1, if $\frac{16}{1250} < \lambda < \frac{16}{1225}$, then BVP (23) has at least one positive solution.

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